COMMERCIAL MATHEMATICS

[With A Short Collection of Commercial Tables]

BY

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WITH A FOREWORD
BY

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FOREWORD

Figures have their own language, which to the initated offer more valuable information than may otherwise be obtained. It is, therefore, unfortunate that the art of calculation has so long been left out of mathematical courses of study, although a small place has of late been given to it in some commercial curricula under the designation Commercial Arithmetic. It is a pleasant surprise now to be told that Commercial Mathematics, demands fuller attention in the training of aspirants in business, and offers adequate materials for a full two years' course in the Intermediate stage. Alertness in calculation, courage to grapple with arrays of figures, ability to read the language of figures, familiarity with the presentation of facts and tendencies with the help of graphs, facility in the use of Tables of Logarithms, Exchange etc.—these should surely be an asset to all desirous of following commercial careers.

The absence of a reliable book on the subject may also have been partly responsible for the neglect with which it has been looked upon in the past. The present book, it is hoped, will go a long way towards removing the disabilities of the subject. The inclusion of Commercial Mathematics as a full subject will surely improve a commercial curriculum of the Intermediate Standard.

3, Government Place, West
Calcutta, 1st July, 1937

N. K. MAJUMDER

INTRODUCTION

The computers' art is at the hub of all transactions for profit. And yet, training in this useful and practical art finds no place in any scheme of education. The range covered in this treatise will show that Commercial Mathematics may very well be taken as a full two years' course in the Intermediate stage. While such a subject will be of considerable practical value, its educative value will be in no way inferior to that of the usual subjects of study at this stage.

Commercial calculations have to be swift and accurate, the degree of accuracy being determined by the practical value of an answer. Theoretical accuracy alone—the result of arithmetical operations without heed to requirements—leads to results like $3\frac{3913}{4536}$ pies, which are not only useless but wasteful of time and energy. Methods have been indicated herein, without setting a taboo on algebraical methods as such—many for the first time to the knowledge of the author—that will eliminate waste and increase speed. Emphasis has been laid for obvious reasons on methods that are specially adapted to Indian conditions.

The subject is best approached in reference to business operations on their arithmetical side. A familiarity with business methods and routine, adequate to a correct apprehension of business problems, is to be engendered in the student along with a mastery over the language of figures. Portions of the book that may be considered too specialised may be omitted ; on the other hand, the teacher may use his discretion in discussing involved theories leading to useful practical formulae. The author does not think it ordinarily necessary for the general student to master the theories of finance, or thoretical considerations of the comparative advantages of different methods of charging overheads in costing etc., beyond what has been givén in the following pages, for efficient practical work. The growing use of Graphics, and increasing attention paid to Business Statistics have necessitated the inclusion of three elementary Sections at the end which should be generally sufficient, and prove useful as a grounding to those desrious of pursuing the subject further.

The methods of Mental Arithmetic have been discussed in some detail. They should be extensively practised, and freely used to curtail written work. The answers to Examples for oral work have been given at the end, other answers being noted against the sums for facility of reference.

Practice in the making and use of Tables should be encouraged and it is suggested that Tables should be allowed to be used even in examinations ; for nothing is more effective in attaining speed and accuracy. The Tables appended are in no way exhaustive, but they comprise most of the tables in general use in their useful ranges.

The book is the outcome of long years of teaching the subject with an avowedly practical bias. It is hoped that the book, besides being useful to students of commerce, should prove a handy book of reference in business houses, financial institutions and to professional accountants. The author however, will thankfully receive suggestions for making it more useful.

R. GUPTA.

Government Commercial Institute
Calcutta, 1st July, 1937

PREFACE TO THE SECOND EDITION

The book has been throughly revised, with an eye to making it more useful both as a text book and as a book of reference. The Section on Income Tax had to be entirely rewritten, owing to the change over to the Slab System. Certain portions dealing with special topics have been expunged, being of little interest to the general reader. More difficult portions have been marked with an asterisk, and may be passed over at first reading.

A Table of Reciprocals and a Table of Antilogorithms have been added for quicker calculation. A number of useful algebraic formulae, and certain mathematical constants have been tabulated for accurate calculation. A new Decimal Table for Rupee-Sterling Exchange has also been added.

R. GUPTA.

Government Commercial Institute
Calcutta, 28th September, 1941

PREFACE TO THE THIRD EDITION

The book has again gone through extensive revision. A new Section has been added on Probabilities and Assurances. Considerable changes have been made at different places to bring the work in line with the syllabuses of different Universities and Education Boards, who have recommended it for class work. A representative selection from questions set at their examinations have also been incorporated in response to requests from nemerous quarters. Some useful changes have also been made in the Tables.

The exigencies of War economy, however, have curbed the growth of this book. More elementary portions have been cut out and practice in short methods and oral work has to be offered separately in the complementary volume—Rapid Calculations. This small book may be profitably used by those who do not care to be thwarted by apparently complicated calculations, and aspire for speed and accuracy in computation.

R. GUPTA.

Government Commercial Institute
Calcutta, 11th July 1945

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A SHORT COLLECTION OF COMMERCIAL TABLES

Special Symbols :

$\rightarrow$	Corresponds to, proportional to, etc.
$\rightrightarrows$	Nearly equal to, approaches, etc.
$\equiv$	Equivalent to.
—	A bar below a figure indicates the number of parts into which the *immediately preceding* value is aliquotised. In analogy $\underline{2}\%$ or $\underline{2}‰$ indicates 2% and 2‰ respectively of the immediately preceding value (*Vide* p. 67).
+, −	These signs after a result indicate that the real value is just above or below that given.

Some Useful Series :

1. *Arithmetical Progression* : $a, (a+d), (a+2d), (a+3d), \ldots$
   n^{th} term $= a + (n-1).\,d.$

Sum of n terms $= n \times$ mean of first and last terms

$$= \frac{n}{2}\Big[2a + (n-1)d\Big].$$

Middle term (n *odd*) is $\frac{n+1}{2}$th term.

Natural Numbers ; $1, 2, 3, 4, \cdots\cdots$

Sum of n terms $= \frac{n(n+1)}{2}.$

2. *Geometrical Progression* : $a, ar, ar^2, ar^3, \cdots\cdots$

   n^{th} term $= a.r^{n-1}$. Sum to n terms $= a.\frac{r^n - 1}{r - 1}.$

   If r be a proper fraction, sum to infinity $= \frac{a}{1-r}.$

3. Series : $1^2, 2^2, 3^2, 4^2, \cdots\cdots$

   Sum of n terms $= \frac{n(n+1)(2n+1)}{6}.$

4. **Series :** $1.2 + 2.3 + 3.4 + 4.5 + \cdots$

Sum of n terms $= \frac{n}{3}(n+1)(n+2)$.

5. Expansion of $(1+x)^n = 1 + nx + \frac{n(n-1)}{2.1}x^2$

$$+ \frac{n(n-1)(n-2)}{3.2.1}.x^3 + \frac{n(n-1)(n-2)(n-3)}{4.3.2.1}.x^4 + \ldots\ldots$$

Also, $(1+x)^{-n} = \frac{1}{(1+x)^n} = 1 - nx + \frac{n(n+1)}{2.1}.x^2 - \frac{n(n+1)(n+2)}{3.2.1}.x^4$

$$+ \frac{n(n+1)(n+2)(n+3)}{4.3.2.1}.x^4 - \ldots\ldots$$

Mathematical Constants :

$\sqrt{2} = 1{\cdot}41421$;. $\sqrt{3} = 1{\cdot}73205$; $\sqrt{5} = 2{\cdot}23607$

$\pi = 3{\cdot}141592654 = 3\frac{1}{7} - {\cdot}\underline{4}‰ = \frac{355}{113} - \frac{1}{10000}$ of $\underline{9}‰$.

$\frac{1}{\pi} = {\cdot}318310 = \frac{1}{4} + \underline{4} + \underline{11} + \underline{40}$; $\sqrt{\pi} = 1{\cdot}772454 = 1 + \underline{2} + \underline{2} + \underline{9}\%$

Diam. of circle to area $1 = \sqrt{\frac{4}{\pi}} = 1{\cdot}1283792$

$= 1 + \underline{8} + 2\% + \underline{3} + \underline{5}\%$.

Diam. of sphere to volume $1 = \sqrt[3]{\frac{6}{\pi}} = 1{\cdot}2407 = 1 + \underline{5} + \underline{5} + \underline{2}\%$.

$\log \pi = 0{\cdot}4971499$; $\log\left(\frac{1}{\pi}\right) = \bar{1}{\cdot}5028501$.

COMMERCIAL MATHEMATICS

SECTION ONE

SOME GENERAL PRINCIPLES

Commercial Mathematics deals with problems arising in course of the daily routine of business. Most of these problems are essentially those of calculation; but a reading of such problems would require more than a passing acquaintance with the methods and machinery of business. It is therefore necessary that the subject be studied against a background of business operations with an eye to the requirements of the counting house.

The chief objective would of course be to gain quickness in calculations without sacrificing accuracy. In business houses the degree of accuracy is determined by custom, the limits being ordinarily indicated by the lowest units current. Wherever this limit has been crossed in the following pages, the student should convince himself of the necessity therefor, and the sufficiency of the result. In freight calculations, for example, it is the practice to work correct to the nearest anna; but in obtaning the rate of dividend in a bankruptcy problem the rate may have to be computed to 4 or 5 decimal places after the pie, depending on the value of the claims on which it has to be calculated.

The question naturally arises as to the admissibility of ignoring errors. Whenever the unit has been selected—that would depend on usage or other requirements, say, $\frac{1}{32}$nd of a penny in the London-Calcutta rate of exchange—errors less than half the unit may be safely ignored in the results. A quick apprehension of the effect of ignoring errors in different stages of calculation will be of valuable assistance in ensuring accuracy of short methods of calculation. It should be easy to see that in converting *tons* and *cwts* into *maunds*, it should be sufficient to carry on to *seers* as the *cwt* is a much larger unit than a *seer*, and the accuracy of smaller units would be fictitious.

As calculations are to be made for business purposes, it would be waste of time and energy to try to obtain unpractical values, if shorter methods are available for practical answers. The student is referred to the complementary volume on Rapid Calculations for a fuller discussion of numerous simple methods of shortening work, while throughout the following pages he would come across the use of easy methods which he must assiduously cultivate.

A more generous use of Tables of Logarithms, decimalised values and the contracted methods also require to be cultivated. These would very greatly reduce routine work, and, used intelligently, ensure any degree of accuracy.

There should also be little hesitation in making use of algebraic symbols and the methods of elementary algebra and trigonometry for tackling problems, if that would lead most easily to their solution. Certain special methods have been indicated in the following pages, which should also provide a powerful machinery for the purpose.

§ **Algebraic Symbols & Formulæ.** A formula is a simple single statement in general terms of a whole series of particular facts, expressed in terms of algebraic symbols.

A retailer marks up his wares R% above cost. Representing list prices by L and cost prices by C, the relationship is expressed as $L = C \cdot \frac{100+R}{100}$. This formula may now be used to represent the whole series of prices entered in his catalogue ; and also to work out the list price corresponding to any given cost price. At the same time, it would enable the determination of cost prices when the list prices are known.

The interpretation of symbolical expressions should be noted in every case. If W is the weight in lb. of feed water per hour and P the electric power in kilowatts given out at an electric station, $\frac{W}{P}$ would indicate the quantity of feed water per hour for production of unit power ; whereas $\frac{P}{W}$ would indicate the rate of production of power corresponding to every lb. of feed water supplied.

§ It becomes necessary at times to make out algebraical statements of data and obtain desired values by solving equations or by elimination. In evaluation from formulae, attention should be given to the units of the various items, cancelling *same* units only from the numerator and the denominator, and also indicating the proper unit of the result obtained.

Illustration. Determine the stress f in a boiler plate from the formula $f=\frac{pd}{2t}$, where the pressure $p=160$ lb. per sq. in., the diameter of the boiler $d=8$ ft. and the thickness of the boiler plate $t=\cdot 63$ in.

Separating out numerals and units $f=\frac{160\times 8}{2\times \cdot 63}\cdots\frac{lb.}{(in.)^2}\times\frac{ft.}{in.}$

To evaluate this formula, the *ft.* in the numerator should be multiplied by 12 for conversion into inches, and the *lb.* divided by 2240 for conversion into tons.

Thus, $f=\frac{160\times 8\times 12}{2\times \cdot 63\times 2240}\times\frac{ton\times in.}{(in.)^2\times in.}=5\cdot 44$ tons per sq. in.

§ **Equations.** An equation expresses a relationship of equality involving variables and constants. It should be remembered that the number of equations required for the complete determination of the unknowns is the same as the number of unknowns. Following are certain elementary rules.

1. Transposition of *terms.* If *terms* are transposed from one side of an equation to the other, they change their signs. Thus, if $3x+46=64$, $3x=64-46$; whence, $3x=18$ and $x=6$.

2. Transposition of *factors.* When a *factor* is moved from one side of an equation to the other it changes its position, the numerator in one case becomes the denominator in the other, and *vice versa.* Thus, if $cx=ay$, $x=\frac{ay}{c}$ and $y=\frac{cx}{a}$.

3. Equations of second and higher orders may be solved by factorization. Thus, if $(x-a)(x-b)(x-c)=0$; each of the factors $x-a$, $x-b$ and $x-c$ may be equal to 0. $\therefore\ x=a, b$ or c.

4. The direct way of finding comparative values of 3 unknowns from two equations is by the method of cross multiplication. Thus, if,

$$\left.\begin{array}{r} ax+by+cz=0 \\ \text{and }\ a'x+b'y+c'z=0 \end{array}\right\} \frac{x}{bc'-b'c}=\frac{-y}{ac'-a'c}=\frac{z}{ab'-a'b}.$$

5. Where indices are involved, the procedure is as follows.

$$\text{If} \qquad x^n=a, \quad x=\sqrt[n]{a} \quad \text{or} \quad (a)^{\frac{1}{n}}.$$

6. The roots of the quadratic equation $ax^2+bx+c=0$ are given by

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

§ The First Four Rules. In performing the ordinary arithmetical operations, the student should develop an alertness in detecting scope for shortening labour in all stages. Thus, (i) to multiply a number by 125, imagine 3 zeros at the end and divide by 8 ; (ii) to find the cost of 243 articles @ 3¾d each, remember that the cost of 240 = £3¾ and add the cost of 3, and so on and so forth. Again, to subtract the sum of a number of items from a single item, perform the subtraction at the end of addition of every column—ticking off for a complete unit of the next column, and while borrowing therefrom for enabling the subtraction. In multiplication, it would be useful to start from the left to the right in the multiplier (as shown in the next section), and in division to adopt the Italian Method ; and in all cases to carry on work in the *standard form*.

Revision Exercises

1. The asterisks in the following multiplication sum represent figures. Replace them by the correct figures. [*Ans.* 2635

```
      5 3 7 8
      2 * 3 *
    ---------
    2 6 * * *
  * * * * *
* * * * *
* * * * *
-----------
*4 1 7 1 0 * *
```

implify :— $\frac{1 \cdot 5}{\cdot 075} \times \frac{3\frac{1}{4}}{1\frac{1}{2}} + \frac{1 \cdot 875}{2 \cdot 1} \times \frac{3 \cdot 5}{3 \cdot 75} - \cdot 1\dot{6}$ [*Ans.* 44

3. Work by the shortest method :—

(a) Given 15% of an amount is £27 10s 6d. How much would be 16½% thereof ? [*Ans.* £30 5s 6½d

(b) $3 \cdot 142 \, (7 \cdot 86^2 - 5 \cdot 93^2)$ correct to 5 significant figures. [*Ans.* 83·623

(c) ·8689 × 99·5 —P.S.C. [*Ans.* 86·45555

4. A water pipe L yards long, d inches in diameter, with one end H ft. higher than the other carries $\sqrt{(3d)^5 \times H + L}$ gallons of water per minute. Evaluate, when the pipe is a mile long, and has a diameter 4½ inches, while one end is 38 ft. higher than the other. [*Ans.* 98·42 gallons a min.

5. A yard stick is ⅛-inch too short. Find the error in the measurement of a piece of cloth measuring 29½ yards by it. [*Ans.* Error 1 ft. 2⅝ in. too long.

6. A warehouse-keeper uses a stone (14 lb.), a lb. too heavy, when entering potatoes in his stock ; and uses another stone, a lb. too light, when selling. If a dealer puts in 182 stones of potatoes with the warehouse-keeper, according to the latter's weights, how much does the latter clear by charging a commission of 3 pies per stone, and selling out the entire stock at the dealer's price of 7as per stone ? [*Ans.* Rs 15 1a 6p

7. In a long division sum the dividend is 529565, and the first two remainders (before taking down the next figure from

the dividend) are 246 and 222, and the third and final remainder is 542. Find the divisor and the quotient. —I. Com.

[*Ans.* 561 ; 943

8. The area of a field is 30479·805 sq. ft. Its length is twice its breadth. Find its length. —I. Com. [*Ans.* 246·9 ft.

9. Determine the smallest quantity that must be added to the following to make the sum the square of a whole number.

$$1\frac{21}{64} \times \frac{19\frac{1}{2} + 1\frac{3}{10}}{\frac{5}{8} \text{ of } \left(1\frac{1}{4} - \frac{7}{11}\right)} + \frac{0\cdot7\dot{8} \text{ of } 3\cdot\dot{5}7142\dot{8}}{3\cdot\dot{8}4615\dot{3}}$$

—I. Com. [*Ans.* 36

10. An educational body requires 60 book prizes at 5s each, 40 at 4s 6d each, 45 at 4s each, and 70 at 3s each. One bookseller offers the 5s prizes at 4s, the 4s 6d at 3s 10d, the 4s at 3s 3d and the 3s at 2s 6d. Another offers a uniform discount of 20%. Which offer is cheaper, and what difference would it make upon the entire cost of the prizes required ?

—I. Com. [*Ans.* Second offer cheaper by 18s 7d

11. A dealer sets aside 1453lb. of tea to be distributed in at least 10800 sample packets. (*a*) How much tea could be put into each packet to the nearest ⅛-oz. ? (b) If tea is to be put in these packets to the largest possible ⅛-oz. and the remaining stock is put in small packets of ⅛-oz. each, how many such smaller packets can be distributed ? [*Ans.* 2⅛oz. ; 3296

12. A carpenter was engaged for a number of days for Rs 15 15as ; but having absented himself he was paid only Rs 8 7as. Show that his daily wages could not exceed 15as.

13. Find the square root of $\frac{54\cdot6121}{45\cdot4276}$ [*Ans.* 1·096 +

14. A square plot of land measuring 182¼ sq. yd. is halved by drawing a diagonal across. The owner of a half-share subsequently acquires adjoining lands to make his plot a square on the diagonal. Find the area of this acquired plot.

[*Ans.* 273⅜ sq. yd.

15. A voluntary fund is raised on the basis that every person present is to contribute as many annas in the rupee as he has

rupees in his pocket. 15 persons have Rs 12 each; 18 have Rs 10 each. How much has each of the remaining 20 persons, if the amount thus raised is Rs 327 8as? [*Ans.* Rs 8

16. A woman buys some eggs at 3 an anna, and also the same number at 5 an anna. She sells them at 3 pies each. Does she gain or lose, and how much %? —S. C. [*Ans.* Loss 6¼%

17. A tradesman sells two articles together for Rs 46 making 10% on one and 20% on the other. If he had sold each article at 15% profit, the result would have been the same. At what price did he sell each article?—S.C. [*Ans.* Rs 22; Rs 24

18. The capital cost of a small waterworks is Rs 50,000. It can pump 1000 gallons of water an hour and it works 16 hours a day. Running costs amount to Rs 140 a month and there is an annual charge of Rs 320 for water-testing etc. The plant depreciates in value by 8% p. a. Calculate the actual cost of supplying 100 gallons of water a day to a consumer for one month. If a charge of Rs 5 is made for this quantity, find the interest earned by the capital.—B.C.S. [*Ans.* Rs 3 2as; 7·2% p.a.

19. The top of the new Howrah Bridge will be 350 ft. above ground level, and it will be visible from ground level at a distance of 23 miles in a direct line from the top. Find the radius of curvature of the earth, assuming it to be a sphere. Give your answer to the nearest 100 miles.—B. C. S. [*Ans.* 4000 miles

20. Two persons going to the same place had 8 md. of luggage between them and were charged excess for the luggage at Rs 8 and Rs 4 respectively. Had all the luggage belonged to one person he would have been charged Rs 14 for excess. Find how much luggage is allowed free, and how much each had. —B. C. S. [*Ans.* 1 md.; 5 md.; 3 md.

21. A straight railway track crosses a straight high road at a place X. There are two stations on the railway at distances of 3 miles and 12 miles respectively from X and on the same side of it. A man wants a site for a farm which should be equally distant from the two stations and the high road. Show how the

site may be determined by a geometrical construction. Point out two possiblities. —B. C. S.

22. The correct measurements of a rectangle are 3·2 yards by 2·4 yards. A boy told to find its area entered the measurements as 3·2 metres and 2·4 metres. Find his error in square yards and also the percentage error, given 1 metre = 39·37 in. —I. Com. [*Ans.* 20⅝% app.

23. A man travelled 60 miles in 3 hours partly by train and partly by car. If he had gone all the way by rail he would have arrived 1 hour sooner and saved ⅖-ths of the time he was in the car. How far did he travel by car ?—I. Com. [*Ans.* 45 miles

24. Two men and 5 boys can do half of a piece of work in 5 days, and 3 men and 4 boys can do ⅓-rd of it in 3 days. How many days will 9 men take to do it all ?—I. Com. [*Ans.* 5 days

25. A bicycle was sold at a loss of 40% on the cost price; a second one was bought with the proceeds plus 5 guineas paid ; this also was sold at a loss of 35% ; a third one was bought with the proceeds plus £5 15s 3d. The third bicycle cost £16. What was the cost of the first one ?—I. Com. [*Ans.* £17 10s

26. Two workmen are engaged on a piece of work for which they are to receive Rs 28. One workman works for 3 days of 9 hours each, and the other 3 days of 8 hours ; but the second does as much in 2 hours as the first does in 3 hours. How should the amount be divided between the two workmen ? —I. Com. [*Ans.* Rs 12 and Rs 16

27. There are two one-pint decanters, containing liquor and water in the ratios 11 : 4 and 10 : 5 respectively. The contents of the two decanters are poured into a two-pint decanter and the mixture is sold at Re 1 a pint, thereby realizing a profit of 2as per pint. Find the cost price of a pint of the liquor.—I. Com. [*Ans* Re1 4as

28. Reduce $\frac{5{\cdot}1183}{0{\cdot}0141}$ of $22{\cdot}\dot{2}$ of $0{\cdot}0\dot{9}$ of $0{\cdot}2\dot{8}\dot{4}$ to a vulgar fraction in its lowest terms. —I. Com. [*Ans.* 189$\frac{4}{7}$

29. A rectangular grass plot the length of whose sides are in the ratio 5 to 6 costs £17 15s $6\frac{2}{3}$d for turfing at the rate of 5d per sq. yard. Find the length of its sides. —I. Com.
[*Ans*. $26\frac{2}{3}$ yd. ; 32 yd.

30. A manufacturer sold an article to a merchant thereby gaining $7\frac{1}{2}$%. The latter sold it to a retailer thereby gaining $12\frac{1}{2}$%, and the retailer sold it to a customer therby gaining 22%, these percentages being based on the cost price in each case. Find how much per cent the customer's price was greater than the cost price of the manufacturer. If the customer's price be a guinea what is the actual cost price ?
—I. Com. [*Ans*. $47\frac{1}{2}$% app. ; $14\frac{2}{7}$ sh.

31. A tennis court is 26 yd. long and 12 yd. wide. A man owns a rectangular field 126 yd. long and 95 yd. wide and decides to mark out as many courts as possible, the netting being 6yd. from the base line and 3yd. from the side lines. If he lets all the courts at Rs 123 per season each, what would be the rental of the field per acre ?
—I. Com. [*Ans*. Max. no. of courts 15 ; Rs 746 app.

32. A man spent 20% of his money, then £50, and then 15% of the remainder. If he had £1698 6s left, what was his original money ? —I. Com. [*Ans*. £2560

33. *A* sold an article to *B*, thereby gaining 13·7% on his outlay. *B* sold it to *C* and gained 7·4% on his outlay. *C* sold it to *D*, who paid a sum which was 28·3% greater than its cost to *A*. What percentage profit on his outlay did *C* gain ?
—I. Com. [*Ans*. 5% app.

34. A man in business loses in the first year 5% of his capital ; but in the second year he gains 6% of what he had at the end of the first year ; and his capital is now Rs 350 more than at the commencement. Find his original capital.
—I. Com. [*Ans*. Rs 50,000

35. A triangular field has its sides of length 111yd. 2 ft. ; 119 yd. 4 inches, and 134 yd. respectively. A farmer wishes to have a wire fence round the field, and supports are to be fixed at equal distances apart all round the field. What is the greatest distance apart of the supports in order that one should be fixed at each corner ? Also, how many supports would there be and what would be their cost at Rs 2 11as 9p each ?

—I. Com. [*Ans.* $7\frac{4}{9}$ yd ; 49 ; Rs 133 15as 9p

36. A dealer sold 3 pianos for Rs 300, Rs 450 and Rs 500 respectively, thereby gaining 5% on the total selling price. He lost 4% by the sale of the first, and gained $7\frac{1}{2}$% by selling the second, the percentages being based on selling prices. What per cent gain did he obtain by selling the third ?

—I. Com. [*Ans.* 8·15%

37. Two stations A and B are 150 miles apart. A goods train leaves A at 8-20 A. M. and travels towards B at $17\frac{1}{2}$ miles per hour. An express train leaves B at 9-40 A. M. on the same day and travels towards A at $57\frac{1}{2}$ miles per hour. When and where will they meet ? —I. Com.

[*Ans.* At 11 hr. 21 m. 20 sec. A.M. at $52\frac{8}{9}$ miles from A.

38. *A* bought the following lots of goods :—

311 tons @ Rs 113 4as p. ton ; 128 tons @ Rs 127 9as p. ton 73 tons @ Rs 128 11as p. ton. Find at what rate the entire stock should be sold to enable *A* to earn 10% profit.

—I. Com. [*Ans.* Rs 130 14as 11p nearly p. ton.

39. Find the square root of 557·196025—I.Com. [*Ans.* 23·605

40. Simplify :—$1\frac{1\frac{3}{8}+\frac{1}{2}}{\frac{4}{5}-\frac{1}{5}}+9\frac{32}{35}-\frac{5+4\times\frac{3}{4}}{5\times4+\frac{3}{4}}$

-I. Com. [*Ans.* $13\frac{5321}{8448}$

SECTION TWO

APPROXIMATION AND CONTRACTED METHODS

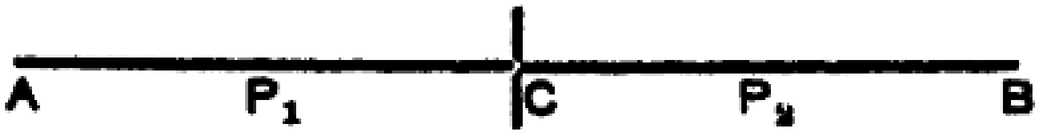
§ **Approximation.** Arithmetical calculations, in certain respects, represent some of the most accurate operations of the human mind. At the same time, they frequently lead to results which have an appearance of accuracy which they do not possess, being vitiated either by errors of observation of the data or by over-reaching the limits of practicability. At least in the business world, theory is not allowed to preponderate over the requirements of practice, and an answer of the type 4s 7$\frac{8}{9}\frac{3}{7}\frac{1}{2}$d is evidently unwanted, although it might have been arrived at after close computation. The simple reason is, that one does not receive nor make payment of this amount. What is wanted is an approximate quantity, which is also real and tangible.

Even in the most accurate scientific calculation, errors of observation, mechanical inaccuracies and many other considerations demand an expression of approximation within certain limits. In our every day work, also, we are constantly making approximations, consciously and unconsciously, in connection with quantitative ideas.

These approximations, again, are subject to assessment, and in the following pages will be discussed the common arithmetical methods thereof.

Quantities are compared when referred to selected units, but these units may vary between very wide limits. While the physicist in his laboratory worries over a difference of $\frac{1}{1000}$th part of an inch, the astro-physicist ignores distances of millions of miles; and while the joiner is exact to the nearest 32-nd part of an inch, the furniture dealer sells his wares measuring to the nearest $\frac{1}{2}$-foot only.

In theory, then, every one of us is making mistakes, big and small, in our daily calculations; and we are all prepared to condone some of them, varying in size according to the unit of reference for the time being.



A point P moving from A to B is nearer to A than to B, so long as it is to the left of the middle position at C. As soon, however, it passes C, it is nearer B than A.

Thus, within the range 1 to 2, say, from A to B, any quantity like 1·3, which is to the left of the mid-point at 1·5, is nearer 1 than 2. For the same reason 1·8 is nearer the upper limit 2 than 1. While the moving point is still to the left of C, being very close to it, say, at 1·49999, it is still nearer the lower limit 1. On the other hand, as soon as it reaches beyond C, by even a very narrow margin, say at 1·500001, it is nearer the upper than the lower limit.

Just at C, the mistake of taking 1·5 to be approximately equal to the lower limit 1 or to the upper limit 2 is the same, being half the range itself. It is, however, the practice to take it as being approximately equal to the upper limit, as in most cases 5 may be followed by other digits.

In every approximation there is presumed to be an error; and in a good approximation this error should not exceed half the value of the unit of reference; the maximum error, occurring just at the mid-position, being equal in value to half the unit.

If, for example, the unit of reference be ·01, that is a hundredth, the number 32·374 lies between 32·37 and 32·38, and is approximately equal to (⇒) 32·37. While, 32·376, lying between the same limits should be approximated to 32·38 correct to two decimal places as in the last case.

Hence the uuual rule is to observe *the digit immediately following the place under approximation* in a number in the decimal system of notation. If this is less than 5, it is ignored in the approximation; and if it is 5 or more, it is cancelled to increase the figure under approximation by 1.

The unit of reference is a matter of choice, and will depend on the requirements of a particular problem. In business calculations, as has already been noted, the unit is usually the lowest value of current measures. For example, it is generally useless to take into consideration a unit smaller than a farthing in Great Britain, or the pie in India ; and it may frequently serve the necessary purpose to make computations correct to the nearest penny or to the nearest 3-pie (pice). In India, the present tendency is to make calculations correct to the nearest anna. Banks and Government Revenue Departments still make certain calculations to the nearest pie, although the pie has long disappeared from currency.

When, however, the value to be obtained is a rate, it will be necessary to keep in view the likely multipliers to ascertain the degree af accuracy of the result. Thus a Bankruptcy dividend may have to be obtained correct to 4 or 5 decimal places after the pie, if necessary.

§ The first few figures of a number, not being zeros, are known as *significant figures*. Thus 8,531,234 ⇛ 85×10^5 correct to 2 significant figures ; and ·00023658 ⇛ ·00024 also correct to 2 significant figures. Correct to the nearest million the first number would be approximately equal to 9 millions.

§ To obtain the value of say, 4$\frac{21}{68}$d to the nearest farthing, it should be observed that numerators 17, 34 and 51 in the fractional portion respectively stand for $\frac{1}{4}$d, $\frac{1}{2}$d and $\frac{3}{4}$d. The given numerator 21 lies between 17 and 34 and and is nearer to 17, and the amount is equal to 4$\frac{1}{4}$d to the nearest farthing. To express, say 1s 4·6184...d to the nearest 32-nd of a penny, the decimal portion ·6184 is multiplied by 32 to the nearest integer to obtain 20 ; and the amount ⇛ 1s 4$\frac{20}{32}$d *i.e.*, 1s 4$\frac{5}{8}$d to the nearest 32-nd of a penny. $ 4·8631 is similarly equal to $ 4·86$\frac{20}{64}$ correct to the nearest 64-th of a cent.

§ **Standard Form.** A multiplication or division is said to be in the *standard form* when the first figure of the multiplier or the divisor is in the unit place. Since, A × B is equal to

$(A \times 10^n) \times \left(\frac{B}{10^n}\right)$ for all values of n, the product 41583·67483 × ·0046423 = 41·58367483 × 4·6423 in the standard form. And since $\frac{A}{B} = \frac{A \times 10^n}{B \times 10^n}$, the quotient of ·083642 ÷ ·0009024 = 836·42 ÷ 9·024 in the standard form. Hence the rule is to move the decimal point in the multiplier and multiplicand through the same number of places in opposite directions to obtain the standard form; and to move it in the same direction through the same number of places in the divisor and the dividend for the same purpose.

§ **Contracted Methods.** When products and quotients are required to a desired degree of accuracy, the actual work of multiplication or division may be contracted to avoid unnecessary work.

§ *Multiplication.* 1. Multiply 84026·38642 by ·00618436 correct to 2 decimal places.

Reduce to the standard form and arrange figures according to their place values.

3 decimal places in the multiplicand are retained in the work, the third place being necessary for approximating the result to 2 decimal places. Places beyond the third are cancelled in the multiplicand.

$$
\begin{array}{r}
\cancel{8}\cancel{4}\cdot\cancel{0}\cancel{2}\cancel{6}\cancel{3}\cancel{8}\cancel{6}\cancel{4}\cancel{2} \\
\underline{6\cdot18436} \\
504\cdot158 \\
8\cdot403 \\
6\cdot722 \\
\cdot336 \\
\cdot025 \\
\underline{\cdot005} \\
519\cdot64\cancel{9}
\end{array}
$$

∴ Product = $\underline{519\cdot65}$ app.

The multiplication commences from the left to the right, the product figures of the first multiplication by 6 being arranged immediately under the corresponding figures of the multiplicand.

At every succeeding step, a figure to the left in the multiplicand is cancelled out and the product figures are arranged to end with the first product line.

At every stage, however, great care should be taken to ascertain the correct figure that may be carried from the cancelled figures under the rules of approximation. Thus, in the first line $6 \times \cancel{5}\cancel{8}\ldots$ the cancelled digits would give 2 to carry.

In the end the last figure 9 of the product is cancelled out, altering the second decimal place from 4 to 5 for approximation.

2. Multiply 8436·586414 by 402·643 correct to 2 sig. figures.

Reduce the work to the standard form; and retain 2 significant figures in the multiplicand to obtain 3 sig. figures in the product, the extra place being taken for approximation. Proceeding, as before, the first 2 sig. figures of the product are obtained to be 34. It is now estimated that if the multiplication were completed by ordinary methods, the number of integral places in the product would be 7. Of these 7, the first two have been obtained to be 34, and the final product is seen to be approxmiately equal to 34×10^5, where the zeros are put in to indicate the place values of the figures obtained.

$$\begin{array}{r} \cancel{8}\cancel{4}\cancel{3}\cancel{6}\cancel{5}\cancel{8}\cdot\cancel{6}\cancel{4}\cancel{1}\cancel{4} \\ 4\cdot02643 \\ \hline 337 \\ 2 \\ \hline 33\cancel{9} \end{array}$$

i.e. $\underline{34 \times 10^5}$ app.

3. Multiply 4983·642 by 712·6142 correct to the nearest thousand.

Reduce the work to the standard form. Express the multiplicand in thousands. Retain only 1 decimal place in the work for approxmiation.

Proceed as before, noting that every product line is now in thousands.

$$498364\cdot2 \times 7\cdot126142$$

$$\begin{array}{rl} \cancel{4}\cancel{9}\cancel{8}\cdot\cancel{3}\cancel{6}\cancel{4}\cancel{2} & \text{thousands} \\ 7\cdot126142 & \\ \hline 3488\cdot5 & \text{thousands} \\ 49\cdot8 & \text{"} \\ 10\cdot0 & \text{"} \\ 3\cdot3 & \text{"} \\ \hline 3551\cdot3 & \text{"} \end{array}$$

i.e. $\underline{3551}$ thousands app.

§ *Division*. 1. Divide 58642·8142 by 923·6182 correct to 1 decimal place.

Reduce to the standard form. Retain in the dividend 1 place of decimal in excess of requirement for approximation, cancelling out the rest. Note that not more than 4 figures are necessary in the divisor for the first division. Cancel the remaining figures of the divisor. Arrange the

$$\begin{array}{r|l} & 63\cdot49 \\ 9\cdot2\cancel{3}\cancel{6}\cancel{1}\cancel{8}\cancel{2} & 586\cdot42\cancel{8}\cancel{1}\cancel{4}\cancel{2} \\ & \underline{55417} \\ & 3225 \\ & \underline{2771} \\ & 454 \\ & \underline{369} \\ & 85 \\ & \underline{83} \end{array}$$

Quotient $= \underline{63\cdot5}$ app.

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figures of the quotient on the top of the dividend, placing the decimal point immediately above the point in the latter.

The first figure of the quotient is obtained by trial to be 6 (being 58 ÷ 9), which is placed on the top of 8 which is the last figure required for the trial division. At each succeeding step of the division cancel a figure to the left in the divisor, instead of bringing down a figure from the dividend.

The actual work of division may be contracted further by using the Italian method, that is by omitting to write down the product line at each stage. This is illustrated in the following.

2. Divide 40264·6142 by ·0021463 correct to the nearest million.

Reduce to the standard form, and then express the dividend in millions, the required unit. Retain 1 decimal place in the dividend for approximation, and proceed as before.

$$\begin{array}{r|l}
 & 18\cdot\cancel{8} \quad \text{millions} \\
2\cdot\cancel{1}\cancel{4}\cancel{6}\cancel{3} & \underline{40\cdot2}\cancel{6}\cancel{4}\ldots\text{millions} \\
 & \underline{\;18\,8\;} \\
 & \;\;1\,7
\end{array}$$

Quotient = 19 millions app.

3. Divide ·0084641 by 342·643 correct to 2 sig. figures.

$$\begin{array}{r|l}
 & \cdot000024\cancel{7} \\
3\cdot\cancel{4}\cancel{2}\cancel{6}\cancel{4}\cancel{3} & \underline{\cdot0000846}\cancel{4}\cancel{1} \\
 & \qquad\;\;\underline{161} \\
 & \qquad\;\;\;\;24
\end{array}$$

Quotient = ·000025 app.

4. Estimate a rough value of :—

$$\frac{486\cdot3846 \times \cdot0483 \times \cdot1864}{\cdot00863 \times 92\cdot48 \times 97\cdot25}$$

Reducing to the standard form, the expression

$$= \frac{4\cdot863846 \times 4\cdot83 \times 1\cdot864}{8\cdot63 \times 9\cdot248 \times 9\cdot725}, \text{ which is } \textit{roughly}$$

$$= \frac{5 \times 5 \times 2}{9 \times 9 \times 10} = \cdot06.$$

Note. The rough value may also be obtained by collecting place values of the first significant figure of each component. Thus, the expression is roughly $= \frac{5\times5\times2}{9\times9\times10} \times \frac{10^2\times10^{-2}\times10^{-1}}{10^{-3}\times10\times10} = \frac{5\times5\times2}{9\times9\times10} = \cdot06$ app.

The working of the second half may also be abbreviated by the omission of the 10's. Thus, $\frac{2-2-1}{-8+1+1}=\frac{-1}{-1}$ which cancels out.

To obtain the result, however, correct to a specified number of decimal places (say 4), it is noticed that 4 decimal places in the answer will mean 3 significant figures. The work will therefore require 5 significant figures in the numerator and 4 or 5 significant figures in the denominator, as necessary.

Given exp. $=\frac{43{\cdot}795}{776{\cdot}2}=\frac{{\cdot}43795}{7{\cdot}762}$ in the standard form

$={\cdot}05642={\cdot}0564$ correct to 4 dec. places.

Note. It is advisable in such casess to use Log Tables.

§ **Mixed Fractions.** It is sometimes useful to abbreviate the writing of decimal fractions by the use of vulgar fractions that would, when decimalised, give the substituted figures in the decimal number. Thus ˙38125 may be written to be $={\cdot}38\frac{1}{8}$, the faaction $\frac{1}{8}$ reproducing the figures 125, after ˙38, when decimalised. Similarly, ${\cdot}560625={\cdot}56\frac{1}{16}$; ${\cdot}138\dot{3}={\cdot}138\frac{1}{3}$ etc. It will be noticed that this notation satisfies the conditions of ordinary arthmetical operations. Thus, ${\cdot}038\frac{1}{4}+{\cdot}052\frac{1}{4}={\cdot}090\frac{1}{2}$; ${\cdot}63\frac{1}{8}-{\cdot}53\frac{3}{8}={\cdot}09\frac{3}{4}$; ${\cdot}58\frac{1}{4}\times 4=2{\cdot}33$ ($4\times\frac{1}{4}=1$, which is carried and added to $4\times 58=232$, and the resulting number duly pointed). A multiplier like ${\cdot}25\frac{1}{4}={\cdot}25+\frac{1}{4}$ of ${\cdot}01={\cdot}25+\frac{1}{4}\%$, may be used by obtaining $\frac{1}{4}(={\cdot}25)$ of the multiplicand and taking 1% of the product and adding. Thus, $23{\cdot}468\times{\cdot}25\frac{1}{4}=\frac{1}{4}$ of $23{\cdot}468$ *i.e.* $5{\cdot}867+1\%$ of $5{\cdot}867=5{\cdot}867+{\cdot}05867$ $5{\cdot}92567$. Also, by the ordinary rules of approximation $3{\cdot}25\frac{3}{8}\Rightarrow 3{\cdot}25$, and $3{\cdot}25\frac{7}{8}\Rightarrow 3{\cdot}26$ correct to two decimal places. In an expression like \$ $4{\cdot}86\frac{17}{32}$, however, the fractional portion should be interpreted as $\frac{17}{32}$-nd part of a cent.

§ **Orders of Smallness.** Of two small quantities one may be comparatively very much greater than the other. Thus ˙005, itself a small quantity, is 1000 times ˙000005. These are then said to be of different orders of smallness. If, for example, $n\times 10^{-2}$, n lying between 1 and 9, is of the first order of smallness, $n\times(10^{-2})^2$ is said to be of the second order of smallness, and $n\times(10^{-2})^3$ of the third order etc. etc. It is evident that in approximations small quantities of higher orders disappear in the presence of one of a lower order.

We thus have, x and y being of the same order of smallness,

$\frac{1}{1\pm x} = 1 \mp x + x^2 \mp x^3 + x^4 \ldots \Rightarrow 1 \mp x$, neglecting $x^2, x^3 \ldots$

$(1+x)(1+y) = 1 + x + y + xy \Rightarrow 1 + x + y$, neglecting xy.

$\frac{1+x}{1+y} \Rightarrow 1 + x - y$, neglecting $xy, xy^2 \ldots$; and

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2} \cdot x^2 + \ldots \Rightarrow 1 + nx$,

neglecting $(nx)^2$, $(nx)^3$......assuming that nx is a small quantity.

$(1+x)^{n_1} \times (1+y)^{n_2} \Rightarrow 1 + n_1 x + n_2 y$, assuming that $n_1 x$ and $n_2 y$ are of the same order of smallness.

Illustrations

(i) $\frac{1}{1{\cdot}005} \Rightarrow 1 - {\cdot}005 = {\cdot}995$, neglecting $({\cdot}005)^2$ etc.

(ii) $\frac{1{\cdot}005}{1{\cdot}007} \Rightarrow 1 + {\cdot}005 - {\cdot}007 = {\cdot}998$, " " "

(iii) $\sqrt[3]{10004} = 10(1 + {\cdot}0004)^{\frac{1}{3}} \Rightarrow 10(1 + \frac{1}{3} \times {\cdot}0004) = 10{\cdot}001 +$

(iv) $\frac{3{\cdot}08 \times 4018}{64{\cdot}79 \times 2934{\cdot}2} = \frac{3 \times 4000}{60 \times 3000} \times \frac{(1 + 2{\cdot}67\%)(1 + {\cdot}45\%)}{(1 + 7{\cdot}98\%)(1 - 2{\cdot}19\%)}$

$\Rightarrow \frac{1}{15}[1 + 2{\cdot}67\% + {\cdot}45\% - 7{\cdot}98\% + 2{\cdot}19\%]$

$= \frac{1}{15}[1 - 2{\cdot}71\%] = \frac{1}{15} \times {\cdot}9729 = {\cdot}065$ app.

Note. When the ultimate multiplier is large, the final result would be rough ; with small multipliers, it is usually fairly accurate. While ordinarily such simplifications should be done with Log Tables, it would be easy to check up results by the above method.

§ **Square Root.** The method of finding the square root of a number is best explained in reference to the illustration below.

To find the square root of 4568·34861 correct to 2 decimal places.

Method. Put dots on the top of the unit place and on the top of every alternate digit, right and left, introducing a zero at the end. Obtain by trial 6 the square root of 45, ending with 5 under the first dot; put it down on the right of the line. Bring down 36, the square of 6, under 45. Subtract and bring down 68, ending with the figure under the next dot. To the left of the left-hand line, bring down *twice* 6, the first figure to the right. Obtain by trial the quotient 7 of 968 divided by 12, putting it down both after 12 and after 6. Multiply 127 by 7 and subtract from 968. That gives 79, after which bring down 38, the next pair of figures. To the left put down *twice* the figures so far obtained on the right i.e. 2 × 67 = 134. Again treat 134 as a trial divisor, obtaining 5 as the quotient figure. Put this 5 down both after 134 and 67 (after a decimal point). Proceed in this manner till the square root is obtained on the extreme right, 67'589...which ⇒ 67'59 correct to 2 decimal places.

$$\begin{array}{r|l}
 & 4\dot{5}6\dot{8}{\cdot}\dot{3}8\dot{6}4\dot{1}\dot{0} \,|\, 67{\cdot}589\ldots \\
 & 36 \\ \hline
127 & 968 \\
 & 889 \\ \hline
1345 & \quad 7938 \\
 & \quad 6725 \\ \hline
13508 & \quad 121364 \\
 & \quad 108064 \\ \hline
135169 & \quad\quad 1330010 \\
 & \quad\quad 1216521 \\ \hline
 & \quad\quad\quad 113489
\end{array}$$

After some practice, the product lines may be omitted as in the Italian method of division.

§ It may be noted that after obtaining a square root up to n sig. figures, $(n-1)$ following figures may be obtained by contracted division after the n-th stage. Continuing the work in the above illustration, the following further figures are obtained.

$$\begin{array}{r|r}
 & 8396 \\
13\cancel{5}\cancel{1}\cancel{6}\cancel{9} & \underline{113489} \\
 & \underline{5354} \\
 & \underline{1299} \\
 & \underline{83} \\
 & 2
\end{array}$$

The square root up to 7 dec. places would be = 67'5898396 app.

EXAMPLES 1

Obtain values to the degree of accuracy noted against each—

1. (i) 6316·516 × ·08164...2 dec. places. (ii) ·00164185 × ·3164...4 dec. places. (iii) 4164·001426 × 61·386...nearest thousand. (iv) 63846·3164316 × 4014·64316...3 sig. fig. (v) 34643·64864 × 4164·36416...nearest million.

[*Ans.* 515·68 ; ·0005 ; 256 × 10^3 ; 256 × 10^6 ; 144 millions

2. (i) ·0643126431 + ·00016438...3 dec. places

(ii) 46·38642684 + 8364·1641...4 dec. places

(iii) 34683·16416 + ·000031413861...nearest million

(iv) ·0064316431 + 84·313641...3 sig. fig.

(v) 46346·31416 + ·000163142...3 sig. fig.

[*Ans.* 391·244 ; ·0055 ; 1104 × 10^6 ; ·0000763 ; 284 × 10^6

3. Simplify—

(i) $\dfrac{368·4361 \times ·006143}{4384·612 \times ·8391}$ ... 4 dec. places [*Ans.* ·0006

(ii) $\dfrac{303·64042 \times 48·36213}{8964216·316}$ ... 4 dec. places [*Ans.* ·0016

4. Transform the following from decimal fractions to mixed fractions and *vice versa*—

(i) ·30625 : ·81375 ; ·002$\dot{6}$; ·15$\dot{1}$4285$\dot{7}$ (ii) ·16$\frac{3}{8}$; ·012$\frac{7}{13}$; ·103$\frac{5}{16}$; ·01$\frac{1}{6}$

[*Ans.* (i) ·3$\frac{1}{16}$; ·81$\frac{3}{8}$; ·002$\frac{2}{3}$; ·15$\frac{1}{7}$ (ii) ·16375 ; ·012$\dot{5}$3846$\dot{1}$; ·1033125 ; ·01$\dot{6}$

5. Extract the square root of the following to the degree of accuracy noted, as required—

268324 ; ·01375929 ; 4·210704 ; ·009921 (4 dec. places) ; 6443·2729 ; 2704·416$\frac{1}{64}$ (3 dec. places)

[*Ans.* 518 ; ·1173 ; 2·052 ; ·0996 ; 80·27 ; 52·004

6. Obtain approximate values—

$573{\cdot}643 \div 1{\cdot}002$; $64{\cdot}6894 \times {\cdot}9997$; ${\cdot}04386 \times 1{\cdot}0008$; $52{\cdot}681 \times 600{\cdot}03$ [*Ans.* 572·50 ; 64·67 ; ·04387 ; 31610

7. Write down approximate values of—

$(1{\cdot}0002)^7$; $36{\cdot}086 \times (1{\cdot}0004)^4$; $23{\cdot}68 \times \dfrac{{\cdot}9992}{1{\cdot}0004}$;

$86{\cdot}24 \times 1{\cdot}003 \times {\cdot}998$ [*Ans.* 1·0014 ; 36·662 ; 23·65 ; 86·326

8. Find the value of—

$\sqrt{3080}$, given $\sqrt{3025} = 55$ [*Ans.* 55·495

5, given $\sqrt[3]{1331} = 11$ [*Ans.* 11·011

$$\left[\textit{Hint.}\quad (1331+4)^{\frac{1}{3}} = (1331)^{\frac{1}{3}}\left[1 + \frac{4}{1331}\right]^{\frac{1}{3}} \approx 11\left[1 + \frac{4}{3 \times 1331}\right] = 11 \times 1{\cdot}001\right]$$

9. Calculate the following values to the number of decimal places noted against each—

2% of $63{\cdot}47\frac{3}{8}$ (2 places) ; 4% of $354{\cdot}10\frac{1}{8}$ (1 place) ; 3% of $41{\cdot}6345\frac{1}{8}$ (2 places) ; $1\frac{1}{2}$% of $56{\cdot}342\frac{1}{8}$ (nearest integer) ; $2\frac{3}{8}$% of $98{\cdot}64\frac{3}{8}$ (nearest integer). [*Ans.* 1·27 ; 14·2 ; 1·25 ; 1 ; 2

10. Find as accurately as possible the weight of a cubical piece of metal the edge of which is 3yd 2ft 3·1 in. long while a cubic foot of the metal weighs 8·8764 cwt. —I. Com.

[*Ans.* 633 ton 6 cwt 3 qr.

11. Standard gold contains $\frac{22}{24}$ fine gold and $\frac{2}{24}$ alloy. 20 lb troy of standard gold are coined into 934 Sovereigns and 1 half-Sovereign. Calculate to 5 decimal places the standard weight of a Sovereign in grains. Also, find the value of 1 oz troy standard gold and 1 oz troy of pure gold, neglecting the cost of the alloy. —I. Com. [*Ans.* 123·27447 gr. ; £3 17s 9d ; £ 4 4s 10d

12. A ship whose average speed was 23·43 knots took $125\frac{1}{4}$ hours on a certain journey. Find to the nearest mile the length of the journey given that 1 knot is equal to 1·15 miles per hour. If another ship took 148 hours on a journey 95 miles longer, what was its average speed in knots correct to 2 decimal places ? —I. Com. [*Ans.* 3375 miles ; 20·39 knots

13. If the population of a town increases every year by 1·8% of the population at the beginning of that year, in how many years will the total increase of population be 30% ? —I. Com.
[*Ans.* 15 years +

[*Hint.* Solve $(1+\cdot 018)^n - 1 = \frac{3}{10}$]

14. Simplify :—

(i) $\dfrac{2}{\sqrt{3}+\sqrt{2}}$; (ii) $\dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}+\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

[*Ans.* ·6357 app. ; 8

SECTION THREE

DECIMALIZATION OF MONEY, WEIGHTS & MEASURES

§ **The Metric System.** The English and Indian systems of money, weights and measures, involve complicated calculations; because the various subdivisions of the different units are unequal fractions. With a view to attaining expedition in calculation, French scientists introduced the idea of sub-dividing units into 10, 100, 1000 etc. equal parts, and adopted the uniform use of prefixes to denote such multiples and sub-multiples. The French unit of length is a *metre*, whence the word *metric* has been derived. Thus:

(i) The unit of length is 1 m. = 39·370113 inches app.

10 m = 1 *Deca*-metre (Dm)   1 metre = 10 *deci*-metres (dm)
10 Dm = 1 *Hecto*-metre (Hm)   1 dm = 10 *centi*-metres (cm)
10 Hm = 1 *Kilo*-metre (Km)   1 cm = 10 *milli*-metres (mm)
10 Km = 1 *Myria*-metre

Myria means ten thousand; *Kilo* means a thousand; *Hecto* means a hundred; *Deca* means a ten. Also, *deci* means a tenth; *centi* means a hundredth; *milli* means a thousandth etc.

These same prefixes are also used in respect of other measures.

(ii) The unit of weight is 1 gramme (gm.)

(iii) The unit of volume is 1 litre = 1000 c.cms.

(iv) The unit of area is 1 Are = 100 sq. metres.

(v) The unit of money is 1 franc which is divided into 100 centimes.

§ The unit of length in trade is the metre; but where long distances are to be measured, the Kilometre is used. In measuring areas, the usual unit is the Are (= 1 Sq. Decametre) or the Hectare (= 100 Ares). The unit of volume used in trade is the litre or the Kilolitre (= 1000 litres); although the Stere (= 1 cu. metre) is sometimes used for measuring solidity. The unit of weight for purposes of retail trade is the Kilo (Kilogramme), heavier weights being measured in terms of Quintals (= 100 Kilos) or Tonneaux (= 1000 Kilos) or the *metric ton*.

§ In the metric system, the unit of volume, capacity and weight are inter-related. While 1 gramme is the weight of 1 cu. cm of pure water at 4°C, 1 litre of such water weighs 1 Kilo. The metric ton is really the weight of 1 cubic metre of water.

The specific gravity of a substance is the weight in grammes of 1 cu. cm. of it.

§ Following is a list of rough equivalents of certain metric units. More accurate values are given in the Tables appended.

1 metre = $1\frac{1}{11}$ yd. ; 1 Km = 5 furlongs ; 1 cm = $\frac{2}{5}$ in.

10 Decares = 1 Hectare = $2\frac{1}{2}$ acres ; 1 sq. Km = $\frac{3}{8}$ sq. mile

1 litre = $1\frac{7}{8}$ pints ; 1 Hectolitre = 22 gallons

1 Quintal = 2 cwt ; 1 tonneau = 1 ton ; 1 Kilo = $2\frac{1}{5}$ lb. (Av.)

Also, 1 acre = 40 Ares ; 1 mile = 16 Km ; 1 gall. = $4\frac{1}{2}$ litres ;

1 cwt = $50\frac{4}{5}$ Kilos ; 1 ton = 1016 Kilos ; 1 cu. ft. = $28\frac{1}{3}$ litres

§ The metric system is in vogue extensively over the Continent, and in Central and South America. The unit of money is different in different countries, but it is almost the universal method (except in U. S. A. and British possessions) to divide all units of money and measures into submultiples of 10 and powers of 10.

The Dollar in U. S. A. and Canada is divided into 100 cents, the ReichsMark in Germany is divided into 100 pfenniges, the Yen in Japan into 100 sens, and even in Ceylon the Rupee is divided into 100 cents.

The effect of all these is to simplify calculations and accounting, the unit being directly adapted to the decimal system of notation.

The equivalents of some foreign units are given in Tables in the Appendix.

Illustrations

1. 1 Hm 3 Dm 2 m 5 dm 8 cm may be directly expressed as 1·3258 Hm, or 13·258 Dm, or 132·58 m, or 1325·8 dm, or 13258 cm, or 132580 mm, and so on.

2. Find the cost of 183·7581 Kg @ 15·25 fr. per Kg.

The multiplication is contracted to give the result correct to two decimal places, that is correct to the lowest extant currency, the centime.

$$\begin{array}{r} 1837\cdot\cancel{5}\cancel{8}\cancel{1} \\ 1\cdot525 \\ \hline 1837\cdot581 \\ 918\cdot791 \\ 36\cdot751 \\ 9\cdot188 \\ \hline 2802\cdot311 \text{ fr} \\ \textit{i.e. } \underline{2802\cdot31} \text{ fr} \end{array}$$

EXAMPLES 2

1. Find the value of a plot of land measuring 518·7568 Ares at the rate of 614·30 francs per Are. [*Ans.* 318672·30 fr

2. The cost of 18678·25 Kg is 49650·00 fr. Find the cost per Kg. [*Ans.* 2·66 fr

3. Find the weight of mercury (Sp. gr. 13·59 gms. per c.cm.) contained in a glass cistern 19·2 cm by 2·5 cm to a depth of 1·25 cm. [*Ans.* 815·40 gm.

4. Find the weight of water contained in a cistern 13·43 m by 5·68 m by 2·50 m. [*Ans.* 190706 Kgm

5. A square sheet of metal 18·3 cm by 18·3 cm is cut into 4 equal squares. If the weight of 1 sq. cm. of the sheet is 5·2 gm, find the weight of one such piece. [*Ans.* 435·36 gm

6. A metal plate 4·6 cm by 2·5 cm of thickness ·15 cm is beaten into another plate of thickness ·05 cm. Find the surface covered by the lamina. [*Ans.* 34·5 sq. cm

7. A tin sheet 32 cm by 10 cm is made into a cubical box without a lid. If 1 c. cm of water weighs 1 gm, how much water can the box contain ? [*Ans.* 512 gm

8. Water is poured into a glass cistern 16 cm by 12 cm by 12 cm, so that it is only half-filled. It is then poured into a glass tube with a cross section of 10 sq. cm. What is the height of the column of water in the tube ? [*Ans.* 1·152 m

9. A metal rod 15 cm long, weighing 483·5 gm is drawn into a thin wire with a section $\frac{1}{100}$th that of the rod. What is the weight of 1 cm of the wire ? [*Ans.* $322{\cdot}\dot{3}$ milligrammes

10. Marble weighs 2·6 gm per c. cm. A marble slab 15·21 m × 6·31 m × 5 cm cracks into two pieces. The weight of a piece is 5910 Kg. Determine the ratio of the superficial area of the two pieces. [*Ans.* 9 : 10 app.

11. Taking 1 litre of gasoline to weigh 1·59 lb and 1 gallon of gasoline to weigh 6 lb, find the depth of a cistern 18 m by ·6 m capable of containing 4770 gallons of gasoline. [*Ans.* 1·67 m

12. A sheet of lead in the shape of a rectangle 18·3 cm by 15·2 cm and 4·2 cm. thick is melted to cover a surface 1·40 m by 7·6 cm. Find the average depth of the lead, and the load per sq. cm, if the specific specific gravity of lead is 11·3.

[*Ans.* 1·1 cm. : 12·43 gm.

13. A piece of timber is measured to be of a rectangular section 40·4 cm by 29·5 cm. If it has a length of 6·50 m, find the largest number of planks 3·25 m by 40·4 cm by ·5 cm that can be obtained therefrom. If the weight of the wood is ·75 gm per c. cm find the weight of each plank. [*Ans.* 118 ; 4·924 Kgm

§ **Decimalization of Money, Weights and Measures.** It becomes frequently necessary to express different units in terms of one of them and decimals. Thus, Rs 8 5as 9p may be expressed as Rs 8·359375. The following rules enable such decimalization directly.

§ **£ s d.** It is observed that 1s = £·05 and 1 farthing = $£{\cdot}001041\dot{6} = £{\cdot}001\frac{1}{24} \rightrightarrows £{\cdot}001$. Hence, the following rule for expressing shillings and pence as the decimal of £1.

1. Reduce pence and farthings to farthings.

2. Write down the unit figure of the farthings in the third decimal place, carrying any remaining figure to be added to the shillings multiplied by 5.

3. Increase the number of farthings by 1, when the number of pence is 6 or more.

This gives the result to three decimal places.

4. Write down the figures, obtained by dividing the number of pence by 6, from the 4th decimal place onward. Where the number of pence is 6 or more, reduce it by 6, before dividing by 6.

Illustrations

$7^s\ 6^d$	$11^s\ 3\frac{1}{2}^d$	$12^s 9\frac{3}{4}^d$
	→	→
£·375	= £·56458$\dot{3}$	= £·640625
←	←	←

§ To reduce decimals of £1 into shillings, pence and farthings, the following rule is to be followed.

1. Divide the first 2 decimal places by 5, the quotient will be shillings.

2. Read the remainder with the next following figure, reduced by 4%, as farthings.

Illustrations

1. £·87346... = 17s 5½d	87 ÷ 5 gives 17 shillings. The remainder 2 followed by 3 *i. e.* 23 reduced by 4% *i. e.* by 1, gives 22 farthings or 5½d.
2. £·06894... = 1s 4¼d	6 ÷ 5 gives 1s. 18 reduced by 4 % *i. e.* by 1, gives 17 farthings or 4¼d.

Note that the first three decimal places are sufficient to give the answer to the nearest farthing. In actual work with decimalized values, therefore, it is sufficient to obtain results correct to 4 decimal places.

In case greater accuracy is required, as in the case of rates of dividend in bankruptcy, any degree of correctness may be obtained by a slight extension of the rule. Thus, to obtain the value of £·658631 correct to the hundredth part of a penny, the number of shillings 12 is obtained by dividing 64 by 5. 48·631 reduced by 4% *i. e.* by 1·945 app. ⇒ 46·686*q* ⇒ 11·67d

§ **Rs as p.** Since, 1*a* = Re·06$\frac{1}{4}$ and 1*p* = Re·0052$\frac{1}{12}$, annas and pies may be expressed in decimals of a rupee as in the following illustrations.

1. 6as ... Re·375 ($\because$ 6 × 6$\frac{1}{4}$ = 37$\frac{1}{2}$)
   7p ... ·0364$\frac{7}{12}$ (7 × ·0052$\frac{1}{12}$)
   ∴ 6as 7p = Re·4114$\frac{7}{12}$

2. 13as Re·8125
   3p 156$\frac{1}{4}$
   ∴ 13as3p = Re·8281$\frac{1}{4}$

Aliter. Treat annas and pies as shillings and pence. Write down the decimal value and add $\frac{1}{4}$th of it to itself. This gives the complete decimalized value. Thus, to decimalize 11as 5p —

First decimal = ·5708$\frac{1}{3}$
$\underline{4}$ = ·1427$\frac{1}{12}$
11as 5p = ·7135$\frac{5}{12}$ Re

§ The rule for transforming decimals into annas and pies is as follows : 1. Double the decimal portion with the decimal point shifted two places to the right. 2. Deduct 4% therefrom. The result is pies.

Re·82643...

82·643 × 2 = 165·286 or, 165
Less 4% ⇒ 6·611 7
158·675 158 pies
i. e. **13as 2·675 pies** *i.e.* **13as 3 pies to nearest 3-p**

§ **Ton cwt qr.** The rule for *sh* and *d* may be modified for decimalizing *cwt* and *qr* into *ton*. Multiply *cwt* by 5 and point off two places, as with shillings. Multiply each *qr* by 12, writing from the 3rd decimal place. Add, and the result will be decimal of a *ton* up to 3 decimal places.

15 cwt ... ·75 ton
3 qr ·036 „ *i. e.* 15 cwt 3qr ⇒ **·786 ton**
·786 ton

§ **Maund Seer Chhittacks.** Multiply *seers* by 2$\frac{1}{2}$, pointing off two places : multiply *chhittacks* by 1$\frac{9}{16}$ writing from the 3rd decimal place and add.

14 sr ... ·35 [14 × 2$\frac{1}{2}$]
7 chh. ·011 [7 × 1$\frac{9}{16}$] *i. e.* **14sr 7chh ⇒ ·361 md**
·361md.

§ Table of Nine Values. Where the same amount or quantity is to be multiplied a large number of times by different numbers, it would be useful to make out a table of multiples as below. For a rate, say, Re 1 6as 7p per gallon the following Table is constructed.

1 gall.	... Rs	1·4114$\frac{7}{12}$
2 galls.	... Rs	2·8229$\frac{1}{6}$
3 galls.	... Rs	4·2343$\frac{3}{4}$
4 galls.	... Rs	5·6458$\frac{1}{3}$
5 galls.	... Rs	7·0572$\frac{11}{12}$
6 galls.	... Rs	8·4687$\frac{1}{2}$
7 galls.	... Rs	9·8802$\frac{1}{12}$
8 galls.	... Rs	11·2916$\frac{2}{3}$
9 galls.	... Rs	12·7031$\frac{1}{4}$

To obtain the value of 573¾ gallons at this rate—

500 galls	...	Rs 705·7292	correct to 4 dec places.
70 ,,	...	,, 98·8021	,, ,, ,, ,,
3 ,,	...	,, 4·2344	,, ,, ,, ,,
¾ ,,	...	,, 1·0586	
		Rs 809·8243	,, ,, ,, ,,
		i. e. **Rs 809 13as 2p**	,, ,, nearest pie

Note. The Table should be constructed with *complete* decimalized values. But it will always be sufficient to carry on the work of calculation from the table correct to 4 dec. places to obtain results correct to the nearest pie or farthing.

EXAMPLES 2A

1. Decimalize—

Rs 8 9as 7p ; Rs 19 2as 11p ; Rs 3 14as 5p ; £ 1 17s 3d ; £2 10s 5¾d : 10s 0¼d ; 2 ton 13 cwt 1 qr ; 1 ton 3 cwt 3 qr ; 3 md 13 sr 10 chh ; 1 md 36 sr 7 chh.

[*Ans.* Rs 8·5989$\frac{7}{12}$; Rs 19·1822$\frac{11}{12}$; Rs 3·9$\frac{1}{96}$; £1·8625 ; £2·523958$\dot{3}$; £·501041$\dot{6}$; 2·662 ton ; 1·186 ton ; 3·34 md ; 1·91 md.

2. The freight on 10 ton 10 cwt over a distance of 500 miles is Rs 1265 12as, find the proportionate cost of carriage of 1 ton over 1 mile. [*Ans.* 3as 10·3p per mile

3. An estate produced 500·7653 tons of rubber valued at £8400. Calculate to 4 significant figures the average price in pence per lb. [*Ans.* 1·797d per lb.

4. A banker charges Rs 15604 10as for a remittance of £ 1203 7s 6d. Work out the rate of exchange to the nearest 32-nd of a penny. [*Ans.* 1s 6½d

5. Work out Tables of Nine Values for the following rates ; and find the value of the quantities noted against each :

(i) @ 1s 10¼d per gallon : 698 ; 315 ; 473 ; 915½ gallons.

[*Ans.* £64 14s 2½d ; £29 4s 0¾d ; £43 17s 0¼d : £84 17s 6d.

(ii) @ 14as 11p per lb : 361 ; 438 ; 926½ ; 1023¼ lb.

[*Ans.* Rs 336 9as ; Rs 408 5as 6p ; Rs 863 12as 3p ; Rs 953 15as 6p

6. In a General Average Adjustment the proportion of the G. A. loss to the total risk involved is ·3563843. Ascertain the shares of the different contributories to the nearest pence with the help of a Table of Nine Multiples :

Ship £15000 Cargo—*A* £346 15s 6d
Freight £ 2850 *B* £918 3s 4d
C £32 13s 9d
D £415 9s 5d

[*Ans.* Ship—£5345 15s 3d ; Freight £1015 13s 11d ; Cargo—*A* : £123 11s 8d ; *B* : £327 4s 5d ; *C* : £11 13s ; *D* : £148 1s 4d

SECTION FOUR

SPECIAL METHODS

§ Variation. When two variables x and y are so related to each other that a change in the value of one results in a proportional change in the other, the relationship subsisting between them may be of the form $x = k.y$ (A), or $x = k\cdot\frac{1}{y}$ (B), where k is a constant independent of the values of x and y. In that case x is said to vary *directly* as y (A), or *inversely* as y (B).

Problems involving variation are to be dealt with by the method of proportion discussed below; but it may be convenient at times to treat problems in *inverse* variation algebraically. The algebraic expressions are $x \propto y$ (A), and $x \propto \frac{1}{y}$ (B).

Illustrations

1. If 5 men can do a piece of work in 15 days, in how many days can 10 men finish it?

Here the total work is invariable, *i.e.* the number of days varies *inversely* as the number of men.

$$\therefore \quad 5 = k.\tfrac{1}{15} \text{ and } k = 75$$

$$\therefore \quad ? \times 10 = k = 75, \text{ and } ? = \underline{7\tfrac{1}{2} \text{ days}}.$$

2. The value of diamond varies as the square of its weight. If a piece weighing $2\frac{1}{2}$ carats cost £50, find the value of another weighing $4\frac{1}{4}$ carats.

Here, $50 = k.(2\frac{1}{2})^2$; and the second piece has a value

$$= k.(4\tfrac{1}{4})^2 = £\frac{50}{(2\frac{1}{2})^2} \times (4\tfrac{1}{4})^2$$

$$= £50 \times \tfrac{4}{25} \times \tfrac{289}{16}$$

$$= \underline{£144\ 10s}$$

3. The price of a stock varies with the dividend that it earns. If the price of $3\frac{1}{2}$% stock is $98\frac{3}{4}$, find the value of (i) 4%

stock, (ii) a share of Rs 100 earning an annual dividend of Rs 8 12as. Answer to nearest anna.

Price of stock $\propto$ dividend

$$i.e.\quad 98\tfrac{3}{4} = k.\,3\tfrac{1}{2} \qquad \therefore\quad k = \frac{98\tfrac{3}{4}}{3\tfrac{1}{2}}$$

(i) Price of 4% stock $= k.\,4 = \text{Rs}\ \dfrac{98\tfrac{3}{4}}{3\tfrac{1}{2}} \times 4 =$ **Rs 112 14as**

(ii) Price of share earning $8\tfrac{3}{4} = k.\,8\tfrac{3}{4}$ $=$ **Rs 246 14as**

4. The time taken in completing a piece of work varies directly as the extent of the work involved and indirectly as the number of men employed in the work and the hours of work put in by each. If 16 men working 6 hours a day can build a road 1½ miles long in 72 days, find how much time 36 men will take to build a similar road 2¾ miles long, working 8 hours a day.

Here time $\propto$ length of road
$\propto$ inversely as no. of men
$\propto$,, ,, hours of work per diem.

$$\therefore\quad \text{time} \propto \frac{\text{length of road}}{\text{no. of men} \times \text{hours of work p. d.}}$$

$$\therefore\quad 72\text{d} = k\cdot\frac{1\tfrac{1}{2}\text{m}}{16\text{m} \times 6\text{h/d}}. \qquad \therefore\ \text{Numerically}\ \ k = \frac{72 \times 16 \times 6}{1\tfrac{1}{2}}$$

$$\therefore\ \text{Reqd. time} = k.\frac{2\tfrac{3}{4}\text{m}}{36\text{m} \times 8\text{h/d}} = \frac{72 \times 16 \times 6}{1\tfrac{1}{2}} \times \frac{2\tfrac{3}{4}}{36 \times 8}\ \text{days}$$

$=$ **44 days.**

§ **Proportion.** *a*, *b*, *c*, *d*, *e*, *f*...... are said to be in proportion if $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$...... These relationships of equality between pairs of ratios, enable the calculation of any one in a group of four quantities when the other three are known. For example, when $\frac{a}{b} = \frac{c}{d}$, if *a*, *b* and *c* are known, *d* can be directly obtained, because $d = \frac{bc}{a}$; if *b*, *c* and *d* are known, *a* can be ascertained from the relationship $a = \frac{bc}{d}$, and so on.

This method of obtaining the fourth term of a proportion, three of which are known, is known as the method of the Rule of

Three, or the method of **Simple Proportion.** The way in which it is applied may be studied in the following illustrations.

Illustrations

1. If a man earns Rs 65 in 31 days, what will be his income in 26 days ?

Arrange data as below :

Rs 65 is earned in 31 days
? ... ... 26 days

It has to be mentally ascertained whether the expected answer will be larger or smaller. In this case, evidently the person will get less than Rs 65 in 26 days, and the expected answer is smaller.

The rule follows from the method of variation and is as follows : Interchange the known items (31 days and 26 days), if necessary, so that *if the expected answer be larger, the larger of the two known quantities goes to the top* ; *if the expected answer be smaller, the smaller number should go to the top.*

The data are now re-arranged thus :

Rs 65	26 days
?	31 days

The answer is obtained by multiplying the top numbers, and dividing by the bottom one.

$$\text{Thus} \quad ? = \text{Rs}\frac{65 \times 26}{31} = \underline{\text{Rs } 54 \text{ 8as } 3\text{p}}$$

The actual written work is quite short, as will be seen from the next example.

2. If the interest on a certain amount at 5 per cent per annum for a given period is Rs 38 4as, what will be the interest on the same sum at 6 per cent per annum for the same period ?

At 5% interest earned is Rs 38¼
6% " " " " ?

Since the expected answer is larger, the figures are re-arranged as below :

6% . Rs 38¼

5% ?

And $? = \text{Rs } \frac{38\frac{1}{4} \times 6}{5}$

$= \text{Rs } \frac{229\cdot5}{5} = \underline{\text{Rs } 45 \text{ 14as } 6\text{p}}$

Note. In actual work the movement of similar items may be indicated by arrow-heads. Thus.

↑ @ 5% interest is Rs 38¼
| @ 6% ... ... ?

$\therefore\ ? = \text{Rs} \frac{6 \times 38\frac{1}{4}}{5} = \underline{\text{Rs } 45 \text{ 14as } 6\text{p}}$

EXAMPLES 3

1. A discount of 10% on a bill amounts to Rs 5 10as 8p. How much would a discount of 12½% on the same bill amount to ? [*Ans.* Rs 7 1a 3p

2. A film 8320 ft long takes 1 hour and 50 minutes to be projected. How long would it take to show a similar film 9152 ft long ? [*Ans.* 2hr 1m

3. 45 labourers can finish putting up a stack of pig iron in 6 hours. How many more men could do it in 4½ hours ? [*Ans.* 15 men

4. A cartload of coal weighing 15md 30sr costs Rs 7 14as ; what would be the cost of 8 md ? [*Ans.* Rs 4

5. A year's charge for a railway siding is Rs 912 8as. What would be the proportionate charge for 172 days ? [*Ans.* Rs 430

6. *A* pays shop rent @ Rs 580 a year for 73 days, when *B* joins to make a partnership with *A*. If *B* agrees to bear ⅜ths of the rent for the rest of the year, how much does *A* pay in rent ? [*Ans.* Rs 406

7. 36 men and 16 boys together do a piece of work in 36 days. In how many days could 45 men and 20 boys do the same work, assuming that 4 boys can work as much as 3 men ? [*Ans.* Within 29 days

8. A sum of Rs 3,50,000 is to be raised annually by a City Municipality in the form of an education cess. What would be the rate at which the money is to be realised on the rateable value of the City amounting to Rs 5·76 crores ? [*Ans.* 1⅙p in Re 1

9. *A* can do a piece of work in 11½ days which *B* can do in 7¾ days. If *A*'s wages are Rs 4 9as per week and *B*'s wages are Rs 6 11as per week, what would *A* have charged for doing some work for which *B* was given Rs 35 8as ?—I. Com.

[*Ans.* Rs 35 15as app.

§ When the number of items in proportional relationship exceeds four, the method may be extended. This is known as the method of **Compound Proportion.**

Illustrations

1. In 11 days, 250 men complete 528 yds of a railway embankment working 9 hours a day. The contract is for building an embankment 1½ miles long in 44 days. How many more men will be drafted to the work, if all agree to work 10 hours a day henceforth ?

250 men	working	9 hours	a day	build ↑	528 yds	in 11 days
?	···	10	···	··· |	2112···	··· 33 ···

The re-arrangement of the numbers is done by considering each pair of similar items in relation to the unknown, *independently of the other items.* Thus, 250 men can do a piece of work working 9 hours a day ; and a *smaller* number of men will be required if they work 10 hours a day. ∴ 9, the *smaller* of the two numbers remains in the top line. Similarly, 528 yds can be done by 250 men ; and a *larger* number of men will be required to do 2112 yds. ∴ 2112, the *larger* of the two, goes to the top. Again, if 250 men take 11 days to do a piece of work, a *smaller* number will do it in 33 days. ∴ 11, the *smaller* of the two, remains on the top.

This re-arrangement is shown below :

250 men	9 hrs	2112 yds	11 days
?	10 "	528 "	33 "

The answer is the quotient of the continued product of the top numbers divided by the continued product of the bottom numbers.

Thus,

$$? = \frac{250 \times 9 \times 2112 \times 11}{10 \times 528 \times 33} \text{ men}$$

$$= 300 \text{ men.}$$

∴ 50 more men will be employed on the work.

2. Five compositors can set in type 600 pages in 16 days of $10\frac{1}{2}$ hours each. Each page has 60 lines with 40 letters in a line. In how many days of 8 hours each will 10 compositors set 1000 pages each of 45 lines with 50 letters in each ?

The problem is stated in the following manner :

Comp.	Pages	Lines/p.	Letters/l.	Days	Hrs.
5	↑ 600	↑ 60	↑ 40	16	$10\frac{1}{2}$
10	1000	45	50	?	8

Every pair is now examined *independently of the others*, in relation to that containing the unknown, in the following way :

If 5 compositors require 16 days to do a certain work
10 ... ... *less* days...... the same work
Again, if 600 pages can be set up in 16 days
1000 ... ... *more*...
Also, if 60 lines ... ... in 16 days
45 ... ... *less*...
and so on.

The work is then re-arranged as follows, or the changes indicated by arrow-heads :

Comp.	Pages	Lines	Letters	Days	Hrs.
5	1000	45	50	16	$10\frac{1}{2}$
10	600	60	40	?	8

$$? = \frac{\cancel{5} \times \cancel{1000} \times \overset{15}{\cancel{45}} \times \cancel{50} \times \cancel{16} \times \overset{7}{\cancel{21}}}{\underset{2}{\cancel{10}} \times \underset{2}{\cancel{600}} \times \underset{2}{\cancel{60}} \times \cancel{40} \times 8 \times \cancel{2}} \text{ days}$$

$$= \frac{525}{32} \text{ days} = 16 \text{ days} + \quad \text{or, } \underline{17 \text{ days.}}$$

3. A contractor engaged 150 labourers to raise an embankment 27 miles long in 90 days. The men agreed to work 8 hours a day for six days in the week. At the end of 60 days it was found that 18 miles of the work had been finished. How should the gang be reduced, if at all, to finish the work within the stipulated time, if the men put in 9 hours work a day, all days of the week ?

150 men	↑	18 miles	60 days	8 hrs	6 days/week
?	|	9 ,,	30 ,,	9 ,,	7 ,,

Re-arranging as before,

$$? = \frac{\overset{50}{\cancel{150}} \times \cancel{9} \times \cancel{60} \times 8 \times \overset{2}{\cancel{6}}}{\underset{\cancel{9}}{\cancel{18}} \times \cancel{30} \times \cancel{9} \times 7} \text{ men}$$

$= 114$ men + i.e., 115 men

Evidently, the work will be completed before time : and the gang may be reduced by 35 men.

EXAMPLES 4

1. 15 men work 8 hours a day to complete a piece of work in 25 days. How many days will be taken by 12 men working 10 hours a day ? [*Ans.* 25 days

2. 13 men are paid Rs 22 4as for 7 days. What will be the wages claimed by 21 men working for 6½ days ? [*Ans.* Rs 33 6as

3. 18 men working 8 hours a day complete a piece of work in 34 days. How many hours a day should 17 men work to finish the same work in 36 days ? [*Ans.* 8 hours

4. 21 men working 7 hours a day for 39 days are paid a remuneration of Rs 119 7as. How much should be paid at the same rate to 35 men working 6 hours a day for 52 days ?
[*Ans.* Rs 227 8as

5. It took 48 men working 8 hours a day to dig a trench 150 yds long 16 ft wide and 6½ ft deep. How many men should be engaged to fulfil a contract for digging a trench 225 yards long 25 ft wide and 8½ ft deep, if they are to work for 9 hours a day, and to finish the work in the same time as the first ?
[*Ans.* Not less than 129 men

6. A labourer was engaged to clear 14 bighas 16 cottahs of land in 15 days at a remuneration of Rs 8. At the end of 10 days he was found to have covered only 9 bighas 5 cottahs working 8 hours a day; and was dismissed. How much should he be paid; and how many hours a day should another man work to complete the clearing in time? [*Ans.* Rs 5 ; 9⅗ hours

7. A contractor undertook to build an embankment 3 miles long in 17 weeks, and engaged 75 labourers to work 8 hours a day. At the end of 9 weeks he found that only a mile and a furlong of the work had been completed. How many more men should he employ, and ask the gang to work 9 hours a day to complete the work in time? [*Ans.* 50 men

8. A contractor agreed to raise on the average 120 tons of coal a day, and employed 40 labourers to do the work. 10 of them were women and could do three-fourths of a man's work each. If the raising is to be increased to at least 150 tons a day, find how many more men, if any, he will have to employ, replacing the women by men, all working 10 hours a day instead of 8 hours as previously. How will his daily wages bill be affected if he pays ⅛-th of the normal wages for overtime?

[*Ans.* 8 men ; Increase : 4¾ normal wages

9. The cost of flooring a square room with patent stone is estimated at Rs 55 2as. What will be the cost of covering another square floor with marble, if a side of the latter is one-third as long again as that of the former, and the rate is increased 5½ times? [*Ans.* Rs 539

10. A typist with a speed of 38 words a minute undertakes to copy a manuscript of 500 pages in 45 days. At the end of 20 days he finds that working 4 hours a day he has been able to finish only 150 pages. He calls in a friend with a speed of 60 words per minute to finish the work in time. How hard should they both work to be within time? [*Ans.* At least 3 hours a day

11. If 75 persons can do a piece of work in 12 days of 10 hours each, how many men will perform a piece of work twice as great in a tenth part of the time, if they work the same number of hours a day, supposing that two of the second set can do as much work in an hour as three of the first set ?—I. Com.

[*Ans.* 1000 men

12. A contractor undertook to make 15 miles of roadway in 40 weeks. In 10 weeks, 3 miles were completed by 180 men working 8 hours a day ; then the men agreed to work 1 hour a day overtime ; and some boys being engaged to assist them, the work was finished in the stipulated time. How many boys were employed, if the work of 3 boys was equal to that of 2 men ?

—I. Com. [*Ans.* 50

13. The freight on 14 md 25 seers carried by a railway over a distance of 550 miles is Rs 38 8as. What will be the charge on 19 md 5 seers for 650 miles at the same rate ?

[*Ans.* Rs 59 8as

14. 15 electric fans and 63 lamps consume electricity, the bill for which averages Rs 55 2as monthly. What will be the estimated cost of using 30 fans and 65 lamps, assuming that on the average a fan consumes 2⅓ times as much current as a lamp ?

[*Ans.* Rs 75 15as

15. A railway company offers to carry a troop of Boy Scouts, not below 50 in strength, at the specially reduced rate of ¾-ths of a single fare for the return journey, and a party of a smaller number at ⅞-ths of a single fare. If their charge for a party of 30 is Rs 223 2as, what would be the charge for a party of 60 ?

[*Ans.* Rs 382 8as

16. 9 fans each consuming 80 watts p. h. for 10 hours a day and 30 lamps of 50 watts p.h. each burning 6 hours a day cost Rs 20 4as a month. What will be the corresponding charge for 15 fans for 9 hours a day and 48 lamps for 8 hours a day, if the rate is reduced by 25% ? [*Ans.* Rs 28 2as

§ **The Chain Rule.** A useful arithmetical device, used largely in the arbitration of exchanges, is the Chain Rule. Its use, however, may very well be extended to various types of problems which involve the establishment of numerical relationship between two items, indirectly related to each other through intermediate links. The following examples will make the Rule clear.

Illustrations

1. Given that Re 1 = 1s 6d, £1 = 80 fr. The relationships are expressed in the form of a chain, and the arbitrated value of the rupee in terms of francs is obtained as below :

Re	shillings	francs
1	$\frac{3}{2}$	
	20	80

$$\text{Re } 1 = \frac{\frac{3}{2} \times 80}{1 \times 20} \text{ fr} = \underline{6 \text{ fr.}}$$

2. Express 4s 6d per yard in francs per metre, being given that 1 metre = 39·37 ins, £1 = 80 fr.

Metre	Inches	Yard	Pence	£	Francs
1	39·37				
	36	1			
		1	54		
			240	1	
				1	80

$$\text{The cost of } 1 \text{ metre} = \frac{39{\cdot}37 \times 1 \times 54 \times 1 \times 80}{1 \times 36 \times 1 \times 240 \times 1} \text{ francs}$$

$$= \underline{19{\cdot}69 \text{ fr.}}$$

It will be noted that the *first item* in the chain is the one of which the value is to be obtained, the *last item* being the one in terms of which the value is to be obtained.

The two items of the link of a chain must be expressed in terms of the *same unit*.

The ultimate relationship between a *unit* of the first item and the last is obtained by *writing down in the numerator the continued product of the right-hand member of each relationship of equivalence, and the continued product of the left-hand member thereof in the denominator.*

The units on both sides should be carefully noted.

3. **Calculate the arbitrated rate of exchange between Calcutta and Berlin from the following data :**

$$\text{Re } 1 = 1\text{s } 6\tfrac{1}{4}\text{d} \; ; \; £1 = \text{M } 20{\cdot}45$$

The relationship between the Rupee and the Mark is expressed in the following manner :

Re			M
1	18¼d		
	240d	£1	
		£1	20·45

Then, according to the Chain Rule,

$$\text{Re } 1 = \frac{18\tfrac{1}{4} \times 1 \times 20{\cdot}45}{240 \times 1}\text{ M}$$

$$\text{and} \quad \text{Rs } 100 = \frac{100 \times 73 \times 20{\cdot}45}{4 \times 240}\text{ M}$$

$$= \underline{155{\cdot}51 \text{ M}}$$

If it were desired to express the value of 100 M in terms of Rupees, the Chain should have been written the other way about, beginning with Marks and ending in Rupees.

4. Determine the par of exchange between London and New York from the following data. £1 contains 123·27447 grains of standard gold which is $\frac{11}{12}$ths fine ; and $10 contain 258 grains of standard gold (U. S. A.) which is $\frac{9}{10}$ths fine. —I. Com.

The Chain is arranged as below :

	Br. stand. gold	Pure gold	U. S. stand. gold	$
£1	123·27447 gr			
	12 „	11 gr		
		9 „	10 gr	
			258 „	10

$$\therefore \; £1 = \$\frac{123{\cdot}27447 \times 11 \times 10 \times 10}{12 \times 9 \times 258} = \underline{\$4{\cdot}86\tfrac{2}{3}.}$$

5. **Find the selling price of an article costing £5, which is marked at an advance of 25%, but sold at a discount of 10%.**

The relationships between C. P., M.P. and S. P. are expressed as follows :

C. P.	M. P.	S. P.
100	125	
	100	90

$$\therefore \quad \text{Reqd. S. P.} = £5 \times \frac{125 \times 90}{100 \times 100} = \underline{£5\ 12s\ 6d}$$

6. A person invests £10,000 in purchasing Stock (1) at 87. On the price falling to 75 he invests the proceeds in another Stock (2) at 160. When the second Stock rises in value to 174, he again sells out, investing the proceeds in Railway Stock at 150. What is his present holding ? Neglect expenses.

The different transactions might be arranged as below :

	St. 1		St. 2		St. 3
87	100				
	100	75			
		160	100		
			100	174	
				150	100

∴ £10,000 invested at first would now purchase

$$£10000 \times \frac{100 \times 75 \times 100 \times 174 \times 100}{87 \times 100 \times 160 \times 100 \times 150} \text{ Stock (3)}$$

$$= \underline{£6250 \text{ Stock (3)}}.$$

EXAMPLES 5

1. Given that an English gallon is 277·274 cu. in., that 6 U. S. gallons are equal to 5 English gallons and that a litre is 61 cu. in., express the U. S. gallon in litres correct to 2 dec. places. —I Com. [*Ans.* 3·79 litres

2. Rs 100 = 77·25 yens Re 1 = 1s 6d. 5737·10 yens = ? £ s d. [*Ans.* £557

3. The value of £1 = 113 and of Re 1 = 8·475 grains of gold. Find the exchange rate between the Rupee and the Sterling. [*Ans.* 1s 6d = Re 1

4. Find the price in rupee per yard, given the price per metre to be 16·50 fr, Rs 100 = 780 fr and 1 yd = 91·43 cm. [*Ans.* Re 1 15as

5. Given £1 = 125·22 fr, Re 1 = 1s 6d, 1 metre = 39·37 in ; 1 Kg = 2·204 lb ; 1 ton = 27$\frac{2}{9}$ md ; 1 md = 82$\frac{2}{7}$ lb, find a multiplier to convert—

(i) shillings per yard into francs per metre [*Ans.* 6·847 fr
(ii) shillings per lb into francs per Kg [*Ans.* 13·799 fr
(iii) rupees per maund into £ per ton [*Ans.* £2·0417
(iv) shillings per lb into Rs per maund [*Ans.* Rs 54·857

6. The London price of silver is 20d per oz. (480 gr). If 1 Tola = 180 gr (Tr.), find the price of silver in India per 100 tolas when exchange is at par (Re 1 = 1s 6d). [*Ans.* Rs 41 10as 8p

7. If the Bombay price of gold is Rs 35 4as per tola, what should be the minimum price of gold in London in £ per oz. ? (Exchange at par ; neglect cost of transport.) [*Ans.* £7 1s

8. What price should a London merchant quote for Jute Firsts in £ per ton which is being quoted in Calcutta at Rs 36 per md. ? (Exchange at par) [*Ans.* £73 10s

9. Find the cost in Rupees of 15 cwts of rubber at 8d per lb exchange being at 1s 6d to Re 1. [*Ans.* Rs 746 10as 8p

10. Aluminium ingots are being quoted in Calcutta at 10as 3p per lb. What should be the corresponding quotation in sterling per ton, with exchange at 1s 4d to Re 1 ? [*Ans.* £95 13s 4d

11. Express a speed 120 miles per hour in yards per second. [*Ans.* 58$\frac{2}{3}$yds/sec

12. Gold is being sold in London at £7 1s 10$\frac{1}{2}$d per ounce and at Rs 35 4as per tola of 180 grains in Bombay. If the day's rate of exchange is 1s 6$\frac{1}{32}$d to Re 1, find the difference between the arbitrated and the actual rate, which represents the cost of shipment, insurance and other incidental charges. [*Ans.* 7$\frac{1}{2}$d per oz.

13. A bookseller takes one anna in the rupee off a bill and still makes 25% on his cost. What is the gross value of the bill, if the books cost him Rs 15 ? [*Ans.* Rs 20

14. A person deposits Rs 24000 in a bank at 5% p. a. At the end of a year he withdraws the principal with interest and buys Post Office Cash Certificates at Rs 75 per Rs 100 with the entire amount. On maturity of the certificates, he invests the proceeds in 3½% Government Stock at Rs 112 per 100. Find his present annual income. [*Ans.* Rs 1050

15. A dealer desires to clear 16% after allowing a discount of 5%. How does he mark an article costing Rs 7 2as ? Answer to nearest anna. [*Ans.* Rs 8 12as

16. Find the cost price of an article marked at 20% above cost and sold at Rs 5 13as 9p after allowing discount at 6¼%.
[*Ans.* Rs 5 3as 3p

17. The price of Ceylon Rubber is Re ·85 per lb. Find the cost of rubber imported into France, allowing costs of transport, insurance etc., at an average rate of 0·35 fr per Kg. Given Rs 100 = 660 fr ; 1 Kg = 2·2 lb. [*Ans.* 12·69 fr

18. Safety razor blades are quoted at German ports at RM 175 per Kg. Find the imported cost of blades per gross, being given £1 = 19·50 RM ; Re 1 = 1s 6d ; 1 Kg = 2·2 lb ; weight of a blade = 2·20 grains ; average cost of transport, insurance etc. = 8as 7·4p per gross. [*Ans.* Rs 3 per gross

19. Determine the par of exchange between London and New York on the basis of the gold content of the Dollar and the Sovereign as on January 31, 1934. £1 contained 123·274 grains of gold $\frac{11}{12}$-ths fine and $1 contained 15·238 grains of gold $\frac{9}{10}$-ths fine. [*Ans.* £1 = $8·2397

20. Coal is sold in England f. o. b. for 17s a ton and in France* the price realised is 30 francs per tonneau (1000 Kg). What is the total amount in francs per tonneau of freight and profit ? £1 = 25·23 fr. —I. Com. [*Ans.* 8·90 fr

§ **Method of Proportional Numbers.** A powerful method of tackling inter-related quantities is to work with proportional numbers, and to obtain actuals by a method of simple proportion. The following illustrations will be helpful.

1. A wholesaler allows a discount of 10% to a retailer who desires to make a profit of 20% on cost. If the retail price of the article be Rs3 6as, what is the wholesale price ?

Now, if Wholesaler's M. P. ... 100 →
Disct. ... 10
Retailer's C. P. ... 90
Profit. ... 18
Retailer's S. P. ... 108 corresponds to Rs 3 6as

The numbers 100, 10, 90, 18 and 108 are proportional to certain real values of which it is given that 108 corresponds to the retail price Rs 3 6as i.e., 54as. ∴ Wholesale Price, which is proportional to 100, will be 50as or Rs 3 2as

Note. In an exactly similar manner, profit, discount or the retailer's cost price may be directly obtained by Rule of Three.

2. The profit in a certain business in 1930 is 20% above that of the previous year. In the following 3 years, the increases are 15%, 10% and 5% respectively over previous year's figures. If the profits in 1933 amounted to Rs 15939, what was the profit earned in 1931 ?

Suppose, 1929 profit is 100
Then, 1930 „ 120
1931 „ 138 →
1932 „ 151·8
1933 „ 159·39 corresponds to Rs 15939
∴ 1931 profit proportional to 138 = Rs 13800

3. Different middlemen in the sale of a plot of land make profits on respective costs at 10%, 10%, 5% and 20%. The land is ultimately sold for Rs 15246. What was its original price ?

1st C. P. 100 → ?
Profit @ 10% 10
2nd C. P. 110
Profit @ 10% 11
3rd C. P. 121
Profit @ 5% 6·05
4th C. P. 127·05
Profit @ 20% 25·41
Last S. P. 152·46 corresponds to Rs 15246
∴ Original price proportional to 100 = Rs 10000

4. If the M. P. be increased by 10%, how should a discount of 10% be altered so that the selling price may remain unaltered ?

Original M. P.	100	Increased M. P.	110
Discount	10	Discount	?
S. P.	90	S. P.	90

Evidently the new discount $= \frac{20}{110}$ *i.e.*

5. If the C. P. is increased by 5%,* how should the listed price, marked to ensure a profit of 10%, be raised in order that the actual profit may remain unaltered ?

Old C. P.	100	New C. P.	105
Profit @ 10%	10	Profit	10
Old M. P.	110	New M. P.	115

The increase on the old M. P. $= \frac{5}{110} \times 100\% = 4{\cdot}55\%$

6. A retailer clears $12\frac{1}{2}\%$ on sales. How much does this represent on his cost ?

S. P.	8
Profit	1
C. P.	7

$\therefore$ Profit % on cost $= \frac{1}{7} \times 100 = \underline{14\frac{2}{7}}$

§ When a problem involves both discount and profit, actual and anticipated, relationships between C. P., S. P. and M. P. are best obtained by the method of the Chain Rule.

7. How should an article costing Rs 9 8as be marked to obtain a clear profit of 10% after allowing discount at 5% ?

The Chain is written out as under :

C. P.	S. P.	M. P.
100	110	
	95	100

$\therefore$ M. P. corresponding to a C. P. of Rs 9 8as $=$ Rs 9 8as $\times \frac{110}{95} =$ Rs 11

§ **Adjustment of differences** helps to solve a very common type of problems.

Illustrations

1. An article is marked to sell at a profit of 10%. If it be sold for Rs 7 8as less, there will be a loss of 5%. What is the C. P. ?

Obviously a difference of Rs 7 8as in the realisation means a difference of 10% + 5% i.e., 15% on the C. P. ∴ C. P. = Rs 50

2. A rise in the rate of income tax from 5p to 6p in the rupee reduces a person's income by Rs 8 4as a year. Find his monthly income.

The difference of 1 pie on every rupee taxed makes a total difference of Rs 8 4as which is equal to 1584 pies.

∴ Annual income = Rs 1584 ; and Monthly income = Rs 132

EXAMPLES 6

1. The earnings of a business were apportioned as follows: Rs 10,000 to reserve ; Rs 30,000 to profits shared by the partners ; Rs 2000 to bonus to employees ; Rs 3500 to writing off of bad debts ; Rs 4500 to a sinking fund. Give the apportionment in percentages of the total earnings. [*Ans.* 20% ; 60% ; 4% ; 7% ; 9%

2. Depreciation on plant and machinery valued at Rs 183,000 @ 10% represents 2·75 per cent of the gross profits. What will be the amount transferred to reserves, if it is to be 13·75 per cent of the gross profits? [*Ans.* Rs 91,500

3. *A*, *B*, *C* enter into a partnership with Rs 5000, Rs 3000 and Rs 2000 respectively, agreeing to share profits in proportion to capital. *A* gets a remuneration of 20%, and *B* 10% of the gross profits. If at the end of a year *A* receives altogether Rs 600 more than *B*, how much did the partners receive?

[*Ans.* *A* : Rs 1375 ; *B* : Rs 775 ; *C* : Rs 350

4. At what price should an article costing Rs 8 8as be marked so as to allow a clear profit of 12½% on sales, after deducting discount at 10%? Answer to nearest anna. [*Ans.* Rs 10 13as

5. An alloy contains 80% of gold and the rest silver. How much gold has to be mixed to sell 100 grains of an alloy 90% fine? [*Ans.* 50 gr. of gold to 50 gr. of first alloy

6. A sum of Rs 84 is to be distributed in alms as follows: 1 anna to a child, 2as to a woman and 3as to a man. Find the number of children, if there is one child to every adult, there being the same number of men as women. [*Ans.* 384

7. A person tenders Rs 490 in currency notes for change. He wishes to get 3 eight-anna pieces, 15 four-anna pieces and 4 two-anna bits to every rupee that he gets in coins. Find how many of each kind he gets. [*Ans.* 70 Rupee coins

8. A dealer altered his trade discount from 15 to 10 per cent. By what percentages were the selling prices altered, the percentage of cash discount remaining the same as before ?—I. Com. [*Ans.* $5\frac{15}{17}\%$

9. An article is marked at 25% above cost. If a discount of 4% allowed on the list price resulted in a profit of Rs 2 8as, what was the cost price ? [*Ans.* Rs 12 8as

10. How should an article be marked to give a profit of 10%, even after a discount of $8\frac{1}{3}\%$ is allowed off the list price ? [*Ans.* 20% above cost

11. 20% on the sales is the profit expected by a retailer of hardware. How much does this represent on his cost price ? [*Ans.* 25%

12. A general discout of $6\frac{1}{4}\%$ off the lists is announced by a watchdealer. He is allowed a commission of 30% on these watches. What profit does he make on the sale of a watch for Rs 121 14as net ? [*Ans.* Rs 30 14as

13. What was the prime cost of an article which on being sold at a gain of $7\frac{1}{2}\%$ yields Rs 4 11as more than if it were sold at a loss of $12\frac{1}{2}\%$? —I. Com. [*Ans.* Rs 23 7as

14. If the manufacturer makes a profit of 20%, the wholesale dealer a profit of 25% and the retailer a profit of 40%, what was the cost of manufacturer of an article which was bought at the shop for Rs 350 ?—I. Com. [*Ans.* Rs 166 10as 8p

15. A property changed hands 3 times, each agent who sold charging 1% as commission. If each time it was sold for the net amount obtained at the previous sale, calculate to the nearest rupee its original value if the third sale realised Rs 1,00,000 net. —I. Com. [*Ans.* Rs 1,03,061 app.

16. A tradesman professes to retail his goods at a profit of 10%, but he adulterates them by adding $\frac{1}{4}$-th of their weight of an inferior article which costs him $\frac{4}{5}$-ths the price of the better. How much per cent profit does he make ? Also, in what proportion must he mix the two kinds so as to gain 20% ?—I. Com.

[*Ans.* $14\frac{7}{12}$% ; 7 : 5

17. Genuine milk contains 88·75% by bulk of water, 2·75 of fat and the remainder is non-fatty solids. A purchaser buys 7 gallons of milk at 3d a quart ; the milk on being analysed is found to contain 90·84% of water, 2·24 of fat, the residue being non-fatty solids. Find whether any thing besides water has been added to it, and find the sum of which the purchaser has been defrauded. —I. Com. [*Ans.* No ; 1s $3\frac{1}{2}$d

18. If 7% of the sale price of an article is equal to 8% of its cost price, and 9% of the sale price exceeds 10% of the cost price by half-a-crown, at what price is the article bought and sold ?

—I. Com. [*Ans.* £43 15s ; £50

19. During a sale a shopkeeper reduced his goods 50% below marked prices, which had originally been fixed to allow 25% profit on selling price after deducting 10% discount for cash. What per cent does he gain or lose ?—I. Com. [*Ans.* Loss 26% app.

20. A manufacturing firm is accustomed to allow to trade customers a discount of 15% on the prices in it's published list. This has given it a profit of 19% on the cost of manufacture. The cost goes up 12% and the firm issues a new price list with all the prices put up 10%. If the firm continues to allow its customers the same rate of discount, what percentage profit will it now make on the cost of manufacture ?—I. Com. [*Ans.* $16\frac{7}{8}$%

21. If the cost of manufacturing a certain article is increased by 16%, and the catalogue price is increased by 15% what profit per cent (to nearest tenth) on the cost of manufacture does a firm gain after allowing a trade discount of $12\frac{1}{2}$% ? The profit per cent on former cost of manufacture was 20%.

—I. Com. [*Ans.* 19%

22. Three persons contribute sums of £250, £500 and £750 respectively towards a venture on the understanding that the profits of the venture shall be divided in such a way that the rate of interest which each receives shall be in proportion to the amount of his contribution. If the profits for a year amount to £245, how much will each receive ?—I. Com. [*Ans.* £17 10s ; £70 ; £157 10s

23. I sell articles at 6s 6d each and thereby make a certain percentage of profit. I increase the price to 7s 1d and the percentage of profit is increased by 10. What profit have I made at the latter price ? —I. Com. [*Ans.* 21$\frac{3}{7}$%

24. A shopkeeper can buy his goods at a price which gives him a gross profit of 20% on the turnover. If the price at which he has to buy rises by 10% while he increases the price to his customers by 5% only, what will then be his percentage gain of gross profit on turnover ? —I. Com. [*Ans.* 16·2%

25. What price should be marked on an article which cost £2 6s 8d so that a profit of 10% may be made after allowing a a discount of 12½% for cash ? —I. Com. [*Ans.* £2 18s 8d

26. A man embarks his whole capital in four successive ventures ; in the first he obtains 100% and in each of the others he loses 20%. Show what percentage he has gained on his original capital. —I. Com. [*Ans.* 2·4%

27. A jeweller, being out of stock of a certain make of watch-chain, took one from a neighbouring shop, and sold it at 10% below the catalogue price. The second dealer allows the first a commission of 25%, and himself gets a commission of 33⅓% from the manufacturers. If the first dealer cleared Rs 45 on this transaction, what profit was made by the second dealer ? Compare their respective pofits. [*Ans.* Rs 25 ; 20% ; 12½%

28. The expenses of a railway were the following percentages of the gross receipts in a year :

Overhead charges	...	10
Permanent way	...	5
Loco and rolling stock	...	45
Line staff and suprevision	...	30

The following year the receipts go down by 15%, expenses remain unaltered, except those on loco and rolling stock which go down by 20%. If the balance is carried to a reserve every year, estimate what proportion the amount set aside in the second year is of the first. [*Ans.* 40%

29. In a competitive examination, Bengal sent up 20% of the candidates, Bombay 15%, Madras 25%, U. P. 12%, the Punjab 16%; and the rest hailed from the remaining parts of India. There was a success of 40% among the Madrassis, Bengal claiming a success to the extent of 80% of the number of successful Madrassis. What per cent of those sent up from Bengal were successful? If between them Bengal and Madras claimed 72% of the total success, what was the total percentage of success? [*Ans.* 40%; 25%

30. An all round increase of 15% in the costs of production resulted in the list price, which was marked 30% above cost, being raised by 20%. If the same rate of discount at 5% continued to be offered, and meant a difference of Rs 247 in the net selling price, how was the article originally marked? [*Ans.* Rs 1300

31. Two grades *A* and *B* of lubricating oil are mixed in the proportion 3 : 2. After 25% of this has been sold from stock, sufficient quantity of *A* is mixed with the remainder to raise the proportion to 5 : 3. If the stock is now 400 gallons, what was the quantity of the original mixture; and how much of grade *A* was added to make the new mixture? [*Ans.* 500 galls; 25 galls *A*

32. An increase of 10% in the cost of raw materials results in an increase of 5% in the total cost of production. If labour absorbs 30% of the total cost, find how the latter will be affected by a rise of 5% in the only remaining item, the supervision charges? [*Ans.* 1% increase

33. A property changes hands three times. Brokerage is paid by the buyer at 5% on each of the first two transactions, and at $7\frac{1}{2}$% on the last transaction. If each sale was effected at cost price, and the brokers between them got Rs 2222 4as on the three transactions, what was the original price of the property?

[*Ans.* Rs 12000

34. Two-thirds of a consignment was sold at profit of 6%, and the remainder at a loss of 3%. If the total amount cleared was Rs 540, what was the value of the consignment ? [*Ans.* Rs 18000

35. The cost of manufacture of an article increases 16%, and the catalogue price is marked up 15%. What profit (to the nearest tenth) is made on cost after allowing a trade discount of $12\frac{1}{2}$% ? The article was originally marked at 20% above cost. [*Ans.* 4·1%

36. Tea is sold at Rs 2 3as per packet of 2 lb at an all-in profit of 25%. If a cess of 2as per lb is removed, and the price is reduced by 4as per packet, what profit would the seller make ?

[*Ans.* $29\frac{1}{6}$%

37. A cotton mill purchases 40 candys of cotton (500 lb per candy) at Rs 180 per candy, and manufactures yarn of 20 counts in packets of $\frac{1}{2}$-lb each. The packets are sold at Rs 18 12as per 100. There is a wastage of 20% in the manufacture, labour represents $12\frac{1}{2}$% and machinery charges another $12\frac{1}{2}$% of the cost of the raw material. Find the total profit and the profit per cent.

[*Ans.* Rs 9000 ; 100%

38. Linseed with 5% refraction can be had in the market for Rs 9 8as per md. What would be the maximum offer on a lot with 10% refraction ? [*Ans.* Rs 9 per maund

39. Artificial silk weighing 5 oz per reel is used in the manufacture of silk goods along with silk yarn which weighs $3\frac{1}{2}$ oz for the same length. If an entire piece of cloth weighs 5 oz, with 25% of artificial silk, what would be the weight of a similar piece made entirely of silk ? [*Ans.* $4\frac{5}{8}$ oz.

40. A tradesman imported a quantity of art buttons on which he paid duty at 5% *ad valorem*. He sells them at a loss of $5\frac{5}{7}$% ; a month earlier he could have cleared $2\frac{6}{7}$% by selling his stock for Rs 1242 more. What was the invoice price of his stock ? [*Ans.* Rs 13800

§ **Mixture.** A type of problems arises when two or more kinds of a commodity are mixed together by way of adulteration or for purposes of blending, as in in the case of tea etc., or for the purpose of reducing the strength of a spirit or an acid. It is straightforward work to find the average cost price of the mixture or to find the selling price thereof at a stated rate of profit or loss.

EXAMPLES 7

1. 18 lb of tea at 14as per lb are mixed with 40 lb of an inferior quality at 9as per lb. What is the mixture worth ? [*Ans.* 10as 6·6p per lb.

2. 30 md of cocoanut oil at Rs 17 8as per md are mixed with 5 md Neem oil at Rs 12 per md to make a soap base. What is the average cost of the mixture ? [*Ans.* Rs 16 11as 5p per md.

3. A dealer mixes 90 gallons of wine containing 10% of water with 60 gallons containing 20% of water. What is the percentage of water in the mixture ? [*Ans.* 14%

4. Lubricating oils of qualities *A* and *B*, containing 3% and 5% of a certain mineral are mixed in the proportion of 5 : 3. What is the percentage of the mineral in the mixture ? [*Ans.* 3¾%

5. Australian Mixed Tallow costs Rs 21 per cwt and Calcutta Mixed Tallow costs Rs 14 8as per md. If they are mixed in the proportion of 7 : 5, find the cost of the resulting mixture. [*Ans.* Rs 15 0a 8p per md.

6. Pure milk contains 89% of water. If a sample of milk is found to contain 90% of water, what is the extent of adulteration in 22 seers of milk ? [*Ans.* 2 seers

7. In a sample of 50 lb of glycerine, there is found to be adulteration to the extent of 20%. If the glycerine is to be made up to a standard of 5% impurity, how much pure glycerine is to be mixed with it ? [*Ans.* 150 lb.

8. In a 35 lb mixture there is 60% quinine. How much of pure quinine should be added to make the adulterant 35% of the total ? [*Ans.* 5 lb.

9. A mixed paint contains 4 gallons of paint in 40 gallons, and the rest oils etc. How much of paint should be added to make the oils 87½% of the total ? [*Ans.* 1⅐ galls.

10. The same quality of rice was purchased from two persons, 150 md at Rs 3 8as 6p per md and 250 md at Rs 3 9as 3p per

md. At what price per md should the rice be sold so as to ensure a profit of at least 25% ? [*Ans.* Rs 4 7as 3p

11. A dealer indents 2000 bags of flour at Rs 9 6as per bag. As a result of an outbreak of fire 500 bags were damaged and were sold at Rs 2 8as per bag. Another 750 bags had also to be sold at a loss of 20%. Find his gain or loss per cent on outlay, if he could sell his remaining stock at a profit of 10%. [*Ans.* Loss $22\frac{1}{12}$%

§ The method of attacking inverse problems is entirely different. Suppose, a quantity of tea at 7as per lb is to be mixed with a better quality at 13as per lb to obtain an average cost price of say, 11as per lb. To find the proportion in which they have to be mixed, we arrange the differences in the price of each quality and the average price as below, writing them diagonally ; then the ratio of these differences represents the ratio required.

	Price per lb		Differences	Ratio
1st quality	7as		2as	1
Average price		11as		
2nd quality	13as		4as	2

The reason is, that if the first quality were sold at the average price of 11as, there would be a profit of 4as on every lb ; on the other hand, if the second quality be sold at the average price there will be a loss of 2as per lb. The price of 11as per lb is, however, one at which there is neither loss nor profit ; so that to balance the loss by gain, to every lb of the first quality sold, 2 lb of the second have to be disposed of. In other words, the cost price of a mixture of the two qualities in this proportion will be just 11as per lb.

It should be noted that this method is based on the *cost prices* of the different items. When, therefore, a problem involves profit or loss at different prices, their net cost prices have to be obtained before using this method.

Illustrations

1. **25oz. of alcohol 80% pure is to be made up to the standard of 90% purity. Ascertain the quantity of pure alcohol required to be mixed.**

In 25oz. alcohol there is 20%, *i.e.*, 5oz. impurity. This quantity of impurity remains unaltered when the new mixture becomes 90% pure, that is, it then becomes 10% of the total quantity.

∴ The total quantity of new mixture is 50 oz., which indicates that 25oz. of pure alcohol have been mixed.

2. A grocer sells one quality of tea at 11as per lb and a better quality at Re 1 7as per lb making a profit of 10% and 15% respectively. He wishes to blend these two qualities so that the mixture may be sold at Re 1 2as per lb at a profit of 12½%. How should he mix them ?

$$\text{C. P. of 1st quality} = 11 \times \frac{100}{110}\text{as} = 10\text{as per lb.}$$

$$\text{,, ,, 2nd ,,} = 23 \times \frac{100}{115}\text{as} = 20\text{as ,,}$$

$$\text{,, ,, mixture} = 18 \times \frac{100}{112\frac{1}{2}}\text{as} = 16\text{as ,,}$$

Arrange the differences in cost prices as below .

			Diffces.	Prop.
1st qual.	10as		4as	2
Mixture		16as		
2nd qual.	20as		6as	3

We find that the proportion in which the two qualities are to be mixed is $\underline{2:3.}$

3. **A grocer sells one kind of tea at 2s 6d a lb and loses 5%, and another at 3s 4d per lb and gains 14%. How should they be mixed so that the mixture may be sold at 3s per lb at a profit of 8% ?**

Qual. 1. S. P. = 2s 6d at a loss of 5%.

$$\therefore \text{ C. P.} = \overset{6}{\cancel{30}} \times \frac{100\text{d}}{\underset{19}{\cancel{95}}} = \frac{600}{19}\text{d}$$

Qual. 2. S. P. = 3s 4d with profit at 14%.

$$\therefore \quad \text{C. P.} = \overset{20}{\cancel{40}} \times \frac{100}{\underset{57}{\cancel{114}}}\text{d} = \frac{2000}{57}\text{d}$$

Mixture. S. P. = 3s with profit at 8%.

$$\therefore \quad \text{C. P.} = \cancel{36} \times \frac{100}{\underset{3}{\cancel{108}}}\text{d} = \frac{100}{3}\text{d}$$

Arranging prices and differences as before, we get

			Diffces.	Prop.
Qual. 1.	$\frac{600}{19}$d		$\frac{100}{57}$d	1
Mixture		$\frac{100}{3}$d		
Qual. 2.	$\frac{2000}{57}$		$\frac{100}{57}$d	1

Teas to be mixed in equal parts.

§ The method may be extended to cover cases in which the number of items is more than two. In such a case, a proportion is obtained for pairs of prices, one larger and the other smaller, taken in reference to the middle (or average cost) price.

A final proportion may be obtained by taking the total number of parts taken of each quality, as shown by the ratios of the difference of each pair of prices.

Illustration

How should water be mixed with wines of different qualities at 10s, 12s 6d, 10s 6d, 18s a gallon to obtain a mixture worth 11s 9d per gallon ?

		Price p.g.	Av. Price.	Diffces.	Diffces.	Diffces.	Prop.
Qual.	1	120d		$\underset{25}{\cancel{75}}$d			25
,,	2	150d			$\overset{5}{\cancel{15}}$d	$\overset{47}{\cancel{141}}$d	52
,,	3	126d	141d		$\underset{3}{\cancel{9}}$d		3
,,	4	216d		$\underset{7}{\cancel{21}}$d			7
Water		0				$\underset{3}{\cancel{9}}$d	3

A final proportion should then be 25 : 52 : 3 : 7 : 3.

Note. The possible number of correct answers in case of more than two qualities is *infinite*; for theoretically, the proportion of each pair may be taken in multiples, before being added up. *E.g.* in the above example, without altering their values, the different pairs of proportional numbers may be written as below :

				Final Prop.
Qual. 1.	50			50
,, 2		25	47	72
,, 3		15		15
,, 4	14			14
Water			3	3

In which case the proportion is 50 : 72 : 15 : 14 : 3.

EXAMPLES 8

1. Chicory costs 2d per lb and cocoa 1s 1d per lb. How should they be mixed in order that the mixture may cost 10d per lb ? [*Ans.* 3 : 8

In what proportion should they be mixed, if by selling the mixture at 10½d per lb the dealer expects to make a profit of 5% ? [*Ans.* 3 : 8

2. A grocer sells tea at 11as per lb, and a second quality at 8as per lb making a profit of 10% on the first and 6⅔% on the second. How should he mix them so that he may sell the mixture at 9as per lb making a profit of 12½% ? [*Ans.* 1 : 4

3. 150 maunds of rice at Rs 3 8as per md is to be mixed with rice at Rs 5 8as per md to be sold at Rs 5 8as per md at a profit of 10%. How much of the better quality is to be used ? [*Ans.* 450 mds

4. An investor divides his money for investment in two different stocks yielding 4⅛% and 4⅓%. If he desires to get an average yield of 4¼%, how should an investment of Rs 40,000 be divided ? [*Ans.* 1st investment Rs 16000 ; 2nd investment Rs 24000

5. How much water should be mixed with 32 pints of wine at Rs 2 12as per pint, so that the mixture may be sold at Rs 3 per pint at a profit of 50% ? [*Ans.* 12 pints

6. Chinese tea worth Rs 2 12as per lb is mixed with tea at 6as per lb and another quality at 13as per lb ; and the mixture is sold at Re 1 per lb at a profit of $14\frac{2}{7}$%. In what proportion were the different qualities mixed ? [*Ans.* 1 : 3 : 6

7. A Kyanite quarry yields three different varieties of rocks containing 72, 68 and 58 per cent of *alumina*. How should they be stacked to yield an average *alumina* content of 65% ?

[*Ans.* 7 : 7 : 10

8. A rice dealer has two qualities of rice which he sells at Rs 3 8as and Rs 4 8as per md, making a profit of 12% and 8% respectively. How should he mix the two qualities, if he desires to sell the mixture at Rs 4 2as a maund at a profit 10% ?

[*Ans.* 2 : 3

9. A retailer sells tea at 8as per lb and a better quality at 14as per lb, making a profit of 12% on his outlay. A purchaser requests him to blend the two kinds, offering 9as 9p per lb. In what proportion should the teas be blended to assure a profit of 17% ? [*Ans.*7 : 2

10. Three qualities of spirits are sold at Rs 3 7as, Rs 4 2as and Rs 4 5as per gallon, the respective profits thereon being 10%, 20% and 15%. In what proportion should they be mixed, so that the mixture may be sold at Rs 3 15as at a profit of 12½% ?

[*Ans.* 1 : 2 : 2

11. A firm is prepared, during the first year, to set off profits on the sale of a certain commodity against loss on the sale of another, with a view to creating a market for the latter. The first quality is worth Rs 18 2as per ton and the other Rs 18 6as per ton, and either quality is sold at Rs 18 3as per ton. If 610 tons of the second quality is sold in a month, what should be the quality sold of the first kind ? [*Ans.* 1830 tons

12. Dress materials of two qualities, costing 4as 6p and 6as per yd, are to be used in making children's frocks to be sold at 8as 6p per piece. Assuming that on the average a yard of cloth is used in a frock and that the overhead charges and profit are covered by charging 3as 6p per piece, find how much of the second quality is to be used with every 200 yards of the first ?

[*Ans.* 100 yds.

13. A salesman is to dispose off a stock of 4 doz. slightly soiled tennis rackets at the specially reduced price of Rs 20 each which cost the dealer Rs 20 12as each. The salesman is instructed at the same time to push the sale of a certain quality of tennis balls at Rs 11 8as per doz, which cost him Rs 8 8as per doz. How many balls at least should be sold, so that the dealer does not suffer any loss on the whole ? [*Ans.* 12 doz.

14. With tea worth half-a-crown a pound a dealer mixes an inferior quality worth 18d a pound. In what proportion must he mix them so that by selling the mixture at the higher price he may gain 16% ? —I. Com. [*Ans.* 19 : 10

15. A tea dealer blends 75 lb of tea @ Re 1 2as per lb with 45 lb of tea @ Re 1 5as per lb. What is the lowest price at which he can sell the mixture so as to gain at least 25% ? —I.Com.

[*Ans.* Not less than Re 1 8as

16. A merchant buys teas at Rs 2 4as, Rs 2 6as 6p and Rs 2 11as 4p per lb, and mixes them in the proportion of 5 : 4 : 1. At what price must he sell the mixture so as to make a profit of 25% ? —I. Com. [*Ans.* Rs 2 15as 3p app.

§ **Use of Symbols.** The following afford further illustration of the value of symbols in ordinary calculations. In cases where arithmetical methods lead to unnecessary confusion or complication, there should be no hesitation in taking recourse to algebra.

Illustrations

1. A certain wine contains 40% alcohol ; another contains 75%. Find how much of the second kind has to be mixed with 20 gallons of the first kind to make it of 60% strength.

An addition of x gallons of the second kind increases the quantity of alcohol from 8 gallons to $(8+\frac{3}{4}x)$ gallons ; and the total quantity of wine to $(20+x)$ gallons. As the mixture is now 60% strong,

$$60\% \text{ of } (20+x)=8+\tfrac{3}{4}x, \text{ whence } x=\underline{26\tfrac{2}{3} \text{ gallons}}.$$

2. The freezing point of water is marked 0° on the Centigrade and Reaumur scales, and 32° on the Fahrenheit scale of thermometers. The boiling point of water is correspondingly marked 100°C, 80°R and 212°F. Convert 48°C to the other two scales.

The distance between the freezing and boiling points of water is divided into 100, 80 and 180 equal parts respectively on the three scales. Representing corresponding readings on the three scales by C, R and F,

$$\frac{C}{100}=\frac{R}{80}=\frac{F-32}{180}.$$

$$\therefore \quad \text{When } C=48^\circ,\ R=\frac{80}{100}\times 48^\circ=38^\circ\cdot 4$$

$$\text{and } F=32+\frac{180}{100}\times 48^\circ=118^\circ\cdot 4$$

3. *A* sold *B* goods costing Rs 1000. *B* sells them to *C* for Rs 1440, making the same rate of profit as *A*. Find how much *B* paid for the goods. —P. S. C.

Assume that the rate of profit made by each is $r\%$ on cost. Then, *A* sells for Rs $1000\left(1+\frac{r}{100}\right)$, *i.e.*, Rs $(1000+10r)$; and *B* sells for Rs $(1000+10r)\left(1+\frac{r}{100}\right)$, *i.e.*, for Rs $\left(1000+20r+\frac{r^2}{10}\right)$, which is given to be Rs 1440.

$$\therefore \quad 20r+\frac{r^2}{10}=440\ ;\ \text{or, } r^2+200r-4400=0\ ;$$

$$\text{or, } (r-20)\,(r+220)=0.$$

$\therefore$ $r=20$, the other value -220, being absurd.

$\therefore$ *B* paid for the goods Rs 1200.

4. A rectangular pedestal of marble is to be polished on all faces except that resting on the ground. By putting three faces down in turn, the total surfaces to be treated are 412, 394 and 404 square feet, respectively. Find the dimensions of the pedestal. —P. S. C.

If the edges are a, b and c ft respectively, the data give :

$$2(ab+bc)+ca=412\ ;\ 2(bc+ca)+ab=394,$$

$$\text{and } 2(ab+ac)+bc=404.$$

Adding, $ab+bc+ca=242$. Subtracting this value from each of the above relationships,

$$b(a+c)=170\ ;\ c(a+b)=152\ ;\ a(b+c)=162.$$

Adding the first two, $a(b+c)+2bc=322$. $\therefore$ $2bc=322-162$ $=160$ and $bc=80$.

Similarly, $ab=90$ and $ca=72$.

$$\therefore\quad a=9\text{ ft}\ ;\ b=10\text{ ft}\ ;\ c=8\text{ ft}.$$

EXAMPLES 9

1. On a certain railway the second and third class fares are 7 pies and 3 pies per mile respectively. A person travels 100 miles in all, partly in the second class and partly in the third, and spends Rs 2 6as 8p. How far did he travel in the second class? [*Ans.* 41 miles

2. A rectangular verandah has an area of 51 sq. ft. If it were 2 ft wider but 2 ft shorter in length, the area would be 75 sq ft. Find its length and breadth. [*Ans.* 17 ft ; 3 ft.

3. The capital of a partnership between *A* and *B* is $\frac{5}{3}$ times the difference between their respective capitals. How many times is *A*'s capital of *B*'s? [*Ans.* 4

4. A traveller is paid a fixed salary *plus* a certain percentage on business introduced by him. In three successive months, the sales amounted to Rs 1000, Rs 1200 and Rs 1600 ; and the traveller received 4% in the second month, and 12% in the third month or Rs 30, above his first month's earnings. Find the fixed salary, and the rate of commission. [*Ans.* Rs 200 p.m. ; 5%

5. A milkman estimates that by selling 3 pints less than the milk he has for sale, he will realise $\frac{7}{8}$-th of what he would obtain by selling 2 pints less than his stock. How much would he realise by selling his entire stock at 2as per pint? [*Ans.* Re 1 4as

6. A man goes for a walk and does half the distance at $3\frac{1}{2}$ miles per hour ; and the other half at $4\frac{1}{2}$ miles per hour. If he had walked the whole way at 4 miles per hour, he would have taken 5 minutes less. What is the distance covered? [*Ans.* 21 miles

7. A number is first divided by 4, and then by 7, the respective remainders being 2 and 3. Find the remainders when the same number is divided first by 7 and then by 4. [*Ans.* 0 ; 2

8. An A. R. P. Warden has a number of volunteers under his control. He estimates that if he divides them into batches of 4, 2 will be left over. If the batches are placed on duty every day in the week in equal numbers, he finds that he has no work for 3 batches. If he divides up the total number of volunteers so as to have an equal number serving every day, find how many will be left over. If he drafts them in batches of four, find how many such batches will have no work. [*Ans.* 0 ; 2

9. In a factory adult workmen are paid proportionate wages for overtime on the basis of an 8-hour day ; children, however, are not permitted to work overtime. An adult is paid 8as a day, and a child 5as a day ; and the wages bill for a normal day is Rs 100. On a certain day, the adults work 2 hours overtime, and the number of boys employed is reduced by half ; and the wages bill stands at Rs 78 2as. Find the number of boys originally employed. [*Ans.* 200

10. Show that the product of any four consecutive numbers increased by 1 is a perfect square.

If this sum is 841, find the smallest number of the series.

[*Ans.* 4

11. The gradient of an inclined plane G is given by the formula $G = f.\frac{F-E}{F+E}$, where f is the co-efficient of friction, F is the total load, and E is the weight of the empty set. Find the best gradient for a horse track where the tubs weigh 5 cwt and carry 11 cwt of coal, the co-efficient of friction being equal to $\frac{1}{60}$.

[*Ans.* 1 in 115

12. A locomotive engine without a train can go 24 miles an hour, and its speed is diminished by a quantity which varies as the square root of the number of wagons attached. With four wagons its speed is 20 miles an hour. Find the greatest number of wagons that the engine can move. —I. P. S. [*Ans.* 143

13. A cistern holding 1200 gallons is filled by 3 pipes *A*, *B* and *C* in 24 minutes. The pipe *A* requires 30 minutes more than *C* to fill the cistern, and 10 gallons more run through *C* per minute than through *A* and *B* together. What time would each pipe take to fill the cistern by itself ?

[*Ans*. *A* : 70 ; *B* : 420 ; *C* : 40 minutes

14. Two workmen *A* and *B* are paid the same rate of wages per hour. They commence work on the same day, and each on that day works for the same number of hours.

On each succeeding day, *A* works for a quarter of an hour less and *B* works for 20 minutes less than on the day before. At the end of 5 days, the rate of *A*'s earnings to *B*'s is 31 : 30. How long did they work on the first day ? [*Ans*. 5 hr 40 min.

15. By running 900 yards and walking another 900 yards, a man covered 1800 yards in 10 minutes 30 seconds. If he had run at a rate 20 per cent slower and walked at a rate 20 per cent faster, he would have taken 10 minutes exactly. Find how long he took to run 100 yards and how long to walk 100 yards.

[*Ans*. 20 sec., 50 sec.

16. Show that the difference between any two numbers comprising of the same digits is exactly divisible by 9.

17. The salary of a traveller consisted partly of a fixed sum and partly of commission, which was proportional to the value of the orders obtained. In two consecutive years he obtained orders to the value of Rs 31,500 and Rs 45,000 respectively, and he received Rs 2545 and Rs 2950 respectively. If his salary for the next year was Rs 3200, what was the value of the orders obtained ? If he had obtained orders to the value of Rs 60000, what would have been his salary ? —I. Com.

[*Ans*. Rs 53333⅓ ; Rs 3400

18. A traveller gets a fixed salary, and also a commission of 1¾% on all orders he secures. He thus receives an annual income of £670. If he secures orders to the extent of about £407 per week, on an average, what is his fixed salary per year ?—I. Com.

[*Ans*. £300

19. A workman is able to save $12\frac{1}{2}$% of his wages ; but if his wages were raised 2s a week and his expenses were increased by 10%, his annual savings would be diminished by 17s 4d. What are the man's weekly wages, a year being taken as exactly 52 weeks ?—I. Com. [*Ans.* 26s 8d

20. What are eggs a dozen when two more in the shilling reduces the price by $\frac{4}{5}$d per doz ? —B. C. S. [*Ans.* 8d

21. A person started at half-past two and drove to a neighbouring village arriving there when the steeple clock indicated a quarter-past three. After staying 25 minutes, he drove back by a road $\frac{1}{4}$-th as long again at a rate twice as fast, reaching home at four o'clock. Determine how much the steeple clock was wrong. —I. Com. [*Ans.* 5 minutes too fast

22. A dealer buys oranges of two qualities, one at 1 shilling a dozen and the other at 8d a dozen. These are mixed up and he sells them at 15 per shilling and thereby makes 5% on his total outlay. Find the ratio of the number of oranges of the two kinds. —I. Com. [*Ans.* 2 : 5

23. Find a prime number of 3 digits, such that the digit in the unit place is equal to the sum of the other two, and if the other two digits be interchanged we still have a prime number of 3 digits. —I. Com. [*Ans.* 167

24. A man borrowed £20 from a money-lender agreeing to pay £24 in monthly instalment of £2 beginning at the end of the first month. Reckoning simple interest at r% *p. a.*, find the sum to which £20 amount in 1 year, and show that the money repaid together with interest, amounts to £$24 + \frac{11r}{20}$. —I. Com.

25. The driving wheel of a locomotive engine is $27\frac{1}{2}$-ft. in circumference and the fore-wheel 16 ft. Two particular spokes, one in each wheel, are observed pointing vertically upwards For what time must the engine travel at $13\frac{3}{4}$ miles per hour before the same two spokes are again simultaneously in the same upward vertical position ? —I. Com. [*Ans.* $43\frac{7}{11}$ sec.

26. Travelling at 30 m.p.h. a car covers 35 miles per gallon of petrol used. At a speed of 40 m.p.h., only 30 miles per gallon is covered. The car uses 13 gallons in all on a journey from *A* to *B* at 30 m.p.h. and returning from *B* to *A* at 40 m.p.h. How long did the complete journey take ?—P. S. C. [*Ans.* 12¼ hours

27. In a motor car race of 1 mile length, car *A* beats car *B* by 5 seconds, whilst car *B* beats car *C* by 110 yd. Had the speed of *C* been increased by 20% it would have tied with *A*. Find the speeds of the three cars in feet per second.—P. S. C.
[*Ans.* 132, 117⅓, 110 ft/sec.

***28.** An apple dealer sells to the first customer a number of apples equal to half of her stock plus half an apple. To the second she sells half her remaining stock plus half an apple, and similarly to the third, fourth and fifth. She then had 3 apples left. How many had she at first ? —P. S. C. [*Ans.* 127

***29.** The electric resistance of a wire of given material is directly proportional to its length and inversely proportional to the square of the diameter of its circular cross section. Calculate the length and diameter of a wire which is to have double the resistance, but only ⅓-rd the weight of a wire of the same material 53 ft long and 0·024 in. diameter.—I. Com.
[*Ans.* 43¼ ft ; ·0153 in. app.

30. A manufacturer has priced certain pieces of furniture. The largest sells at Rs 400 and the smallest at Rs 90. He wishes to alter the prices so that the largest will sell at Rs 260 and the smallest at Rs 60. Assuming that the new price *P* and the old price *Q* are connected by the relation $P = a + b.Q$, find the new prices of the furniture valued at Rs 125, Rs 150 and Rs 175.
—I. Com. [*Ans.* Rs 82 9as ; Rs 98 11 as ; Rs 114 13as app.

31. Where a railway goes around a curve the outer rail is raised to keep the train on the line. When the curve has a radius of *R* feet, the distance between the two rails is *W* feet, and the customary speed of the train is *V* miles an hour then the outer rail should be $\frac{W \times V^2}{1\cdot25 \times R}$ inches higher than the inner rail. Calculate how much higher the outer rail should be on a curve of radius 580 ft, if the rails are 4 ft 8½ ins. apart and the customary speed is 40 miles per hour. (Answer correct to 2 decimal places.) —P. S. C. [*Ans.* 10·39 ins.

*32. Two trains are travelling in opposite directions at uniform speeds. They each pass a telegraph post in the same time, and they pass each other in 5 seconds. If the lengths of the trains added together be 264 yards and the faster train's speed exceed that of the slower by 25%, find the length and speed of each train. —P. S. C.

[*Ans.* 48 m.p.h., 60 m.p.h. ; 352 ft. ; 440 ft.

SECTION FIVE

RATES

§ **The Method of Aliquot Parts.** When a multiplier is broken up into a number of parts, each of which is a factor of the preceding, the successive portions are known as aliquot parts. Thus, 37½ = 30 + 5 + 2½, in which 5 is ⅙-th of 30 and 2½ is ½ of 5. The aliquotisation will be denoted hereafter as follows :

$$37\tfrac{1}{2} = 30 + \underline{6}^* + \underline{2}^* \text{ ; also,}$$

$$17\tfrac{3}{4}\% = 10\% + \underline{2} + \underline{2} + \underline{10}.$$

Illustrations

1. Find the cost of 58½ cwt @ 18s 9¾d per cwt.

Here, 18s 9¾d = £1 *less* 1s 2¼*d* = £1 − (1s + $\underline{6}$ + $\underline{8}$)

Cost of 58½ cwt. @ £1		£58	10s	
Less	@ 1s	£2	18s	6d
	$\underline{6}$		9	9
	8		1	2⅝
		£55	0	6⅜

2. Find the cost of 241 tons 15 cwt 3 qr 17½ lb @ £8 13s 7¾d per ton.

Remembering that 241 = 12 × 20 + 1, and 15 cwt 3qr 17½ lb = 10 cwt (½ ton) + $\underline{2}$ + $\underline{10}$ + $\underline{2}$ + $\underline{2}$ + $\underline{4}$, we have,

	£	s	d
	8	13	7¾
			241
	2092	8	7·75
$\underline{2}$ of 1 ton	4	6	9·88
$\underline{2}$	2	3	4·94
$\underline{10}$		4	4·09
$\underline{2}$		2	2·05 −
$\underline{2}$		1	1·02 +
$\underline{4}$			3·26
	£2099	6s	8·99d

Note. The work is taken to two decimal places after the pence to obtain the result correct to the nearest *farthing*.

Reqd. cost = £2099 6s 9d

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*To be read *2 bar* ($\underline{2}$) &c., meaning ½ of the *immediately preceding* value; or $\underline{6}$ meaning ⅙-th of the immediately preceding value &c. &c.

Vulgar fractions should not be used, unless they are very simple; and the final result should invariably be given to the lowest current unit in trade, unless otherwise required.

§ Conversion of Measures. The following approximate equivalents may be used for conversion of measures—

1 Seer = 1 Kg − $\underline{15}$	1 lb = $\frac{1}{2}$ seer − $\frac{2}{9}$ chh.
1 Seer = 2 lb + 1 oz − $\underline{12}$	1 cwt = 1 md + $\underline{4}$ + $\underline{3}$ + $\underline{3}$ = $(1 + \frac{1}{4} + \frac{1}{9})$ md
1 Md. (Standard) = $\frac{1}{2}$ cwt + $\underline{2}$ − $\underline{16}$	1 Km = $\frac{1}{2}$ mile + ($\underline{4}$ − $\underline{3}$%)
1 ton = 20 md + $\underline{3}$ + $\underline{12}$	1 Metre $\overset{=}{\rightarrow}$ 1 yd + $\underline{12}$ + $\underline{8}$

Note. For conversion of the ordinary units of trade see Tables.

§ Rates. A fraction should be interpreted as a rate. Thus $\frac{6}{11}$ means 6 out of every 11; it may be Rs 6 in every Rs 11, or £6 in every £11, or 6 men in every 11 men, and so on.

Particular cases in which the denominator is 10 or a power of 10 are written in the form of *decimal* fractions. Thus $\frac{7}{10}$ is written as ·7, $\frac{783}{1000}$ as ·783 etc. In the special case where the denominator is 100, the fraction is written as a *percentage*. Thus $\frac{7}{100}$ is written as 7%, $\frac{83}{100}$ as 83%; and sometimes also fractions with denominator 1000 are written as so much *per mille*, meaning a thousand. Thus $\frac{17}{1000}$ is written as 17 per mille or simply as 17‰.

When fractions have denominators composed entirely of 9's they are written as *recurring decimals*. Thus $\frac{5}{9}$ is written $\cdot\dot{5}$, $\frac{19}{99}$ as $\cdot\dot{1}\dot{9}$. It is well known that their value is to be obtained by repeating the recurring portion to infinity, and can be directly written down to any degree of approximation.

§ Calculation of Rates. The calculation of Rates forms one of the most useful applications of arithmetic in everyday business. Rates are usually quoted in percentages, as in the cases of Discount, Brokerage; sometimes in vulgar or decimal fractions, as in Average Adjustments in marine insurance; and quite frequently in terms of the currency of the country, as in the case of Income Tax, Customs, etc. It should be recognised that all fractions represent merely rates, which are mutually interchangeable. Thus the fraction $\frac{1}{4}$, *i.e.*, 1 in every 4, might be treated either as ·25, or 5s in £1 or 4as in Re 1 or 25 cents in a Dollar or 25% or 250 per mille.

The following well known relationships may be noted :

$5\% = \frac{1}{20} = 1\text{s in £1}$ $\qquad 6\frac{1}{4}\% = \frac{1}{16} = 1\frac{1}{4}\text{s in £1}$

$2\frac{1}{2}\% = \frac{1}{40} = \frac{1}{2}\text{s in £1}$ $\qquad 3\frac{1}{3}\% = \frac{1}{30} = 8\text{d in £1}$

$1\frac{1}{4}\% = \frac{1}{80} = \frac{1}{4}\text{s in £1}$ $\qquad 3\frac{3}{4}\% = \frac{3}{80} = 9\text{d in £1}$

Also, $6\frac{1}{4}\% = \frac{1}{16} = 1\text{a in Re } 1$; $1\frac{1}{4}\% = \frac{1}{80} = \frac{1}{5}\text{th of } 6\frac{1}{4}\%$;

$8\frac{1}{3}\% = \frac{1}{12}$; $10\% = \frac{1}{10}$; $20\% = \frac{1}{5}$; $25\% = \frac{1}{4}$;

$33\frac{1}{3}\% = \frac{1}{3}$; $12\frac{1}{2}\% = \frac{1}{8}$; $16\frac{2}{3}\% = \frac{1}{6}$.

Further, $\frac{2\text{ as}}{\text{Re } 1} = \frac{1}{8} = 12\frac{1}{2}\%$; $\frac{4\text{as}}{\text{Re } 1} = \frac{1}{4} = 25\%$ etc. ;

$\frac{5\text{as } 4\text{p}}{\text{Re } 1} = \frac{1}{3} = 33\frac{1}{3}\%$; $\frac{2\text{as } 8\text{p}}{\text{Re } 1} = \frac{1}{6}$; $\frac{1\text{a } 4\text{p}}{\text{Re } 1} = \frac{1}{12}$.

And, $\frac{4\text{s}}{£1} = \frac{1}{5} = 20\%$; $\frac{5\text{s}}{£1} = \frac{1}{4} = 25\%$ etc. ;

$\frac{6\text{s } 8\text{d}}{£1} = \frac{1}{3}$; $\frac{3\text{s } 4\text{d}}{£1} = \frac{1}{6}$; $\frac{1\text{s } 8\text{d}}{£1} = \frac{1}{12}$;

$\frac{2\text{s } 6\text{d}}{£1} = \frac{1}{8}$; $\frac{7\text{s } 6\text{d}}{£1} = \frac{3}{8}$ etc. etc.

§ The easiest way usually to calculate rates is to break them up into suitable portions by the method of *aliquot parts*. Thus, to calculate $7\frac{3}{4}\%$, we break it up into $5\% + 2\frac{1}{2}\% + \frac{1}{4}\%$, *i.e.*, $5\% + \underline{2} + \underline{10}$. Integral rates may be calculated by multiplying the amount by the rate and pointing off two places.

Illustrations

1. 2% on Rs 844 1a = Rs 16·881 = Rs 16 14as 1p

[Multiply by 2 and shift the decimal point 2 places to the left]

2. $2\frac{1}{2}\%$ on Rs 6484 = Rs 162 1a 7p [Divide by 40]

3. $2\frac{1}{2}\%$ on £564 = 282s = £14 2s [Take $\frac{1}{2}$s in £1]

4. 5% on £836 5s 4¾d
5% on £836 = £41 16s [1s in £1]
5% on 5s 4¾d = 3¼d [$\frac{1}{20}$ of 64¾d] to nearest farthing.
£41 16s 3¼d

5. (i) 6¼% on Rs 8645 = Rs 540 5as [1a in Re 1]
(ii) 6¼% on Rs 538 10as 6p
6¼% on Rs 538 = Rs 33 10as
On 10as 6p, *i.e.*, 126p = 8p [$\frac{1}{16}$ of 126p]
Rs 33 10as 8p to nearest pie.

6. 5% on Rs 8385 6as 9p [5 = 6¼ − 1¼]
@ 6¼%... Rs 524 1a 5p
@ 1¼%... 104 13as 1p [1¼ = ⅕ of 6¼]
Rs 419 4as 4p
Or directly, 5% = $\frac{1}{20}$...Rs 419 4as 4p

§ It will be necessary for the student to be trained in writing down the practical result at sight, ignoring errors that cannot be taken into account in practice. It may be useful to the beginner to note + or − sign at end of a result, if the error neglected is near about a quarter of the range,—without a preponderance of either sign—and the errors balanced in the end. In commercial calculations it is sufficient to work at every stage correct to the lowest unit of calculation; although, in theory, such a course might slightly throw out the final result by the accumulation of errors. For greater accuracy the work should be continued to 1 or 2 decimal places after the smallest unit; or, the work conducted entirely in decimals correct to 4 decimal places at least, and the answer finally converted into trade units.

Illustrations

1. Find 6¼% on £83 6s 5½d [6¼% = 5% + ¼]
@ 5%...£4 3s 3·88d [∵ $\frac{1}{20}$th of 6s 5½d $= \frac{77\frac{1}{2}}{20}$d ⇒ 3·88d]
¼... £1 0s 9·97d [∵ ¼th of 3s 3·88d $= \frac{39{\cdot}88}{4}$d ⇒ 9·97d]
∴ @ 6¼%...£5 4s 1·85d *i.e.*, £5 4s 1¾d to nearest farthing

2. Find 3¾% on £38 14s 1½d $[3\frac{3}{4}\% = 3\% + \underline{4}]$

@ 3%... £1·1612
$\underline{4}$ ·2903
£1·4515 *i.e.* £1 9s 0¼d

3. Find 2¾% on Rs 6183 5as 7p, which = Rs 6183·349

@ 2½% *i. e.* $\frac{1}{40}$ ...Rs 154·5837
$\underline{10}$ 15·4584
Rs 170·0421 *i. e.* Rs 170 0a 8p

4. Find 7¾% on Rs 5813 10as 6p $[7\frac{3}{4} = 6\frac{1}{4} + \underline{5} + \underline{5}]$

@ 6¼%... Rs 363 5as 7·9p
$\underline{5}$... 72 10 8·8
$\underline{5}$... 14 8 6·6
Rs 450 8as 11·3p *i. e.* Rs 450 8as 11p

5. Find 8½% on Rs 868 6as 8p $[8\frac{1}{2} = 10 - \overline{1 + \underline{2}}]$

@ 10%··Rs 86 13as 5·6p Or, Rs 86·8417 in decimals
less 1% 8 10 11·4 − 8·6842
$\underline{2}$ 4 5 5·7 − 4·3421
Rs 73 13as 0·5p − 73·8154
i. e. Rs 73 13as

6. Find 2as 6p% on Rs 8084 $[2\text{as } 6\text{p}\% = \frac{1}{8}\% + \underline{4}]$

@ ⅛%··· Rs 10·105
$\underline{4}$ 2·5263
Rs 12·6313 *i.e.* Rs 12 10as 1p

7. Equivalent of Rs 8582 14as 9p @ 1s $6\frac{5}{32}$d to Re 1.

@ $\frac{1\text{s}}{\text{Re } 1}$ we get 8582·922.. shillings
i.e., £429 2s 11d
@ $\frac{6\text{d}}{\text{Re } 1}$... $\underline{2}$ 214 11 5½
@ $\frac{4}{32}$d/Re 1 4 9 4¾ +
@ $\frac{1}{32}$d/Re 1 1 2 4¼
£649 6s 1½d

8. Convert Rs 8654 10as into sterling @ 1s 6¼d to Re 1.

@ 1s to Re 1···		£432 14s 7½d
@ 6d	··· 2	216 7 3¾
@ ¼d	24	9 0 3¾
∴ @ 1s 6¼ to Re 1		£658 2s 3d

9. Calculate freight on a package 16 c.ft. 3^{I}, 4^{II} @ 27s 6d per steamer ton (40 c. ft.)

@ 40s per ton	...16s 3¼d	[*i.e.*, @ 1s per c. ft.]
∴ @ 20s...	... 8 1½d	
@ 5s...	... 2 0½d	
@ 2s 6d	... 1 0¼d	
@ 27s 6d	...11s 2¼d	

N. B. Notice that work is started with an imaginary rate 40s per ton (40 c. ft.) *i.e.* 1s per c. ft. or, 1d per Prime, for simplification of subsequent calculations.

§ Frequently rates are quoted, comprising of two or more successive rates. These, if used in sets in a large number of cases in the same office, may generally be reduced to single equivalent rates. Thus, interest @ 3% p.a. for 4 months is equivalent to 1%; a discount of 20% and 10% is equivalent to 28%; an insurance rate of ⅜% less 20% and 10% is equivalent to ·27%, and so on.

§ The current practice in most cases is to calculate correct to the nearest anna, sometimes to the nearest quarter-anna (*i.e.*, 3-pie) and rarely to the nearest pie. In writing down results the value of the errors made by taking the lower or the upper limit can be safely adjudged mentally in reference to the last working unit, as has been indicated in the above illustrations.

EXAMPLES 10

Find the value of the following correct to the nearest farthing or 3-pie.

1. 4% on £6843 10s; £202 8s 10d; £73 15s 9¾d; Rs 568; Rs 512 6as; Rs 2028 3as 9p; Rs 6142 12as 6p

[*Ans.* £273 14s 9½d; £8 1s 11½d; £2 19s 0½d; Rs 22 11as 6p; Rs 20 8as; Rs 81 2as.; Rs 245 11as 6p

2. 3½% on £516 2s 3¾d ; £208 9s 6½d ; Rs 242 5as 6p ; Rs 516 13as 9p

[*Ans.* £18 1s 3¼d ; £7 5s 11¼d ; Rs 8 7as 9p ; Rs 18 1a 6p

3. 4¾% on £21 3s 9¾d ; £73 18s 10½d ; Rs 648 10as 9p ; Rs 346 14as 6p

[*Ans.* £1 0s 1½d ; £3 10s 3d ; Rs 30 13as ; Rs 16 7as 6p

4. 3¾% on £15 6s 8¾d ; £92 13s 0⅛d ; Rs 284 12as 3p ; Rs 516 10as 9p

[*Ans.* 11s 6d ; £3 9s 5¾d ; Rs 10 10as 9p ; Rs 19 6as

5. 4½% on £20 18s 9½d ; £43 15s 8¼d ; Rs 2684 3as 3p ; Rs 5638 10as 9p

[*Ans.* 18s 10d ; £1 19s 5d ; Rs 120 12as 9p ; Rs 253 11s 9p

6. 7¾% on Rs 516 6as 6p ; Rs 2168 3as 6p

[*Ans.* Rs 40 0a 3p ; Rs 168 0a 6p

7. 7½% on Rs 281 2as 9p ; Rs 364 6as 9p

[*Ans.* Rs 21 1a 6p ; Rs 27 5as 3p

8. 5¾% on Rs 614 11as 6p ; Rs 508 7as 3p

[*Ans.* Rs 35 5as 6p ; Rs 29 3as 9p

9. 9½% on Rs 381 2as 9p ; Rs 643 5as 6p

[*Ans.* Rs 36 3as 6p ; Rs 61 2as

10. 12¾% on Rs 6418 5as 6p ; Rs 2086 12as 9p

[*Ans.* Rs 818 5as 6p ; Rs 266 1a

11. @ 9p in the rupee on Rs 6168 10as [*Ans.* Rs 289 2as 6p

12. @ 10½p in the rupee on Rs 15088 2as 3p [*Ans.* Rs 825 2as

13. @ 1s 9d to the rupee on Rs 648 2as 9p [*Ans.* £56 14s 3½d

14. @ 2s 3d in the £ on £430 18s 11¾d [*Ans.* £48 9s 7½d

15. @ 1s 6¼d to Re 1 for Rs 6184 4as [*Ans.* £470 5s 2½d

16. @ 1s $6\frac{27}{32}$d to Re 1 for Rs 5938 8as [*Ans.* £466 5s 3¾d

17. @ $ 4'90⅜ to £1 for £681 3s [*Ans.* $ 3340'19

18. @ 105'75 fr to £1 for £21 10s 6d [*Ans.* 2276'27 fr

19. Freight on 28¼ c.ft. @ 37s 6d on 50 c.ft. [*Ans.* £1 1s 2¼d

20. Freight @ 21s 6d per 50 c.ft. on 42½ c.ft. ; 23¼ c.ft. ; 61½ c.ft. [*Ans.* 18s 3¼d ; 10s ; £1 6s 5¼d

§ **Insurance.** In consideration of a *premium*, an *insurer* agrees to indemnify the *assured* in the event of certain contingencies to the extent of the *sum assured.*

Insurance companies collect the premiums, out of which they pay claims as they arise, and thus do the great social service of distributing the cost of such losses over the very much larger population of the goods and persons assured. Premiums are fixed up by actuaries who seek to provide for the cost of such claims according to theories of probability, the expenses of the insurance company and also some remuneration to it for the service. (*Vide* Section Eleven)

In life insurance, the contingency provided for usually is the assured reaching a specified age or earlier death. Death, of course, is a certainty, but the time of death cannot even be conjectured in a particular case. Premiums are worked out with the help of complicated statistical methods, and data of past experience of mortality, the anticipated rate of interest etc.

It is customary to quote such rates per Rs 1000 of sum assured per annum ; and to make an extra charge if premiums have to be accepted quarterly or monthly. In the case of a claim, the *entire sum* assured falls due together with any *bonus* that may have been declared during the tenure of the policy by way of distributing a share of the profit earned by the company amongst policy-holders.

§ The contract in all other forms of insurance is of a different character, the sum assured indicating the *maximum limit* of payment on a policy against actual or constructive loss. It is then conceivable that in many cases there will be no claim, and when there is a claim, the actual value of the loss sustained, on account of a risk that has been insured against, is payable up to the *limit* of the assured sum. An *average clause* usually provides that, in case of under-insurance, only that proportion of the actual loss may be claimed that the sum assured bears to the real value of the goods assured. In case of over-insurance, however, no amount greater than the sum assured can be claimed in case of loss. In case of *marine insurance only*, established usage sanctions the reimbursement of cost of insurance and 5% to 10% by way of anticipated profits.

Another fundamental difference between life insurance and other forms of assurance lies in the fact that a life contract is spread over a large number of periods, whereas the other contracts are made for short periods ; say, during a voyage or during a period awaiting shipment, or despatch hinterland &c. In marine insurance cargoes and freight are insured for definite voyages, ships are insured for voyages or for definite periods : and in fire insurance the rates quoted are annual, while special short period rates are quoted at certain fractions of the annual rates.

§ **Marine Insurance.** Certain special features in marine insurance require elucidation. The contract is for indemnity against perils of the sea ; and losses may be—(i) *Total Loss* ; (ii) *Particular Average loss* ; and (iii) *General Average loss*. In case of total loss, absolute or *constructive*, the insurer indemnifies the insured for the full value of the goods up to the sum assured, taking charge of the goods that may be salvaged. Cover is sometimes secured against *t. l. o.* i.e. total loss only, other kinds of losses described below being excluded from the contract. Partial losses in marine insurance are known as particular average losses. No partial loss can be covered in respect of corn, fish, salt &c. which are particularly liable to spontaneous damage on a sea voyage. In case of heavy machinery &c. the chances of any appreciable partial loss are remote. In both these cases, insurance is effected on *f. p. a.* (*free of particular average*) terms. Other commodities, ship and outstanding freight (which cannot be claimed if the cargo does not reach its destination) are usually insured on *w. p. a.* or, *w. a.* (*with particular average*) terms, covering the risk of partial losses during transit. Even in such cases, petty and vexatious claims are avoided by stipulating that no claim below 3% (in some cases like sugar, tobacco, rice &c. not under 5%) would be entertained. The extent of the indemnity is assessed on the difference in the value of the goods when sound and the damaged value. In a peril which endangers the safety of the ship, the cargo and the freight at the same time, the master of the vessel is entitled to incur extraordinary sacrifices or expenses for saving all the interests. The corresponding loss is known as a *General Average* (*G. A.*) loss and, under international custom, it is to be shared *pro rata* between all the parties exposed to the peril, in proportion to the respective values of their interests. The G. A. contribution called up from the different parties are re-imbursed by insurance companies under all types of insurance policies except those on *t. l. o.* terms.

Special risks like *War Risks* (*W. R.*), *Riots and Civil Commotion* (*R. C. C.*) risks &c. are also covered by insurance companies in return for special additional premia on the declared value in the policy.

§ The Indian practice is to calculate premia correct to the nearest anna. For obvious reasons, the amount assured is declared correct to the nearest rupee : and, when the rate is small, to the next higher ten-rupee.

In most cases the premiums are quoted with discounts. It may be convenient at times to calculate with the help of equivalent rates.

Illustrations

1. Calculate premium on Rs 5847 worth of tea shipped from Calcutta to London at $\frac{5}{8}$% less 20% and 10%.

Now $\frac{5}{8} = \cdot625$
less 20% $= \cdot125$
$= \cdot5$

The equivalent rate
$= \frac{1}{2}$% less 10%

@ $\frac{1}{2}$% Rs 5847 = Rs 29·235
less 10% = 2·9235
Rs 26·3115 Prem. = Rs 26 5as

Note. The calculation is usually made in *as p* and not in decimals. Decimal calculations, however, may be practised to attain quickness combined with accuracy.

2. A rate $\frac{3}{8}\%$ *less* 20% may be verified to be $\equiv 3‰$

„ $\frac{5}{16}\%$ *less* 10 & 5 „ „ „ $\rightarrow 3‰ - \underline{9}$

„ $\frac{3}{32}\%$ net $\equiv 1‰ - \underline{16}$

&c. &c. &c.

3. Jute worth Rs 58000 was insured for Rs 40000. Find the amount payable by the insurance company if a loss by fire is assessed at Rs 29000.

$$\text{Ratio} = \frac{\text{Rs } 40000}{\text{Rs } 58000} = \frac{20}{29}.$$

$$\therefore \quad \text{Loss made good} = \tfrac{20}{29} \text{ of Rs } 29000$$
$$= \underline{\underline{\text{Rs } 20000}}$$

4. For how much should a shipment worth Rs 19900 be insured to cover both cost of goods and insurance at $\frac{1}{2}\%$ net?

If the goods be insured for 100

Premium is $\underline{\frac{1}{2}}$

Corresponding cost of goods $99\frac{1}{2}$

$$\therefore \quad \text{Sum assured} = \text{Rs } 19900 \times \frac{100}{99\frac{1}{2}}$$
$$= \text{Rs } 20000$$

5. Goods of invoice value Rs 8973 10as are insured to cover both cost of goods and insurance at $\frac{5}{16}\%$ less 10% *w. a.*, and *W. R.* at $\frac{7}{16}\%$. Find the declared value.

If the declared value be $\underline{100}$

w. a. premium $\frac{5}{16}\cdot\frac{9}{10}$

W. R. „ $\underline{\frac{7}{16}}$

Corresponding cost of goods $99\frac{9}{32}$

$$\therefore \text{ Declared value} = \text{Rs } 8973{\cdot}625 \times \frac{100}{99\frac{9}{32}}$$
$$= \text{Rs } 8973{\cdot}6\ [1 + \tfrac{23}{32}\%] \text{ nearly (See p. 18)}$$
$$= \text{Rs } 8973{\cdot}6\ [1 + \tfrac{1}{2}\% + \underline{2} - \underline{8}]$$
$$= \text{Rs } 9038$$

Note. In practice the value declared will usually be Rs 9040, correct to the next higher ten-rupee. If the premium calculated on the premium is < Rs 5, it would be sufficient merely to add the premium on the cost to the cost, and take the next higher ten-rupee.

6. **Particular Average Adjustment.** 40 bales of jute bound from Calcutta to Dundee were insured for Rs 13000. At destination it was found that 3 bales had been damaged. These fetched £30 @ 1s 6d and the prevailing price was £25 per bale. Estimate the amount claimed under Particular Average.

Value of 3 bales, sound	...	£75
Do damaged	...	30
Loss		£45

$$\text{Ratio of Loss to Total Value} = \frac{£45}{£40 \times 25} = \frac{9}{200}$$

$$\therefore \quad P.\ A.\ \text{claim} = \frac{9}{200} \times \text{Rs } 13000 = \textbf{Rs 585}$$

Note that the amount of the claim is less than the actual loss, Rs 600, as there has been under-insurance.

7. **General Average Adjustment.** A steamer, damaged in a storm, was towed into port. Part of the cargo was jettisoned, and part was damaged, as indicated below. The G. A. adjustment was arranged by the shipowners who also advanced all expenses.

Contributories		*G. A. Statement*	
Ship, less cost of repairs	£15,000	Cost of towage G. A. statement etc.	£2,000
Freight	1,000	Cargo *A* jettisoned	200
Cargo *A*	800	„ *B* damaged : sound value	500
Do. *B*	1,200	„ *C* lost	1,300
Do. *C*	2,000		

Total *G. A.* loss = £4,000; total value of contributories = £20,000.

$$G.\ A.\ \text{ratio} = \tfrac{4000}{20000} = \tfrac{1}{5}$$

Shipowner pays	$\frac{1}{5}$ of £15000 =	£3000 ;	receives	£2000
Charterer (freight)	$\frac{1}{5}$ of £1000 =	200	„	——
Cargo *A*	$\frac{1}{5}$ of £800 =	160	„	200
„ *B*	$\frac{1}{5}$ of £1200 =	240	„	500
„ *C*	$\frac{1}{5}$ of £2000 =	400	„	1300
	Total	£4000		£4000

Net amount paid by shipowners		£1000
,, ,, ,, ,, charterer		200
		£1200
Net amount received by	*A*	£ 40
,, ,, ,, ,,	*B*	260
,, ,, ,, ,,	*C*	900
		£1200

Assuming that all risks were fully insured, the shipowner and the charterer would be re-imbursed by their respective insurers. In practice, the insurers will pay up all contributions due from owners of the ship, the cargo and freight ; and collect from the Average Adjuster all payments due to them, including sale proceeds of salvaged goods.

EXAMPLES 11

1. A person aged 30 wishes to insure his life for Rs 10,000. What will be the annual premium on—
   (i) a 20 years' Endowment Policy @ Rs 50 10as per mille ;
   (ii) a 20 years' Limited Payments Whole Life Policy @ Rs 32 10as per mile ;
   (iii) a Whole Life Policy @ Rs 30 10as per mille ?
   [*Ans.* (i) Rs 556 4as (ii) Rs 326 4as (iii) Rs 306 4as

2. Calculate quarterly and monthly premiums on (i) above if an addition of 5% is to be made on the former, and of $7\frac{1}{2}$% on the latter. [*Ans.* Quarterly : Rs 146 ; Monthly : Rs 49 13as

3. If a person aged 30 years is prepared to spend up to Rs 54 a month, find the maximum cover that he might obtain (correct to the nearest Rs 500) on the above terms.
   [*Ans.* (i) Rs 11,500 (ii) Rs 20,000 (iii) Rs 21,000

4. Calculate marine insurance premium (*w.a.*) on a shipment of Rs 15,800 worth of cotton textiles from Bombay to Chinese Ports @ $\frac{5}{8}$% less 5% *plus* $\frac{1}{4}$% to cover risk of craft. For how much should the shipment be insured, so that both the goods and the cost of insurance may be covered ? [*Ans.* Rs 133 5as ; Rs 15,934

5. Find the marine insurance premium (*f. p. a.*) on a shipment of wheat worth Rs 14,825 from Karachi to United Kingdom @ $\frac{7}{16}$°/o less 5% *plus* *W. R.* @ $\frac{3}{16}$%. Find the declared value and the premium thereon, if the cost of insurance is also to be covered ? [*Ans.* Rs 89 7as ; Rs 14,918 ; Rs 90

6. What is the marine insurance premium (*w. a.*) on a shipment of Kandahar Wool from Karachi to Hamburg valued at Rs 12,650 ? The rate is $\frac{11}{16}$% less 5% *plus* $\frac{1}{8}$% for Transhipment Risk. [*Ans.* Rs 98 7as.

7. A ship worth Rs 18,000 is wrecked ; $\frac{1}{8}$th belonged to *A*, $\frac{1}{4}$-th to *B* and the rest to *C*. Find what loss each of them will sustain if the ship is insured to the extent of $\frac{3}{5}$-ths of its value. —I. Com. [*Ans.* *A*-Rs 900 ; *B*-Rs 1800 ; *C*-Rs 4500

8. What premium at 3% will be paid for insuring a ship worth Rs 12,000. What sum must be insured so that in case of loss the value of the ship and the premium may be recovered ? —I. Com. [*Ans.* Rs 360 ; Rs 12371

9. Calculate the total fire insurance premium payable in respect of cotton mill (Preparing, Spinning & Weaving Department).

	Value Rs	Rate %
Pucca Buildings used for Doubling, Reeling, Bundling, Baling &c.	18000	3as
Contents of above	20000	4as
Kutcha Yarn Conditioning Cellar	1500	6as
Contents of above	6500	4as
Carding Rooms	8000	6as
Contents of above	10000	7as

[*Ans.* Rs 179 6as.

10. From the details below calculate the total fire insurance premium on Jute Godowns and/or Sheds not forming any part of, nor used in connection with Pressing or Assorting Premises.

	Value Rs	Annual Rate % As
(1) Buildings (Class I) for storage of jute in pressed bales only.	32 000	28
(2) Buildings (Class II) for storage of Jute loose and/or in drums	10 000	48
(3) Peels of Jute consisting of Bales only in Open Sheds.	10 000	72
(4) Extra on above for not being protected by a covering of paulins.		8

The cover on items (1) and (2) will be for 1 year ; and that on (3) and (4) for 2 months, the rate thereof being $\frac{3}{8}$-ths the annual rate. [*Ans.* Rs 1047 8as

11. Cargo worth Rs 46,500 was insured for Rs 30,000. Estimate the amount payable by the Insurance Company on a claim made in respect of a loss of Rs 10,695. [*Ans.* Rs 6900

12. A floating policy is issued in respect of a Shellac Factory for Rs 50,000. The rate chargeable is 20% above that applicable to the most highly rated risk in the range, which is 8as % on buildings (and contents) where grinding etc. are carried on. A loss occurs by fire in a portion of the buildings, valued at Rs 10,000, the rate for which by itself would have been 6as%. If the loss is assessed at Rs 8640, and the value of the stock and buildings is Rs 60,000, find the loss really borne by the assured.
[*Ans.* Rs 1462 8as

13. An exporter of goods valued at Rs 15,500 insures them for Rs 20,000, paying a premium of ⅝% less 20% and 10%. There is a total loss, and the company agrees to pay the full value of the goods *plus* 5% thereon as estimated profit. Find the loss to the shipper, if any. [*Ans.* Gain Rs 754 12as

14. In January 1940, the London Underwriters' Schedule of rates for War Risks insurance was as follows : East and South Africa including and up to Cape Town—Allied flag 20s per cent, Neutral flag 15s per cent, East and Far East—Allied or Neutral flag 15s per cent. Calculate *W. R.* premia on the following shipments from London :

Cape Town	£3400	Allied Vessel
Gulf of Guinea Ports	£4650	Neutral Vessel
Aden	£5680	Allied Vessel
Calcutta	£4340	Neutral Vessel

[*Ans.* £34 ; £34 17s 6d ; £42 12s ; £32 11s

15. A reinsurance on Rs 50,000 is effected at the tariff rate of 5as % subject to a discount of 40%, *plus* an extra anna 1⅛% for cover against risk of Riots and Civil Commotion. Calculate the total amount of the premium. [*Ans.* Rs 128 15as

16. Cargo worth Rs 24,200 is damaged at sea and realises Rs 10,000 at destination. It was insured for Rs 18,000 @ 10as%

plus War Risk insurance at 3½%. Calculate the saving in premium effected by under-insurance, and the loss suffered by the shippers on that account. [*Ans.* Rs 255 12as ; Rs 3638

§ Customs, Port Charges etc. When cargoes are awaiting shipment at the docks, or are to be cleared from the docks, the space they occupy, and the services rendered by the Port Authorities during the time, are charged for.

For purposes of state revenue, and for the purpose sometimes of affording protection to indigenous industries, duties are charged on goods imported or exported. The rates are sometimes levied on weight or measurement, when they are called *specific* duties ; and sometimes on the value of the goods, when they are described as *ad valorem*. The rate charged on the value of an import is based on its *c. i. f.* value in the absence of a declared Tariff value. The list of rates is known as the Tariff Schedule, an illustrative extract from which is given in the Tables appended. The U. K. rates in the Tables refer to exports to, and imports from the United Kingdom.

§ Consular Charges. Representatives of foreign Governments are authorised to levy certain fees for *vise*-ing documents of export to their respective countries. Below are given specimen rates charged by some consulates in Calcutta.

U. S. A. : $ 2·50 for consular invoice in quadruplicate.
$ 1·00 for certificates of shipment of American goods back to the country of origin.

France : Fcs. 14·40 for *vise*-ing a Consular Invoice.

Italy : Lire 10 for *vise*-ing documents.
Lire 20 for legalising documents.

The examples following show the nature of such charges. Calculations are usually made correct to the nearest anna. Consular charges are exchanged into local currency at the day's rate of exchange, or at specified rates declared by relative Governments from time to time.

*EXAMPLES 12

1. 1700 bundles of iron weighing 85 ton 8 cwt 1 qr 3 lb are discharged at the Calcutta Jetties. Calculate the following Port Charges thereon :

Landing	@ Re 1 4as per ton or part thereof
River Due	@ 12as ,, ,,
Differential Toll	@ 12as ,, ,,
Surcharge	@ $12\frac{1}{2}$% on River Due.

[*Ans.* Rs 244 9as

[Note. 1 Bdle. has average weight $= \frac{\text{85 tons 8 cwt}}{1700} = \frac{1}{20}$ ton $+ \frac{1}{4250}$ ton + smaller quantities. $\Rightarrow$ 1 cwt $+ \frac{1}{53}$ qr app.]

2. In the above case, 75 bundles were landed on the 1st March 1939, 430 on the 2nd, 960 on the 3rd and 235 on the 4th *idem*. Delivery was taken on the 8th March 1939. The bundles are assumed to be of the same weight. Find the rent charged by the Port Commissioners @ 5as per diem per ton or part thereof up to 3 days after 3 clear days after delivery ; and @ 10as per diem per ton or part thereof thereafter. [*Ans.* Rs 61 4as

3. A consignment of 20 cases measuring altogether 199 c.ft. 8' are discharged at the Calcutta Jetties. Freight having been charged by measurement, calculate the following charges payable to the Port Commissioners :

Landing	@ Re 1 per 40 c. ft. or part thereof.
River Due	@ 60% of Landing Charges.
Differential Toll	@ 60% of Landing Charges.
Surcharge	@ $12\frac{1}{2}$% of River Due.

[*Ans.* Rs 11 6as

4. In the above instance, 8 cases are landed on the 30th August 1936, 4 on the 31st *idem*, and 8 on the 1st September 1936. Delivery is taken on the 6th September. No rent is charged for delivery within 3 clear days of discharge at the jetties ; 5as per diem per 40 c. ft. or part thereof being charged for *the subsequent days up to 3 days, and at double the rate thereafter. Calculate the value of the rent bill. [*Ans.* Rs 5 5as

5. Calculate Port Charges on a shipment of 830 ton 15 cwt of Coal on the following bases :

Shipping Charges	@ 8as per ton or part thereof.
Unloading ,,	@ 3as ,, ,, ,, ,,
Dock Removal Charges	@ 6as ,, ,, ,, ,,
Shipment Rent per week	@ 1anna ,, ,, ,, ,,
River Due	@ 8as ,, ,, ,, ,,
Surcharge	@ 1anna ,, ,, ,, ,,

The stack awaited shipment at the jetties for 4 weeks.

[*Ans.* Rs 1558 2as

6. If the shipment in No. 5 above was under a certificate from the Coal Grading Board, it should be allowed a rebate of 50% of the River Due and exempted from the Surcharge. What would be the Port Charges in these circumstances ?

[*Ans.* Rs 1298 7as

7. Draw up the schedule of Port charges on a shipment of steel sheets in 10 bundles, weighing 1 ton 12 cwt 1 qr each, at the following rates :

Shipping Charges	@ Rs 3 12as per ton or part thereof.
Unloading Charges	@ Re 1 4as ,, ,, ,, ,,
Shipment Rent for 3 weeks	@ 15as (per week) ,, ,, ,, ,,
Dock Removal	@ Rs 2 13as ,, ,, ,, ,,
River Due	@ Re 1 4as ,, ,, ,, ,,
Surcharge	@ 2as 6p ,, ,, ,, ,,

[*Ans.* Rs 204 9as

8. 3600 lbs of tea were warehoused in the Tea Warehouses of the Port of Calcutta, for sale and shipment. Calculate charges thereon at the following rates :

Receiving at warehouse, including wharf toll	per 90 lb......6 pies
Rent, including cost of laying down, opening before and closing after broker's inspection, for the first month or part thereof ...	per 90 lb......3 annas

After the first month per week or part of a week ... ... per 90 lb......9 pies

Shipment Charge ... ... per 100 lb......1 anna

Removal charge equal to 75% of the Shipping charge for Tea removed from a Warehouse to a Shipment Shed ... ...

The Chests were in the Warehouse for 1 month 19 days, after which the lot was shipped. [*Ans.* Rs 18 5as

§ **Freight.** The charges for transporting goods by Rail or Steamer are usually made in accordance with fixed schedules of rates. Bulky goods are charged on the basis of the space occupied, and heavier goods on the basis of weight, the carriers usually reserving to themselves the right to charge the rate giving the higher return.

In marine transport the rates quoted in the Calcutta market, as in many other maritime centres, are in shillings per ton. The *steamer ton*, however, is generally taken to be 20 cwt by weight or 50 cubic feet by space. In particular cases, the *steamer ton* is also taken to vary from 10 to 20 cwt, and it is customary in many European trades to charge 40 c. ft. to the ton. A Tonnage Schedule is to be found in the appended Tables.

In railway transport in India rates are quoted on various bases. Goods are classified, and rates are fixed in pies per mile per maund mainly on considerations of "what the traffic will bear". There are also special rates between important trading centres, which are lower than the ordinary *class rates* so that traffic between them might be encouraged. The lowest rates in this connection are quoted, for obvious reasons, between ports, which sometimes work out at as low a figure as ¼-th of the *class rate*. There are also *schedule rates* for particular commodities which are lower for longer distances.

In India, it is customary to charge for fractions of 10 seers as

10 seers in most railways and inland steamer companies, calculations being made correct to the nearest anna.

The methods of calculation are shown below.

Illustrations

1. Calculate freight on a shipment of 34½ cwt of soap in bags at 18s 6d per *steamer ton* (15 cwts).

15 cwts are charged......	18s 6d
30 „ „ „	£1 17
3 „ „ „	3 8
1½ „ „ „	1 10
∴ Reqd. charge ...	£2 2s 6d

2. Calculate freight on 67½ c. ft. of raw silk at 29s 6d per *steamer ton* (50 c. ft).

@ 50s per ton the charge would be	£3	7s	6d
@ 25s „ „ „ „ „ „	£1	13s	9d
@ 5s „ „ „ „ „ „		6s	9d
	£2	0s	6d
less @ ½s „ „ „ „ „ „			8d
∴ Required charge ...	£1	19s	10d

EXAMPLES 13

Calculate freight in the following cases :

1. Bales of piecegoods weighing 36 md 13 seers from Bombay to Lahore at Rs 3 9as 2p per md. [*Ans.* Rs 130 7as

2. 589 md 18 seers of wheat from Lyallpur to Karachi at 11as 8p per md. [*Ans.* Rs 429 14as

3. Gunnies weighing 35 md 7½ seers from Calcutta to Delhi at 10as 2p per md. [*Ans.* Rs 22 6as

4. Wheat weighing 583½ cwt from Karachi to United Kingdom at 24s per ton (18 cwt). [*Ans.* £38 18s

5. 438½ cwt of cotton seeds from Bombay to London at 21s per ton (13 cwt). [*Ans.* £35 8s 4d

6. The rate of freight from Calcutta to Liverpool is 37s 6d per ton (40 c. ft). Make out a Table of Nine Values and find with the help thereof, or otherwise, charges on the following shipments :

A *Packages measuring*	B *Packages weighing*
(*a*) $75\frac{3}{12}$ c. ft.	(*a*) 15 cwt 3 qr
(*b*) $38\frac{7}{12}$ c. ft.	(*b*) 9 cwt 1 qr
(*c*) $26\frac{6}{12}$ c. ft.	(*c*) 32 cwt 2 qr
(*d*) $19\frac{3}{12}$ c ft.	(*d*) 46 cwt 3 qr
(*e*) $22\frac{5}{12}$ c. ft.	(*e*) 28 cwt 1 qr
(*f*) 15 c. ft.	(*f*) 17 cwt 2 qr

[*Ans*. A. £3 10s 7d ; £1 16s 2d ; £1 4s 10d ; 18s ; £1 1s : 14s 1d
B. £1 9s 6d ; 17s 4d ; £3 0s 11d ; £4 7s 8d ; £2 13s ; £1 12s 10d

7. The rate of carrying a ton of coal over the E. I. Railway from Jherriah to Benares is Rs 2 11as ; and from Jherriah to Bombay it is Re 1 0a 10p ; and to Cawnpore it is Re 1 10as 6p per ton. Compare the cost of coal freightage to a proposed Mill at each of these places on the basis of an average consumption of 15000 tons a year.

[*Ans*. Benares Rs 40312 8as ; Bombay Rs 15781 4as ;
Cawnpore Rs 24843 12as

8. The following is the schedule of rates over the M. S. M. Railway for grains, seeds etc.

For the first 75 miles...$\frac{1}{3}$ pie per md per mile
,, distance between 76-150m...$\frac{1}{4}$,, ,, ,, ,,
,, ,, ,, 151-225m...$\frac{1}{5}$,, ,, ,, ,,
,, ,, ,, 226-300m...$\frac{1}{6}$,, ,, ,, ,,
,, ,, ,, 301-400m...$\frac{1}{7}$,, ,, ,, ,,
,, ,, ,, 401-500m...$\frac{1}{8}$,, ,, ,, ,,
etc. etc.

Calculate freight on the following :

(*a*) 23 md 8 seers over 318 miles [*Ans.* Rs 8 15as

(*b*) 67 md 19 seers over 118 „ [*Ans.* Rs 12 9as

(*c*) 93 md 10 seers over 480 „ [*Ans.* Rs 46 6as

(*d*) 22 md 16 seers over 178 „ [*Ans.* Rs 5 13as

9. Calculate freight and other charges on jute from Chandpur to Chittagong for shipment.

Freight 9as per bale ; Jetty Charge 1a 3p per bale ; and River Due at 4as per bale.

A rebate is allowed on Jute freight over the A. B. Railway at the following rates on fully pressed bales.

20-30 thousand bales in a year (July-June)	6 pies p. bale
30-40 „ „ „ „	1a „ „
40-50 „ „ „ „	1½as „ „
50-100 „ „ „ „	2as „ „
100 thousand bales and above	3as „ „

Calculate also the ralative rebates.

(*a*) 35000 bales (*b*) 89000 bales (*c*) 120000 bales

[*Ans.* Freight etc.—Rs 31171 14as ; Rs 79265 10as ; Rs 106875
Rebate— Rs 2187 8as ; Rs 11125 ; Rs 22500

SECTION SIX

TRADE ROUTINE

§ **Invoicing.** Following is a specimen Invoice, the details and form of which should be carefully noted. It is an export invoice ; an inland invoice is similar in form, but usually requires fewer details.

Invoice of 20 Cases Poplin, shipped per S. S. City of Calcutta from Liverpool to Calcutta, by order and on account and risk of Wagner & Co., Calcutta.

Insurance effected Indent No. 513.

Mark	Serial No.	Particulars	Rate	£	s	d	£	s	d	£	s	d
S. W. Cal.	1-12	12 cases contg. 240 pcs. Self colour Poplin 38" 9540 yds	8d	318	0	0						
	13-18	6 cases contg. 120 pcs. Cord Poplin 34" 4800 yds.	10d	200	0	0						
	19-20	2 cases contg. 40 pcs. Poplin with floral design 34" 1600 yds	9d	60	0	0	578	0	0			
		Charges :										
		Packing 20 cases (4'×3'×4') @ 6s		6	0	0						
		Carriage		3	4	0						
		Dock Dues		1	2	6						
		Loading charges		3	1	0						
		Freight to Calcutta 960 c.ft. @ 22s 6d plus 10%		29	14	0						
		Marine Insce. on £700 @ 5s 6d% plus Stamp		2	0	0						
		Commission @ 2½%		14	9	0	59	10	6	637	10	6
		Six Hundred Thirty-Seven Pounds Ten Shillings Six-Pence only.										

E. & O. E.

18th Feb. 1940. James Wiggins

Manchester.

§ In Inland Trade the following terms of payment are in common use :

Prompt cash—Payment to be made within a day or two, without discount.

C. O. D.—Cash on Delivery. *V. P. P.*—Cash on delivery by post. (Value Payable Post).

Net cash—Payment within about a week, without discount.

2% 1 *mo.*—2% Discount allowed, if payment is made within 1 month.

Prompt Cash less 3 mos.—Discount, equivalent to interest at market rate for 3 months, allowed if payment is prompt.

Monthly account—Accounts to be submitted and settled within the first week of the following month.

§ In Foreign Trade payment is arranged against Bank Credit or by means of Bills of Exchange (B/E). Quotations of prices are made frequently to include some of the charges incidental to exportation. The usual charges are given below.

1. Packing.
2. Carriage to Dock
3. Dock Dues & Duty
4. Loading Charges
5. Freight
6. Marine Insurance
7. Landing Charges.
8. Dock Charges and Duty.
9. Warehousing
10. Transport
11. Commission

The charges are to be borne by the buyer or the seller according to the terms quoted :

loco—Price at place of purchase. Buyer to bear all charges.
f. a. s.—Free alongside ship. Buyer to bear charges from 4 to 11
f. o. b.—Free on board. ... ... 5 to 11
c. & f.—Cost and Freight. ... ... 6 to 11
c. i. f.—Cost, Insurance and Freight. ... ... 7 to 11
Franco—Free of Charges. 11

§ It is, however, customary for the exporter to advance money on charges incidental to shipment up to placing the goods on board the vessel, usually including the cost of freight and insurance, and to realise them on the invoice, which is accompanied by a Bill of Exchange for the amount of the invoice.

§ The above invoice may be examined to obtain prices other than *loco* on the basis of the charges shown.

1 Case Self-colour Poplin 38″

[illegible] pcs..............795 yd..............£26 10s

@ *loco* price ... 8d per yd			
Charges :			
Cost of 1 Case	6s		
Carriage (proportionate)	3s 2·4d		
Dock Dues, say	2s 3d		
Loading chgs., say	3s 1d	15s 6·4d	
f. o. b. cost 795 yd...		£27 5s 6·4d	
∴ *f. o. b.* price			8¼d p. yd.
Freight : 48 c. ft.		£1 9s 8·4d	
c. f. cost 795 yd.		£28 15s 2·8d	
∴ *c. f.* price			8½⅛d p. yd.
Insurance on £30		1s 7·8d	
c. i. f. cost 795 yd.		£28 16s 10·6d	
∴ *c. i. f.* price			not < 8¾d p. yd.

Expenses, not specially covered by the agreed price, are charged on the invoice. Thus, in a *loco* invoice, as in the illustration above, all incidental expenses advanced by the shipper are shown on the invoice. If the terms were *f. o. b.* Liverpool, the price would be enhanced, but in the invoice only freight, insurance and commission could be shown as charges. Similarly, in a *c. i. f.* invoice nothing other than commission will be shown under charges, although the *c. i. f.* price would be a further increase on the *loco* price.

§ When goods are sent on consignment the consignee arranges for the sale of the goods, deducts incidental expenses and his own commission, and remits the balance of the proceeds to the consignor along with an Account Sales (A/S).

Account Sales

To A. B. Co., Delhi.......................................Cr.

A/S of 3 cases of Shoes received from A. B. Co., Delhi, on the 15th March 1941 for sale on consignment.

	Rs as p	Rs as p
50 pairs @ Re 1/-...	50 0 0	
200 „ @ Re 1/4...	250 0 0	
60 „ @ Re 1/8...	90 0 0	390 0 0
Charges :		
Cartage and coolie Rs	6 7 0	
Warehousing	21 3 0	
Commission @ 2%	7 13 0	35 7 0
Net proceeds		354 9 0
Less remitted on 31-3-1941		200 0 0
E. & O. E.		Rs 154 9 0

Dated Calcutta, C. D.
the 17th April, 1941 pp. *Desh Shoe Stores*

§ Adjustments sometimes require to be made between traders for overcharges or undercharges, wrong casting, errors in calculation etc. If the consignee has been overcharged, for example, he is entitled to a Credit Note from the consignor for the corresponding amount. It the difference goes against the consignee, he is sent a Debit Note for the relative amount:

Credit Note

Messrs. R. C. Banerjee & Co., Calcutta
Credited by the Delhi Soap Factory Ltd., Delhi.

	Rs as p
1 case Washing Soap returned not being up to specification	28 7 0
Overcharged in Invoice No. 83 @ 2 as per cwt on 2 ton 10 cwt	6 4 0
	34 11 0
Less Discount @ 12½%	4 5 6
	Rs 30 5 6

Debit Note

Messrs. R. C. Banerjee & Co., Calcutta
Dr. to the Delhi Soap Factory Ld., Delhi.
To undercasting of Invoice No. 83 Rs 10 0 0

EXAMPLES 14

Make out invoices in proper form from the data below :

1. Buyer—The Steam Publishing Co., Sellers—Papers Limited. Terms—2% 1 mo. Delivery—*ex* Seller's godown. 120 Reams of Royal Size Paper (28 lb) @ 4¾as per lb. 10 Reams Royal Size Rough Cover (34 lb) @ 4¼as per lb.

[*Ans.* Rs 1087 13as.

2. Seller—Indian Trading Co., Calcutta. Buyer—Lucknow Co-operative Stores. Terms—Prompt Cash *less* 3 mos. Delivery *ex* Seller's godown. 15 cwt hardware @ Rs 8 8as per cwt, 2 doz English Brushes @ Rs 10 8as per doz. 6 doz. packets of assorted Distempers @ Rs 12 per doz. Cartage to Railway Station Rs 18 12as. Freight Rs 56 4as. Miscellaneous expenses Rs 5.

[*Ans*, Rs 300 8as

3. Buyer—Universal Trading Co., Calcutta. Seller—Exporters Limited, London. Terms—Price *c. & f.* : Insurance effected ; Documents against acceptance ; 3 mos. net cash ; payable in sterling. Shipment Mark—$\dfrac{\Delta}{\text{CAL}}$. Items : 4 bdles of Packing Paper—12 cwts—Price—3¾d per lb. Insurance @ 1⅜% on £20, plus stamp 6d.

[*Ans.* £21 6s

4. R & Co. of Calcutta bought of F & Co. of London the following goods on 13th Sept. 1928, terms 2% cash :—

2 pcs. Bleached Calico 1579...186 yd @ 4½d ; 1 pce. Unbleached Calico 92½yd @ 3½d ; 3 pcs. Flannel 795...198 yd @ 10¾d ; 6 pcs. Flannellette 154...394½yd @ 6½d. Trade discount on first two items 7½%, and 5% on the rest.

—I. Com. [*Ans.* £22 13s 2d

5. Make an Account Sale of 160 Chests of Tea *ex* S. S. Mayurpankhi sold by Messrs H & R for account of Messrs. S. C. & Co. :—

55 chests, net weight 3296 lb @ 2s 1d ; 40 chests net weight 2397 lb @ 2s 1½d ; 30 chests net weight 1804 lb @ 2s 2d ; 35 chests net weight 2099 lb. @ 2s 2½d ; less 3 months' discount @ 5% p.a.

Charges :—Entries, Dock Charges and Rent £16 13s 8d ; Import Duty on Net weight at 1s per lb ; Insurance on £1150 at 4s 2d% ; Brokerage ¼% ; Commission 2½%. —I. Com.

[*Ans.* £485 13s 6½d

§ **Price-fixing.**—Elaborate calculations usually require to be made to determine the cost price of an article to the manufacturer, on the bases of *direct charges* and *indirect charges* incurred in the manufacture.

A certain percentage is added for profit, and goods are sold inclusive to the wholesaler at the price. Between the wholesaler and the consumer a large number of middlemen intervene, each charging his own profit on the cost price. Besides the charges of these people, the goods have also to pay various other charges at different stages before they actually reach the consumers, *e.g.*, transport charges, insurance charges, port and dock dues, customs etc. ; and each addition increases the price.

In quoting prices for the export market, consideration has to be given to the different expenses *ex* warehouse—like cartage, etc.—that may have to be incurred by the exporter, in accordance with the conditions of the sale.

In making a foreign tender, such expenses, local customs and other expenses and usually also customs and other expenses at port of delivery or up to the place of delivery, have to be estimated beforehand and added to the total price of the goods at home.

§ **Profit and loss.** A tradesman has to incur certain expenses before goods are ready for sale. Some of these directly increase the cost price, and are direct charges. Others have the effect of reducing gross profit, which is represented by the difference between cost (C.P) and selling prices (S. P.). This reduced profit is known as the net profit.

In the case of a manufacture, for example, the cost of raw materials, inward freight, cost of power used, wages etc. are Direct Charges. But the salaries paid for supervision and management, rent, rates and taxes of offices and godowns, advertisement and the one hundred and one incidental expenses, not directly chargeable to production, but necessarily incurred for earning a profit in the ordinary course of business, are regarded as Indirect Charges.

In the case of a retailer, expenses incurred for the purpose of making the goods available for sale, may also be partially direct and partially indirect. In Departmential Stores, certain indirect charges are allocated to the different departments, usually in proportion to their turnover.

The retailer's margin of profit is usually in the form of discounts *trade* and *cash*. *Deferred rebates* are sometimes also allowed in certain trades, generally calculated at an agreed rate on the turnover of the previous accounting year.

§ Profits and losses are usually calculated as percentages of the cost price. In retail trade it is sometimes calculated on sales or turnover. The conversion of profit on sales to profit on cost and *vice versa* is simple. If x be the profit % on sales, the percentage profit on cost $= \frac{100x}{100-x}$; and if y be the percentage profit on cost, the corresponding profit % on sale $= \frac{100y}{100+y}$.

Thus, 20% on cost would correspond to $\frac{100 \times 20}{100+20}$ or $16\frac{2}{3}\%$ on sale and 20% on sale would correspond to $\frac{100 \times 20}{100-20}$ or 25% on cost. In case of loss, the signs of x and y are changed.

§ Goods are usually sold at discounts off the listed or marked price (M. P.). The M. P. reduced by discount is the S. P., while the C. P. is increased by the profit to arrive at the S. P. Problems involving profits and losses are best tackled by the Method of Proportional Numbers, choosing a convenient item to correspond to 100.

Illustrations

1. For how much should an article costing Rs 20 be sold to clear 10% ?

C. P......Rs 20
Add profit 10%... 2
S. P......Rs 22

2. If an article marked Rs 25 be sold for Rs 20, what is the discount allowed ?

M. P. Rs 25
S. P. ,, 20
Disct. ,, 5

Rate of discount % = $\frac{5}{25} \times 100 = \underline{20\%}$

§ **Quotations and tenders.** Prices are quoted for different markets on different terms in accordance with custom or requirement. Thus prices of seeds for export are quoted in the Calcutta market on *f.o.b.* terms, while it is customary to quote rates *c.i.f.* Calcutta for foodstuffs, medicines etc. shipped from London.

The basic price is that *ex* dealer's warehouse, from which the different rates are calculated on estimates of the probable costs to be incurred in conection with the sale.

Illustrations

1. Ascertain prices *f.o.b.* London, *c.f.*, *c.i.f.* Calcutta from the following data ; and also the landed price of 24 cases in Calcutta. Also quote price in Calcutta *f.o.r.* Goalundo, assuming an estimated profit of 15% on cost. (A case contains 1 gross bottles).

Basic price @ 11s 8d per doz		Freight	...£2 4s
Cost of cases ...	£3	Insurance	...£3 12s
Cartage ...	12s	Landing charges	...Rs 13
Ry. Freight ...	16s	Customs	...Rs 36
Dock Dues &c. ...	12s	Cartage (*in*)	...Rs 3

Cartage (*out*)	Re 1 4 3
Ry. Freight to Goalundo	Rs 8 12 0

Exchange at 2s to Re 1

Basic Price of 24 gross ...	£168 0s	
Cases	3 0s	
Cartage	12s	
Railway freight	16s	
Dock Dues &c.	12s	
f.o.b. cost of 24 gross ...	£173 0s	
		per gross
∴ *f.o.b.* price per gross		= £7 4s 2d
Freight	£2 4s	
c.f. cost of 24 gross ...	£175 4s	
∴ *c.f.* price per gross		= £7 5s 10d
Insurance	£3 12s	
c.i.f. cost of 24 gross	£178 16s	
∴ *c.i.f.* price per gross		= £7 8s 10d
c.i.f. cost ...	£178 16s	
@ Exchange 2s to Re 1 ...	Rs 1788	
Landing Charges ...	13	
Customs ...	36	
Cartage ...	3	
Cost to warehouse ...	Rs 1840	
Landed cost *per gross*		= Rs 76 10as 8p
Profit @ 15% ...	276	
	Rs 2116	
Cartage ...	1 4as	
Ry. Freight ...	8 12as	
	Rs 2126	
∴ Price *f.o.r.* Goalundo per gross		= Rs 88 9as 4p

2. A person intends to submit a tender for repairs to certain roads. He estimates that the cost of materials will be Rs 15591 12as; the cost of labour will be Rs 7815 10as; and supervision charges will be 5% of the cost of labour and materials together. If he wishes to make a profit of 12½% on this work, what should be the value of his tender?

Estimated cost of materials	Rs 15591 12as
„ „ „ labour	7815 10as
	Rs 23407 6as
Add 5% for supervision	1170 6as
	Rs 24577 12as
Add 12½% for profit	3072 4as
Value of tender	Rs 27650 0a

EXAMPLES 15

1. Ascertain landed price in rupees per yard from the following details:

Cost of 853¾ yds Serge @ 3s 6d per yd. Freight £4 13s. Insurance £1 5s. Cartage Rs 5 4as. Port & Dock charges Rs 43 10as. Duty @ 25% on *c.i.f.* price. Exchange @ Re 1 = 1s 4d. [*Ans.* Rs 3 7as 6p per yd.

2. Quote prices *c.i.f.* Bombay from the following estimates: Export Price £1 7s 6d per doz. Freight on 3 cases (containing 1 doz. bottles each) 22s 6d. Insurance 12s 6d. 3 cases @ 2s 6d each. Dock charges at port of shipment 5s per case. Exchange Re 1 = 1s 6d. [*Ans.* Rs 31 1a 9p per doz.

3. A continental buyer notices that certain lines of goods are cheaper in Belgium than in Paris. Where should he make his purchases so that the import price may be the lowest, on the basis of the following estimates?

Belgium	France	
11s 6d	12s 9d	Cost per cwt.
8s 3d	6s 9d	Port Dues and Customs per cwt.
£1 7s 3d	£1 5s 2d	Freight per ton.
12s 6d	15s 6d	Insurance on £100.

What difference do the prices mean to a Calcutta importer who desires to place orders for 3 ton 10 cwt of hardware? Exchange Re 1 = 1s 4d. [*Ans.* France ; Rs 17 nearly

4. How should the retail price of coal per maund be fixed on the following basis on an indent of 27 tons ?

Price *f.o.r.* Colliery	...	Rs 2 6as per ton
Rent etc. at Ry. siding (delivery)	...	1 anna per ton per day
Cartage to godown	...	40 carts @ 8as each
Rent of godown (15d/s)	...	Rs 30
Selling Expenses	...	Re1 6as per ton.
Estimated Profit	...	25% on final cost.

The stack was cleared from the railway siding at destination in 4 days. (Take 1 ton on rail to be equal to 27 md at godown.)

[*Ans.* Not less than 4as 3p

5. An English Engineering firm desires to submit a tender for the construction of waterworks in an Indian town. It is estimated that the cost of materials, including shipping and other charges, will be £15640 12s ; the cost of labour will be £10845 18s and overhead charges and incidental expenses will be £4645. Taking exchange at 1s 6d to Re 1, calculate the value of the tender in sterling. How will the Indian authorities be affected if exchange stands at 1s 4d to Re 1, when payment is made on completion of work? [*Ans.* £31131 10s ; Less Rs 51885 13as 8p

6. Ascertain the landed cost of shirting per yard in Calcutta, given the following details of purchase of a bale containing 1600 yards.

Cost *f.o.b.* London	9d per yard
Freight	25s
Marine insurance	13s 6d
Dock dues in Calcutta	Rs 11 2as
Exchange	@ 1s 4d to Re 1

At what price can the shirting be sold in Calcutta to clear 15% ? [*Ans.* Not less than 11as per yd.

7. A consignment of 8400 lb of yarns was invoiced *f.o.b.* Kobe @ Rs 13 12as per 100 lbs. The following expenses were incurred :

Freight	Rs 435 *less* 20%
Insurance	Rs 10 12as
Duty	Rs 456 4as
Dock dues etc.	Rs 18 8as
Cartage	Rs 3 8as

Find the imported price per lb to the nearest 10th of a pie, and the selling price, leaving a margin of 12½% for profit.

[*Ans.* 3as 9·5p ; 4as 3p

8. A petroleum dealer so constructs his syphon that $\frac{1}{12}$th of a gallon of petroleum is left over in the hose every time petrol is delivered. If the total daily sale is 680 gallons to 200 customers on the average, work out an estimate of profit represented by this fraudulent practice, if petroleum is selling at Re 1 5as a gallon. [*Ans.* Rs 21 14as daily

9. From the following data calculate *f.o.b.* and *c.i.f.* prices to the nearest farthing and make out an *f.o.b.* and a *c.i.f.* invoice :

Buyer—Riverside Traders, Calcutta.

Seller—Clydesdale Manufactures Ltd. Birmingham. 450 pcs. enamelware, catalogue No 468 @ 2s 6d each. Packing 15s ; cartage 4s 3d ; railway freight to Liverpool 13s ; loading charges 6s ; freight on 42¼ c.ft. @ 90s per ton (40 c.ft.) ; Insurance @ 17s 6d% and stamp 9d.

[*Ans.* 2s 7¼d ; 2s 10d ; *f.o.b.* invoice—£63 18s 11d
c.i.f. invoice—£63 15s

10. 75 cases of a commodity were sold to an importer for Rs 8450 *f.o.b.* at the foreign port, each case measuring 4′3″ × 3′4″ × 2′6″. The importer has to pay for the following :—

Freight Rs 23 12as per ton of 40 cu.ft. *plus* 10% primage ; Marine Insurance on Rs 9000 @ Rs $5\frac{11}{16}$% *plus* 15% ; Customs Entries Rs 2 7as ; Landing Dues 11as 9p per case.

Opening for inspection by Customs—7 cases at Re 1 3as 6p per case *plus* 20% ; Delivery @ Re 1 3as per case ; Rent for warehousing 5 weeks at 3as per week ; Insurance 2 months on Rs 10000 at Rs 3 1a 6p % per annum ; Cartage Re 1 1a per case for 30 cases and Re 1 9as 6p per case for 45 cases.

Find the total cost to the importer. —I. Com.

[*Ans.* Rs 11155 12as 9p

11. Equal numbers of eggs are bought at the following rates : 12as per score, 8as per dozen and 1a 3p per pair. They are sold at 9as per dozen, and the total profit amounts to Rs 21 8as. Find the number bought at each of the above prices. [*Ans.* 960

12. Equal amounts are spent in purchasing two lots of oranges at 4as per dozen and at 5as per score. The entire lot is sold at Rs 2 5as 6p per 100. The profit made on the transaction is Rs 225. Find the amount invested. [*Ans.* Rs 720

§ **Discount** is allowed in business in consideration either of payment of dues in ready cash, or with a view to providing a margin of profit to the different agents in the distribution of goods. In the first instance, the discount is known as Cash Discount and is taken off the net value of a bill. The other kind is known as Trade Discount, being allowed to the trade either at a flat rate or by way of successive discounts. In Insurance, other than Life, it is customary to quote rates frequently subject to two or more successive discounts. In such cases, it may be useful to work with equivalent rates as shown in some of the following examples. In calculations, the rates should be split up into suitable aliquot parts, and all other devices shown in Section VI should be used as occasion arises.

In business offices Tables are generally used to facilitate computation. Tables given at the end of this book may also be used as Tables of Nine Values in speeding up work.

§ **Successive Discounts.** It is customary in trade to offer a series of discounts, each being calculated on the last reduced value. The value of an invoice for Rs 3585 less 20%, 25% and 5% would be obtained as below :

Bill	Rs 3585
Less Disct. 20%	717
	2868
Less Disct. 25%	717
	2151
Less Disct. 5%	107 9as correct to nearest anna.
	Rs 2043 7as

§ A single discount equivalent to 20%, 20% and 5% could be found as under :

Say, Bill	...100
1st Disct.	... 20
	80
2nd Disct.	... 16
	64
3rd Disct.	... 3·2
	60·8

Thus,

Bill	... 100
Reduced value	... 60·8
Equiv. Disct.	... 39·2%

§ If the successive discounts per cent are d_1, d_2, a simple algebraic solution would give the equivalent discount to be

$$=(d_1+d_2)-\frac{d_1d_2}{100}$$

For three successive discounts d_1, d_2, and d_3 per cent, a single equivalent would be

$$100-\frac{(100-d_1)(100-d_2)(100-d_3)}{10^4}\text{ per cent.}$$

$$=(\Sigma d_1-\frac{1}{100}\Sigma d_1d_2+\frac{1}{10^4}d_1d_2d_3)\,\%$$

Thus (i) a discount of 10% and 5% is equivalent to a single discount $(10+5)-\frac{10\cdot 5}{100}$ or 14½ per cent ; and

(ii) a discount of 10%, 10% and 5% is equivalent to a single discount $100-\frac{90\times 90\times 95}{10^4}$ or $100-76{\cdot}95$ or 23·05%.

Short Methods

1. When the amount is in £'s any percentage may be obtained directly by multiplying the £'s by twice the rate, pointing off one decimal place, and calling the result shillings.

3% on £45 = 27·0s = £1 7s
4½% on £20 = 18·0s = 18s
1% on £7 = 1·4s = 1s 4¾d to nearest farthing.

2. When the amount is in Rupees, 6¼%, being equivalent to 1 anna in the rupee, may be taken as the basic rate.

Discount @ 5% on Rs 538 4as
@ 6¼%.........Rs 33 10as 3p
@ 1¼%.....5...Rs 6 11as 9p
∴ @ 5%.........Rs 26 14as 6p

EXAMPLES 16

Calculate discount, commission or brokerage in the following cases :

1. 8¼% on Rs 918 13as 9p [*Ans.* Rs 75 12as 9p
2. 12¾% on Rs 1015 3as [*Ans.* Rs 129 7as
3. 3⅛% on Rs 756 2as 6p [*Ans.* Rs 23 10as
4. 9⅜% on Rs 5134 12as [*Ans.* Rs 481 6as
5. 10%, 10% & 5% on £63 10s [*Ans.* £14 12s 9d
6. 15% & 2½% on Rs 850 [*Ans.* Rs 145 9as.
7. 16⅔% & 10% on Rs 3400 [*Ans.* Rs 850
8. 17½%, 10% & 5% on Rs 5600 [*Ans.* Rs 1649 14as 6p
9. 12½%, 5% & 2½% on £46 13s 8¾d [*Ans.* £8 16s 11½d

10. Find the net price of a tyre listed at Rs 84 less 10% & 10% & 5%. [*Ans.* Rs 64 10as 3p

11. A trader is allowed a cash discount of 5% in addition to a trade discount of 12½% & 2½%. Find the net amount of a bill for Rs 738 6as. [*Ans.* Rs 598 7as

12. Calculate the equivalent discount, to the nearest hundredth per cent, for the following :

(i) 5% & 1¼% (ii) 10% & 5% & 2½% (iii) 15% & 10% & 5%
[*Ans.* (i) $6\frac{3}{16}$% ; (ii) 16·6⅜% ; (iii) 27·3¼%

13. Which is more profitable to the buyer ; (i) a net discount of 25% ; (ii) a discount of 20% and 5% ; or (iii) a discount of 15% and 10% ? [*Ans.* First

14. A bookseller offers 13 copies to every dozen of the same book ordered and a cash discount of 10%. What is the net value of the discount to the buyer ? [*Ans.* 16·92%

15. A manufacturer of a Patent Medicine despatches 1 dozen extra bottles for every gross ordered. A discount of 25% is also offered to the trade. If a bottle is marked at Re 1 2as, what is the lowest price at which a bottle can be retailed without loss ?
[*Ans.* Not less than 12as 6p

16. The Freight on Jute from Calcutta to Dundee is 37s 6d per ton plus 10% primage less 12½% & 2½%. Under a system of Deferred Rebate a shipper claims Rebate on a total shipment on 4327½ tons in a year at 5% on the account. Find the amount claimed. [*Ans.* £380 14s 6½d

17. A change in the rebate on a bill for the consumption of electric current from 25% to 33⅓% reduces the bill by Rs 6 4as. What is the gross value of the bill ? [*Ans.* Rs 75

18. Calculate an agent's charge for *del credere* in respect of bills valuing £6358 14s 9d @ 3½%. [*Ans.* £222 11s 1½d

19. A trader is offered a discount of 13¾% by one wholesaler and 10% and 5% by another. Find which is cheaper. If he sells at 5% below list, what profit does he make ? [*Ans.* Latter ; $11\frac{1}{9}$%

20. A manufacturer of hair oil allows a discount to retailers of 3as per bottle marked at Re 1 each. Retail shops, under pressure of competition, pass on 50% of their margin to customers. If they obtain 13 bottles for the price of a dozen, find the price paid by customers, and the profit. [*Ans.* 14as ; 16⅔%

21. A rebate of 10% is allowed on net invoice value to retailers who indent 2000 bottles or more annually of a Hair Lotion. The ordinary discount is 10% and 5% ; and a bottle is sold at Re 1 4as although listed at Rs 18 per dozen. Find the amount of rebate, and the profit of a retailer who sells 2000 bottles a year. [*Ans.* Rs 256 8as ; 8·3%

22. A traveller is offered a salary of Rs 100 per month plus a commission of 5% on sales, or a flat rate of 15% in commission only. If the average monthly sales amount to Rs 1050, which terms would he prefer ? [*Ans.* Latter

23. An auctioneer charges 4¾% on the sale proceeds and spends on the average ⅛% on advertisement and other incidental charges. What is his net profit on a sale which realised Rs 5784 ? [*Ans.* Rs 267 8as

24. A land agent charges 1½% on a transaction. A particular plot of land is bought, and sold through him shortly thereafter at a loss of 10%. If the first purchaser bought it for Rs 20000, how much did the agent clear on the two transactions ? [*Ans.* Rs 570

25. In terms of an agreement, the Sole Agents in a certain territory of a manufacturer of Soaps, are to complete a sale of 200000 lb worth Rs 75000 in a year on which they are to be paid a commission of 11¼%. If the Agents desire to spend a quarter of their remuneration for the first year by way of free distribution of samples, how many cakes of 4oz. each will be distributed ? [*Ans.* 22500 pieces

26. A manufacturer of condiments offers a discount of 12½% to the trade for payment within a month, and a further 7½% for cash. For how much will a bill for Rs 1825 10as be settled for payment in cash ? [*Ans.* Rs 1477 9as 9p

27. A motor car salesman is offered (i) a salary of Rs 1500 p.m. or (ii) a salary of Rs 400 p.m. plus commission at 7½%, or (iii) only commission at 10%. If he expects his sales would reach Rupees two lakhs in a year, which terms should he prefer ? [*Ans.* Last

28. A motoralties dealer offers a discount of 15% less 5% and 5% on a certain make of tyre ; another offers 20% and 5%. Which is the cheaper store ? [*Ans.* Latter

29. A hardware merchant offers a discount of 10, 5 and 5 per cent to the trade. Prepare a Table of Nine Values for the equivalent rate. How much should be allowed as discount on a bill for Rs 12186 10as ? [*Ans.* Rs 2288 0a 6p

30. Calculate Marine Insurance Premium on Rs 8560 at 6as% *less* 10% and 2½%, *plus* an additional 2as% net for Riots and Civil Commotion Risks. [*Ans.* Rs 38 14as

31. A deferred rebate of 10% on net receipts is granted to a shipper guaranteeing a shipment of 20000 tons in a year, only 5% being allowed if the total does not reach this minimum. The shipper finds that on the last voyage he cannot complete more than 19582 tons @ 25s per ton less 10% & 5%. Does he gain or lose by fulfilling the contract by sending ballast ?

[*Ans.* Gain £644 7s

32. The following is a schedule of rates of remuneration to a life insurance agent :

20% on first year's premium up to Rs 10000 sum assured
25% „ „ „ „ above Rs 10000 up to Rs 40000
40% „ „ „ „ „ Rs 40000 „ „ Rs 80000
50% „ „ „ „ „ Rs 80000

On Renewal premiums a level rate of 2½%.

He introduces new business to the extent of—

Rs 8000 in the 1st year on which the monthly premium is Rs 24 8a
„ 20000 „ 2nd „ „ „ „ „ „ „ „ 78 12as
„ 50000 „ 3rd „ „ „ „ „ „ „ „ 185 8as
„ 81000 „ 4th „ „ „ „ „ „ „ „ 316 4as
„ 120000 „ 5th „ „ „ „ „ „ „ „ 460 0as

Make out a statement of his remuneration during these years, assuming that all policies issued through his agency continue. What would be the renewal commission during the following year ? [*Ans.* 5th year's income Rs 2941 8as ; Rs 319 8as

§ **Simple Interest.** The formula for the calculation of interest I on a principal P at a rate R per cent per annum for Y years is $I = \frac{PRY}{100}$.

Months and days are converted into fractions of a year before this formula is applied. In many cases, however, it is convenient to obtain an equivalent rate, *i.e.* R. Y., and then apply the ordinary methods of calculating percentages.

Thus, 5% p. a. for 73 days is equivalent to 1%
4% p. a. for 3 months is also ... 1%
3% p. a. for 4 „ „ „ ... 1%
2½% p. a. for 2 years „ „ ... 5%
&c. &c.

Also, $6\frac{1}{4}\% = 5\% + \underline{4}$; $7\frac{1}{2}\% = 5\% + \underline{2}$
$5\% = 6\frac{1}{4}\% - \underline{5}$; $7\frac{1}{2}\% = 6\frac{1}{4}\% + \underline{5}$
&c. &c.

Illustrations

1. To find interest for 1½ years at 5% p.a. on Rs 848 12as.

Equivalent rate = $7\frac{1}{2}\% = 6\frac{1}{4}\% + 5$

Now, @ 6¼% interest = Rs 53 0a 9p
$\underline{5}$ 10 9 9
∴ @ given rate „ = Rs 63 10as 9p

2. To find interest for 1 year 3½ months @ 6% p.a. on £586 14s 8d.

Equivalent rate % $= 6 \times \frac{15\frac{1}{2}}{12} = 7\frac{3}{4} = 5 + \underline{2} + \underline{10}$

Now, @ 5% interest = £29 6s 8¾d
$\underline{2}$ 14 13 4½
$\underline{10}$ 1 9 4
∴ @ given rate = £45 9s 5¼d

3. To find interest on £86 @ 4% p.a. for 6 months.

Equivalent rate = 2%
∴ Reqd. interest = 34·4s = £1 14s 4⅘d

4. To find interest on Rs 6254 8as 3p @ 3¾% p.a. for 8½ months.

@ 2½% for 1 yr on Rs 6254·5156...		Rs 156·3629
@ 1¼% ,, ,, ,, ,,	2	78·1814
∴ @ 3¾% ,, ,, ,, ...		...Rs 234·5443
∴ @ 3¾% for 6 mos ...	2	117·2722
,, ,, ,, 2 mos ...	3	39·0907
,, ,, ,, ½ mo. ...	·4	9·7727
∴ @ 3¾% for 8½ mos ...		Rs 166·1356
i.e.		Rs 166 2as 2p

5. To find interest on £523 5s 9¾d @ 7¾% p.a. for 5 months 10 days.

@ 5 % p.a. for 6 mos. on £523 5s 9¾d...		£13	1s	7·74d
@ 2½%	2 ...	6	10	9·87
@ ¼ %	10 ...		13	0·99
@ 7¾% for 6 mos...	...	£20	5s	6·60d
less ,, ,, 20 days...	9	2	5	0·73
∴ @ 7¾% for 5 mos. 10 days		£18	0	5·87 –
i.e.		£18	0s	5¾d

§ When days are taken into account, ordinary calculations involve complications. In such cases usually Interest Tables are used. A useful Table is given at the end, and may be used as a Table of Nine Values. The following rule may also be used to simplify work when no Table is handy.

Third, Tenth and Tenth Rule :

(i) Obtain the continued product of 2R. P. with the decimal point shifted 5 *places* to the left.

(ii) Multiply (i) by *D*, the number of days, writing down the product correct to 4 decimal places.

(iii) Add to the product obtained in (ii)

(*a*) a third of itself

(*b*) a tenth of (*a*)

(*c*) a tenth again of (*b*

(iv) **From this sum deduct itself with the point shifted back** ***4 places.***

(v) **Convert this into money units to obtain interest required.**

Illustration

Calculate interest on Rs 6436 10as 3p @ 2½% per annum for 125 days.

(i) gives ...	Re ·3218320...
(ii) (i) × 125...	Rs 40·2290
(iii) (a) 3	13·4097
(b) 10	1·3410
(c) 10	·1341
	Rs 55·1138
(iv)	·0055
	Rs 55·1083
(v) Reqd. interest	= Rs 55 1a 9p

§ **Interest on Savings Bank Accounts.** It is customary to allow interest on these accounts on minimum monthly balances. The annual interest is obtained by calculating a month's interest on the total of these minimum balances, usually with the help of a Table of Nine Values for the interest for *one month* at the rate allowed.

§ **Provident Deposit Scheme.** Under this scheme a fixed monthly sum paid by a customer is accumulated at simple interest on monthly balances, till at the end of a fixed period, the entire amount deposited becomes payable with interest.

The total interest earned in a year by such deposits of one rupee a month is 6·5 times the rate of interest *per unit.* If the rate of interest per annum is 4% say, the above interest becomes = 6·5 × ·04 = 0·26. Treating this as a multiplier and with the help of a Table of Nine Values at this rate, it is easy to find out the interest earned in a year at various rates of monthly deposit. The amount of a deposit of Re 1 a month at interest i per unit per annum = $(12 + 6{\cdot}5i)$ in a year. The amount at the end of n years = $(12 + 6{\cdot}5i).S_{\overline{n}|}$. (*Vide* Annuities).

§ **Provident Funds.** These funds are usually accumulated from a stated percentage of the pay of an employee together with another stated percentage thereof contributed by the employer. The calculation of interest on the monthly accumulations during a year is made as above. Frequently, however, such accumulations are invested in Trustee Securities and other safe investments. The profit—or loss in the case of a fall in values—from these investments is ordinarily distributed amongst the different accounts in proportion to the balances at credit.

§ **Interest on Current Accounts.** Interest on these accounts with bankers, as also similar accounts maintained by traders amongst themselves, is calculated on daily balances. The actual method of calculation varies in practice, and is based on the idea of the equivalence of earning capacities of money. Thus, a balance of Rs 200 for 13 days for this purpose, is taken to be equivalent to Rs 2600 for 1 day.

The custom in Banks in this regard is to provide in the Ledgers an *extension* or *products* column on both Dr. and Cr. sides, along with a column in which the number of days intervening transactions are entered. The balance, on either side, at any date is multiplied by the number of days to the date of closing the accounts and the product entered in the *extension* column. At the time of closing the account, the extension columns on Dr. and Cr. sides are casted ; and the balance of these totals is obtained. The interest due on the account is now, merely, the interest on this balance at the given rate per annum *for 1 day*. This may be calculated directly from Interest Tables, or with the help of Tables at the end of this book.

For facility in computation, the day's balance is taken to the *nearest ten-rupee* in India, and the *nearest £* in Great Britain.

In Accounts Current between traders, it is the practice to write up the interest on a day's balance for the days intervening between that date and the end of the period of account (usually a calendar month), directly from the Interest Tables, and entered in a separate column on either side—Dr. and Cr. At the end of the accounting period the Dr. and Cr. interests are balanced to obtain the interest due or payable.

Illustration

Current Account with Interest on Daily Balances @ 2½% p.a.

1936	Debits Rs.	as.	p.	Days to Dec. 31	Products			1936	Credits Rs.	as.	p.	Days to Dec. 31	Products		
Jan. 6	211	8	3	360	7	66	00	Jan. 1	536	7	10	365	19	71	00
Feb. 18	518	3	4	317	16	48	40	6	718	10	–	360	25	92	00
Mar. 23	29	6	6	283		48	90	30	1200	–	–	336	40	32	00
July 30	1018	–	–	154	15	70	80	Feb. 8	400	–	–	327	13	08	00
Aug. 4	526	–	–	149	7	89	70	May 10	1600	3	–	235	37	60	00
Oct. 19	68	–	–	73		43	80	July 23	300	6	9	171	5	13	00
Nov. 2	848	2	3	60	5	10	00	Sept. 21	517	9	2	101	5	25	20
Dec. 4	611	5	6	27	1	64	70	Nov. 6	602	3	–	55	3	30	00
18	500	–	–	13		65	00	Dec. 18	1018	13	6	13	1	32	60
28	813	6	3	3		24	30					–			
	To Bal.								*Interest*						
	1820	7	1					Dec. 31	65	1	11				
	6959	7	2						6959	7	2				
								Total Cr. Product					151	63	80
								Dr. ,,					66	57	60
					56	57	60	Balance ,,					95	06	20

Interest on Rs 950620 @ 2½% p. a. for 1 day = Rs 65 1a 11p

§ **Years' Purchase.** A common phrase used in this connection is the valuation of an asset at so many years' purchase. An estate yielding say Re 1 per annum is said to be valued at 20 years' purchase, if it be assessed at Rs 20. It is evident that calculating simple interest, a 20 years' purchase yields a return of 5% p. a. Similarly, a 25 years' purchase yields 4% p. a. and so on.

Short Methods

1. The interest on a given number of £'s for a complete number of *years* may be obtained directly by multiplying it by twice the rate and the number of years, pointing off one decimal place and calling it shillings.

Interest @ 2% p. a. on £18 for 3 years = 2 × 10·8s = £1 1s 7¼d.

Interest @ 1¼% p. a. on £20 for 4 years = 20·0s = £1.

2. To find interest at 6% p. a. on a given number of £'s for a number of *months*, write down the product of the principal and the number of months.

The unit figure multiplied by ⁶⁄₅ gives the pence, and the remaining figures give the number of shillings.

Interest @ 6% on £135 for 5 months.

Product = 675

Interest = 67s + 5 × ⁶⁄₅d = £3 7s 6d

3. To find interest for a number of *months*, or trade discount similarly calculated, the following basic equivalents may be noted.

Rate % per annum	Equiv. Rate p. m. per £	Equiv. Rate per Rupee	per mensem per cent
1¼	¼d	⅕ pie	1⅔ annas
2½	½d	⅖ pie	3⅓ annas
3¾	¾d	⅗ pie	5 annas
5	1d	⅘ pie	6⅔ annas
6¼	1¼d	1 pie	8⅓ annas

Thus, to find 3¾% on Rs 5864 for 3 months, the equivalent rate is 3 × 5as% = 1% − 16

∴ 3¾% on Rs 5864 for 3 months...Rs 58 10as 3p

less 3 10 9

Rs 54 15as 6p

For other short methods see Rapid Calculations.

EXAMPLES 17

[Answer to nearest 3p or farthing]

Calculate interest on the following :

1. £581 7s 9¾d @ 5% p. a. for 9 months [*Ans.* £21 16s 0½d
2. £4867 9s 10d@ 4% p. a. for 7 months [*Ans.* £113 11s 6d
3. Rs 939 5as 9p @ 6¼% for 10 mos. [*Ans.* Rs 48 14as 9p
4. Rs 7385 6as @ 4% for 5 months [*Ans.* Rs 123 1a 6p
5. Rs 385 8as @ 3½% for 7 months [*Ans.* Rs 7 14as
6. Rs 1987 9as @ 2¾% for 1 year 2 mos. [*Ans.* Rs 63 12as 3p
7. Rs 5385 13as @ 7¾% for 4 months [*Ans.* Rs 139 2as 3p
8. Rs 6068 2as @ 8¼% for 3½ months [*Ans.* Rs 146 0a 3p
9. Rs 51654 @ 6¾% for 1½ months [*Ans.* Rs 435 13as 3p
10. Rs 30384 3as @ 5½% for 7½ mos. [*Ans.* Rs 1044 7as 3p
11. Rs 9897 12as @ 4¾% for 4¼ months [*Ans.* Rs 166 8as 3p
12. Rs 8388 7as @ 7½% for 8 months [*Ans.* Rs 419 6as 9p
13. Rs 654 14as @ 5¼% for 1 year 3 mos. [*Ans.* Rs 42 15as 6p
14. Rs 3814 2as @ 4⅜% for 1 year 3 mos. [*Ans.* Rs 208 9as 3p
15. £608 8s 9d @ 5⅞% for 9 months [*Ans.* £26 16s 3d

Use the *Third*, *Tenth* and *Tenth Rule* to calculate interest on the following :

16. £653 8s 9d @ 3% for 83 days [*Ans.* £4 9s 2d
17. £568 13s 8¾d @ 4½% for 117 days [*Ans.* £8 4s 1d
18. Rs 638 8as @ 2¾% for 51 days [*Ans.* Rs 2 7as 3p
19. Rs 5684 10as 3p @ 5½% for 93 days [*Ans.* Rs 79 10as 9p
20. Rs 9395 2as 6p @ 6¼% for 86 days [*Ans.* Rs 138 5as 9p
21. Rs 18639 15as @ 1¾% for 47 days [*Ans.* Rs 42
22. Rs 25187 3as 9p @ 3½% 75 days [*Ans.* Rs 181 2as 3p
23. Rs 6039 14as 10p @ 7¾% for 59 days [*Ans.* Rs 75 10as 9p
24. Rs 15839 5as @ 1¼% for 19 days [*Ans.* Rs 10 5as
25. Rs 3834 13as @ 5% for 115 days [*Ans.* Rs 60 6as 6p

26. At what workable rate of simple interest would Rs 5856 amount to at least Rs 6000 in 9 months ? [*Ans.* $3\frac{1}{3}\%$ p. a.

27. An advance of Rs 4873 is made on condition that a sum of Rs 5000 will be paid back at the end of a year. What is the return on the lender's investment ? [*Ans.* 2·6% p. a.

28. A person undertakes to pay back a loan of Rs 4000 in monthly instalments of Rs 500 plus interest at 6% on outstanding balances. Find the average rate of interest earned by the lender. [*Ans.* $3\frac{3}{8}\%$ p. a.

29. It is agreed that a loan of Rs 10000 will be paid back in 15 equal instalments of Rs 800 a month. What is the return on the investment ? [*Ans.* 16% p. a.

30. The Cash value of a Typewriter is Rs 420 ; and if an initial deposit of Rs 100 is made, the dealer allows payment to be spread over twelve months following at the rate of Rs 30 a month. What does this accommodation cost the buyer ? [*Ans.* $12\frac{1}{2}\%$ p. a. on the average.

31. An estate consists of house property of gross rental £150 ; but land tax £3 10s, sewers tax £1 15s and in insurance £3 15s are paid. The losses on account of bad tenants amount to $12\frac{1}{2}\%$ of the rental, ordinary repairs to $17\frac{1}{2}\%$ and collection charges to $2\frac{1}{2}\%$. Find the value of the property at 4% simple interest. —I. Com. [*Ans.* £2306 5s

32. A man deposited Rs 5000 on 20 April in a bank paying interest at 2% p. a. He withdrew Rs 3000 on 15 May and deposited Rs 4000 on 6 June. How much interest was due to him on 30 June following ? —I. Com. [*Ans.* Rs 17 2as 5p

33. A tradesman marks his goods with two prices, one for ready money and the other for six months' credit. What ratio should the two prices bear to each other allowing 5% simple interest ? —I. Com. [*Ans.* 40 : 41

34. On 1st Jan. 1936 a man had Rs 750 in a Savings Bank Account, and he puts in Rs 115 at the end of each second month during the year. What interest should be added to his account at the end of the year according to the following rule :—

Interest at 3% p. a. is allowed on every complete Rs 10 deposited and commences on the first day of the month next following the deposit and is calculated to 31st December.

—I. Com. [*Ans.* Rs 31 0a 10p

35. Find the rate per cent at which a customer pays interest in buying the following on credit terms : "We send a cycle on receipt of 1s deposit and payment of 1s per week for 89 weeks, making a total of £4 10s in all, or net price is only £4. —I. Com. [*Ans.* 14⅘% app. (simple)

36. A customer obtains from his banker the loan of £1250 on June 2. He pays £450 or July 2, £600 on Oct. 1 and finally pays £800 on Nov. 1. How does the customer stand with regard to the bank on Dec. 31, assuming that the bank charges 5% p. a. interest on all sums that the customer owes it, but allows interest at 2½% p. a. on all sums standing to the credit of the customer ? —I. Com. [*Ans. Cr.* £621 12s 11d

37. In terms of an agreement, part repayments are accepted towards reduction of the principal earning interest in lumps of Rs 500. Calculate the total amount of interest @ 6% p. a. paid by a person who borrowed Rs 2000 and repaid as follows, with interest on outstanding balances.

Rs 200	at	the end	of	1 month
Rs 300	,,	,,	,,	2 months
Rs 400	,,	,,	,,	3 months
Rs 200	,,	,,	,,	4 months
Rs 500	,,	,,	,,	5 months
Rs 400	,,	,,	,,	6 months

Also, find the average rate of simple interest earned by the lender. [*Ans.* Rs 42 8as ; 4¼%

38. A person lends Rs 5000 to another on the understanding that an average rate of interest at 12% p. a. for 1 year will be calculated, and the total amount repaid in 12 equal monthly instalments. What is the total return on the investment, assuming the lender accumulates the repayments with his banker @ 4% p. a. on monthly balances ? [*Ans.* 19·28%

39. A person pays Rs 25 monthly into a Savings Bank carrying simple interest on monthly balances at 2%. Find the amount at the end of the year. [*Ans.* Rs 303 4as

40. How much should a person pay into a Provident Deposit Fund monthly to assure payment of Rs 5000 at least at the end of 2 years? The answer is to be given in complete rupees, the rate of interest allowed being 5% per annum. [*Ans* Rs 198

41. Balance the following account current with interest at 4% p. a. up to January 31st 1937.

Date	Dr.	Date	Cr.
Jan. 1937	Rs.	Jan. 1937	Rs
12	120	1	450
18	50	2	215
25	200	5	620
		10	500
		18	200
		27	600

[*Ans.* Interest Rs 5 2as 10p

42. Calculate interest on the following account at 2% per annum on daily balances up to December 31st 1937.

Date	Withdrawals	Date	Deposits
1937	Rs	1937	Rs
Jan. 9	200	Jan. 1	2500
May 12	400	Jan. 8	500
Oct. 3	800	March 15	600
Oct. 18	200	Aug. 9	300
Nov. 19	500	Decr. 3	100

[*Ans.* Interest Rs 56 13as 9p

SECTION SEVEN

BUSINESS FINANCE

§ A business is financed chiefly in one of the following ways :

1. A person brings capital on his own account entirely, when he is usually described as a Sole Trader.

2. Two or more persons join in partnership, contributing different amounts to form the Capital of the business.

3. A large number of persons, contributing large or small amounts of money, combine with a view to carry on business, in the form of a Co-operative Society or a Joint Stock Company.

4. A number of organizations form into a Trust or a Kartel.

§ The **Sole Trader** is entitled to the whole of the profits of the business, just as much as he has to bear all losses. It is customary to build up reserves out of profits to provide against lean years.

§ **Partnership.** The division of profits and losses between partners is always a matter of agreement. In the absence of any stipulation in the Partnership Deed, the Indian Partnership Act provides for an equal distribution amongst partners.

A partnership brings together finance, or skill and finance. Some of the usual methods of apportionment of profit are, therefore, on the following bases :

(i) average. investment[*] of capital during an accounting period ; (ii) fixed proportion ; (iii) partly of salary, bonus, commission, etc. to partners and a proportional apportionment thereafter ; (iv) partly of interest on capital investment and apportionment thereafter etc. etc.

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[*] Since R invested for P periods has an earning capacity of R.P. for a unit period, R.P. is taken to be a measure of the average investment.

The division of any surplus between partners, whatever may be the basis, should be made in the Profit and Loss Appropriation Account. Thus interest charged on capital, remuneration to partners &c.—which are meant to make the division of profit just and equitable, considering finance and skill—are not to be debited to the Profit and Loss Account as ordinary business expenses. In case, the profit is insufficient to cover the agreed interest on partner's capital, it is customary to divide it in proportion to their average investment to avoid showing a loss.

It sometimes happens that partners bring into a business capital short of, or in excess of, agreed amounts. In such cases, in the absence of any stipulation in the Deed, interest is charged on the deficit and allowed on the excess at 5% p.a.

The mere receipt of a share of profit does not necessarily make the recipient a partner. An employee may be allowed a share of profits as his remuneration, and yet not be a partner so long as he does not share losses in the same proportion. Such a share of profit should be debited to the Profit and Loss Account as an ordinary business expense.

*§ The Partnership Deed also lays down the manner in which available assets are to be divided between partners upon liquidation. If on complete liquidation of a business there is a profit or a loss it should be shared between the partners in the profit or loss ratio ; but assets are to be distributed in the ratio of the capital brought in by the partners. If the firm goes bankrupt, losses should be shared in the profit or loss ratio (*not* in the capital ratio). If this results in a deficit in the capital of a partner, he would be called upon to make the deficiency good ; but in case the partner concerned is himself insolvent, his share is to be made good by the other partners in proportion to their last agreed capitals.

§ **Joint Stock Companies.** The proprietors in a Joint Stock Company are ultimately the shareholders, who have no further liability in regard to losses beyond the value of the shares held by them. If the shares have been paid in full, their right to a share of the profit, in proportion to the paid up value of the shares held by them, continues during the existence of the company, although they have no liability in regard to losses. The rate at which profits are divided on the shares held is known as *dividend.*

§ **Co-operative Societies.** The proprietors here also are the shareholders, who, however, have unlimited liability in respect of losses, unless the Society is formed with limited liability. There is usually a limit to the number of shares held by a member, and profits are divided in the shape of dividends.

§ **Trusts & Kartels.** These are monster combinations of different units organized to be worked on planned policies, usually with a view to establishing monopolies, entire or partial. The allocation of profits and losses is essentially a matter of previous arrangement.

Illustrations

1. Divide a year's profit between *A*, *B* and *C* from the following data :

	Capital	Salary	Bonus &c.
A	Rs 10,000	Rs 1000	...
B	Rs 4,000	...	10% on Sales.
C	Rs 6,000	Rs 2000	2% on Profit.

Interest on capital	@ 5% p. a.
Year's profit	Rs 12500
Gross sales	Rs 40000

Profits to be divided in proportion 3 : 4 : 3.

Profit		Rs 12500
Interest on Capital		1000
Salary to *A* & *C*		3000
Bonus to *B*		4000
C's remuneration		250
Divisible profit		Rs 4250
A's share $\frac{3}{10}$	...	Rs 1275
B's share $\frac{4}{10}$	...	Rs 1700
C's share $\frac{3}{10}$	...	Rs 1275

	A	*B*	*C*
Interest on capital	Rs 500	Rs 200	Rs 300
Salary, Bonus &c.	1000	4000	2250
Share Profit	1275	1700	1275
Total	Rs 2775	Rs 5900	Rs 3825

2. Distribute a year's profit on the basis of average capital invested between the partners *A*, *B* and *C*.

	Capital Rs	Period mos.	Salary Rs
A	20000	12	3000
B	10000	9	2000
C	15000	8	*nil*

Net profit : Rs 12500

Interest on capital : 4% p. a.

Average Investment Statement

	Equivalents Rupee-month	Proportionals	Interest @ 4% p. a
A	240 000	8	Rs 800
B	90 000	3	300
C	120 000	4	400
		15	Total Rs 1500

	Net profit ...	Rs 12500
less	Salary to partners	5000
	Intt. on Capital	1500
	Divisible Profit	6000

A's share $\frac{8}{15}$ ... Rs 3200

B's „ $\frac{3}{15}$ ... 1200

C's „ $\frac{4}{15}$ ... 1600

	A gets :	*B* gets :	*C* gets :
Salary	Rs 3000	Rs 2000	–
Intt. on Capital	800	300	400
Share of Profit	3200	1200	1600
Total	Rs 7000	Rs 3500	Rs 2000

3. *A* with a capital of Rs 20000 in a business, admits *B* into partnership. *B* can bring in only Rs 5000 as capital, but guarantees a turnover of Rs 100000 annually. They agree to charge interest at 5% on capital. Further, *B* is to get a bonus of 5% on the turnover before profits are divided in the proportion 5 : 3. The following year the turnover amounted to Rs 120000, and profits to Rs 13810. Show the appropriation of profit.

		Rs	
Profit	...	13810	
Bonus to *B*	...	6000	
Intt. on Capital :			
A...Rs 1000			
B... 250		1250	
Divisible profit		6560	
A's share $\frac{5}{8}$: 4100			
B's share $\frac{3}{8}$: 2460			
But of the total profits			
A gets :			
Interest on Capital		1000	
Share of Profit		4100	Rs 5100
B gets :			
Interest on Capital		250	
Bonus		6000	
Share Profit		2460	Rs 8710

4. Apportion a year's profit of Rs 3500 from the data below :

Partners	Capital	Profit proportionals
A	Rs 20 000	3
B	„ 25 000	4
C	„ 40 000	5

Interest on Capital @ 6% p. a.

Amount required to pay interest on Capital :

A	...	Rs 1200
B	...	1500
C	...	2400
Total		Rs 5100

There are ordinarily two ways of dealing with the profit, which is not sufficient to pay the interest :

(i) to apportion the deficit as a loss between the partners in the profit ratio ; or,

(ii) to divide the available profit in the profit ratio.

The Current Accounts will show balances :

Under (i)

	A	*B*	*C*
Intt. on Capital	Rs 1200	Rs 1500	Rs 2400
Share Loss	400	533 5as	666 11as
Balance	Rs 800	Rs 966 11as	Rs 1733 5as

Under (ii)

	$\frac{3}{12}$	$\frac{4}{12}$	$\frac{5}{12}$
Share Profit (Intt.)	Rs 875	Rs 1166 11as	Rs 1458 5as

In the first case the business will show a loss ; in the second a profit.

5. A partnership between *A*, *B* and *C* is voluntarily wound up. The assets were as under : Cash Rs 5800, Fixed Capital Rs 54600, Bills Receivable Rs 16200. The liabilities amounted to Rs 70600. The creditors agreed to accept in full settlement a composition of 12as in the rupee. The fixed capital realised 120% of the book value, but 25% of the bills could not be realised. Divide the balance, if any, between partners. Given that originally *A* brought in Rs 20000, *B* Rs 20000 and *C* Rs 30000 to the business, and profits were to be divided in proportion to capital.

Realization :		Rs
Cash		5800
Fixed Capital	... 54600	
Add 20%	... 10920	65520
B/R	... 16200	
Less Unrealised 25%	... 4050	12150
Total		83470

			Rs
	Brought forward		83470
Disbursement :			
Creditors	...	70600	
Less 4as in Re 1	...	17650	52950
		Balance	30520

Balance divisible in the proportion 2 : 2 3

A gets $\frac{2}{7}$	...	Rs 8720	
B gets $\frac{2}{7}$		8720	
C gets $\frac{3}{7}$		13080	30520

*6. Into a partnership business between *A*, *B* and *C*, the partners agreed to bring in capital to the extent of Rs 10,000, Rs 20,000 and Rs 25,000 respectively. Actually, however, their respective investments were Rs 8,000, Rs 25,000 and Rs 20,000 during the first year. It was agreed that interest @ 6% p. a. will be charged or credited to a partner's capital account on a shortage from, or excess over, the agreed capital contributions. Distribute a year's profit of Rs 8000 in the proportion 3 : 5 : 8, transferring balances to partners' capital accounts.

	Agreed contribution Rs	Shortage Rs	Interest *Dr*	Excess Rs	Interest *Cr.*
A	10 000	2000	120		
B	20 000			5000	300
C	25 000	5000	300		
		Totals	420		300
		Net	120		

Allocation of Profit

Profit	Rs 8000				
Add Interest earned	120				
Divisible profit	Rs 8120				
A gets $\frac{3}{16}$ of Rs 8120		Rs 1522 8as		Rs	as
Debit Intt.		120	Net	1402	8
B gets $\frac{5}{16}$ of Rs 8120		Rs 2537 8as			
Credit Intt.		300	Net	2837	8
C gets $\frac{8}{16}$ of Rs 8120		Rs 4060			
Debit Intt.		300	Net	3760	0

A's Capital Account

Dr.			*Cr.*
To Interest on Shortage of investment	Rs 120	By Balance	Rs 8000
Balance	9402 8	„ Share of Profit	1522 8as
	Rs 9522 8as		Rs 9522 8as

B's Capital Account

Dr.			*Cr.*
To Balance	Rs 27837 8as	By Balance	Rs 25000
		„ Share of Profit	2537 8
		„ Interest on excess investment	300 0
	Rs 27837 8as		Rs 27837 8as

C's Capital Account

Dr.			*Cr.*
To Intt. on Shortage of investment	Rs 300	By Balance	Rs 20000
Balance	23760	„ Share of Profit	4060
	Rs 24060		Rs 24060

EXAMPLES 18

1. *A* and *B* enter into a partnership, with Rs 5000 and Rs 6000 respectively, agreeing to charge interest at 4% per annum before dividing the profits between themselves in proportion to their respective capitals. How much should each receive out of total profits amounting to Rs 4312 ?

[*Ans. A* : Rs 1960 ; *B* : Rs 2352

2. *A* joins *B* in a partnership on condition that *B* is to receive Rs 100 a month for managing the business. The balance of the year's profits, after charging interest on capital at 5% p. a., is to be divided in proportion to their respective capitals. Find the apportionment of gross profits amounting to Rs 4000, if *A* brings in a capital of Rs 10000, *B* having started the business with Rs 4000.

[*Ans. A* : Rs 2000 ; *B* : Rs 2000

3. A partnership has a capital of Rs 40000, of which a half is contributed by *A*, the other half being contributed equally by *B* and *C*. They appoint *D* as manager, agreeing to remunerate him at 10% of the gross profits. *B* and *C* are also paid 5% each thereof, before the final apportionment of profit in proportion to capital. Find how much *A*, *B*, *C*, and *D* should receive out of total profits amounting to Rs 12600.

[*Ans.* *A* : Rs 5040 ; *B* : Rs 3150 ; *C* : Rs 3150 ; *D* : Rs 1260

4. *A* starts a business with a capital of Rs 4000. After 3 months, he is joined by *B* who brings in Rs 6000 ; and at the end of another 3 months *C* joins the business with Rs 10000. The year's profit is Rs 5400. Divide it between the partners on the basis of average invesment.

[*Ans.* *A* : Rs 1600 ; *B* : Rs 1800 ; *C* : Rs 2000

5. *A* owns $\frac{2}{5}$ths of a partnership with *B*. It is agreed that *A* will draw a salary at Rs 250 a month for acting as the manager ; while *B* is to receive an over-riding commission of 2½% on the turnover as the salesman. In a certain year the profits amounted to Rs 12010, and the sales to Rs 80400. What return do the incomes of *A* and *B* represent on their respective capitals of Rs 20000 and Rs 25000 ? [*Ans.* 29% and 24·8%

6. Of the capital Rs 25000 in a partnership between *A* and *B*, the former brought in Rs 20000. After some years *A* retires from active partnership on condition that an annuity of Rs 3000 to him should be a first charge on the income so long as he does not withdraw his capital from business, irrespective of the extent of profit or loss. He is further to be entitled to a 4as share in the net profit of the firm. What average return on investment did this arrangement mean to either, if the gross profits before paying *A* during the following 3 years were Rs 3200, Rs 6400 and Rs 10000 ? [*Ans.* *A* : 19½% app. ; *B* : 53%

7. *C* was admitted to a partnership between *A* and *B*. *A* agreed to give up $\frac{2}{5}$ths of his share, and *B* $\frac{1}{10}$th of his, to *C*. When the profits were divided *C* got Rs 1500. How much did the other partners receive, if originally A's share in the business exceeded B's by $\frac{5}{16}$ths of his own share ?

[*Ans.* A : Rs 1920 ; B : Rs 1980

8. A partnership business was managed by A at Lahore and by B at Srinagar. It was agreed that half the net income from each of the branches should be retained by the local partner as his remuneration. The balances are to be pooled and divided between the partners in the ratio of 5 : 6. Divide between them the income of Rs 10000 at Lahore and Rs 12000 at Srinagar.

[*Ans.* A : Rs 10000 and B : Rs 12000

9. Three men A, B and C go into business ; A contributing Rs 5000, B Rs 3000 and C Rs 2500 of the capital, on the understanding that after allowing $\frac{1}{8}$th of the profits to C as manager, the remainder should be divided amongst them all in proportion to the amount of capital contributed by each. At the end of the year C received Rs 280. What were the total profits of the business, and how much did A and B receive respectively ?

—I. Com. [*Ans.* Rs 840 ; Rs 350 ; Rs 210

10. Two partners start in business, A contributes £12000 and B £18000. B was to have 15% of the profits for his salary as manager. At the end of seven months A withdrew $\frac{1}{3}$rd of his capital, and two months later B withdrew $\frac{1}{2}$ of his. The profits for the year amounted to £3130. What sum of money ought each to receive ? —I. Com. [*Ans.* A : £1054 ; B : £2076

11. A, B and C have respectively Rs 50,000, Rs 35,000 and Rs 25,000 invested in a business. A and B receive respectively 20% and 10% of annual net profits as salary. The residue of the profits is divided among them in proportion to their capital. If at the end of the year A receives altogether Rs 1200 more than B, what does each receive ? —I. Com.

[*Ans.* A : Rs 3181 6as ; B : Rs 1981 6as ; C : Rs 976 12as

12. A, B, C and D, having capitals of Rs 12000, Rs 5000, Rs 2000 and Rs 1000 respectively in their firm, agree to pay interest at 5% p. a. on their capital, and to share profits and losses in the ratios of 4 : 3 : 2 : 1.

At the end of the year it is found, that the firm has incurred a net trading loss of Rs 550. If the partners decide to start the next year without an adverse balance, find what sums should be paid by or to the firm in the case of each partner. —I. Com.

[*Ans.* *A* : Rs 20 ; *B* ; Rs 215 ; *C* : Rs 210 ; *D* : Rs 105 paid

13. *A*, *B* and *C* are in partnership having capitals Rs 7500, Rs 5500 and Rs 3000 respectively. They agree to divide net profits, so that *A*, *B* and *C* will have $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{6}$ respectively. If interest at 4% p. a. be paid on capital, and *B* and *C* received Rs 350 and Rs 250 per quarter for their services to the firm, what would each receive by the end of the working year, if the net profit be Rs 8960 before the interest on capital and salaries of *B* and *C* were paid ? —I. Com.

[*Ans.* *A* : Rs 3260 ; *B* : Rs 3593 5as 4p ; *C* : Rs 2106 10as 8p

14. *A* and *B* enter into a partnership on the following conditions : (*a*) interest @ 6% p. a. will be charged to business on *A*'s capital of Rs 20000 and *B*'s capital of Rs 25000 ; (*b*) partners may draw Rs 250 p. m. each against anticipated profits, which they do ; (*c*) *B*, acting as the manager, will be entitled to a remuneration of 1% on the gross sales ; (*d*) and profits and losses are to be divided in the ratio 2 : 3 ; (*e*) the balances to the credit of partners to be left in the business for 3 years. The total sales in a year amounted to Rs 10,00,000 ; and the gross profit to Rs 47800, before taking *B*'s commission into account. By how much will the partners' accounts be credited at the end of the year ? [*Ans.* *A* : Rs 12240 ; *B* : Rs 29560

15. In terms of an agreement two partners in a business, *A* and *B*, are to be allowed to draw salaries at the monthly rate of Rs 250 and Rs 120 respectively. The capitals contributed by them are Rs 5000 and Rs 4000 respectively, that brought in by *C* being Rs 20000. Interest is agreed to be allowed at the rate of 4% per annum ; and profits are to be divided in the proportion of 1 : 1 : 6. Distribute a profit of Rs 6000 in the first year amongst the partners. [*Ans.* *A* : Rs 3250 ; *B* : Rs 1650 ; *C* : Rs 1100

16. A match factory suprintendent is engaged on the understanding that he is to receive 10% of the net profit of the business as salary, after his salary has been deducted as an expense. The profits for the year were Rs 13200. Find the superintendent's salary for the year. [*Ans.* Rs 1200

17. An advertising expert is taken into a proprietory business on the following terms : 10% on all business obtained in response to adverisements, 15% special on all such business in excess of Rs 30000 in a year ; and a fixed allowance of Rs 100 p.m. Ascertain the average cost of employing the expert, expressing it as a percentage of the average business in the following years, amounting to Rs 10000, Rs 35000, and Rs 45000. [*Ans*: $17\frac{1}{3}\%$

*__18.__ *A* started a business on Jan. 1 1932 with Rs 10000. On May 3 following *B* brought Rs 20000 into the business. On August 6 next *C* was tempted with the prospects of the business and joined it with Rs 15000. Out of a total profit of Rs 20900, *B* and *C* agreed to pay *A* a bonus of Rs 1180 from their share of profits on the basis of average investment, in addition to a salary of Rs 3000 to *A*. Excepting the salary and bonus due to *A*, which were drawn out in full, the whole of the profit was left in the business. Show the balances at the credit of the partners on Jan 1 1933. [*Ans.* *A* : Rs 16100 ; *B* : Rs 27290 ; *C* : Rs 18330

*__19.__ *A* and *B* enter into partnership with a borrowed capital of Rs 2000 for which they are jointly and severally liable. The amount is to be repaid in equal monthly instalments of Rs 100 each with interest at 12% p. a., on the amount outstanding. The partners agree not to divide profits till the loan is paid back in full with interest. If the first two years' profits of the business are Rs 900 and Rs 1400, divide the surplus equally between the partners. [*Ans.* Rs 45 each

*__20.__ A receiver was placed in a charge of a partnership business in voluntary dissolution. His remuneration was to be at the rate of Rs 150 a month, not exceeding Rs 2000 on the whole, as cost of office establishment etc. ; 20% on all realisation on bills receivable and 10% on other assets (except cash). The liquidation lasted for 14 months. The assets realised the following amounts : Cash, Rs 5000 ; Stock-in-trade, Rs 18500 ; Furniture and Fixtures Rs 2200 ; Motor Cars Rs 2300 ; Lorries

Rs 6800 ; Goodwill Rs 1000. The total value of the Bills Receivable was Rs 48000, and proved to be 20% bad. Find the amount received by each of two equal partners, after meeting creditors of Rs 46440. [*Ans.* Rs 7500 each

***21.** The assets of a partnership were valued in the books of account at Rs 44000. *A*'s share of the business was $\frac{3}{5}$ths, the rest being divided equally between *B* and *C*. The assets were divided between the partners before realization. *A* sold his share outright at 10% above its book value. *B* was charged with the realization of *C*'s share on a commission of 5% only on bills collected on *C*'s behalf. He could obtain 25% in excess of the value of the stock, furniture etc. in their share, valued in the books at Rs 10000 ; but could not realise more than 80% of the Bills Receivable. Find how much was received by each partner from the dissolution, and which of the partners made the best bargain. [*Ans.* *A* : Rs 29040 ; *B* : Rs 9442 ; *C* : Rs 9138 ; A

***22.** *A* retires from an active partnership with *B*, leaving his capital of Rs 20000 in the business, on condition that he gets an 8 annas share of the profits of the business, after paying him a guaranteed annuity of Rs 3000. *C*, the firm's manager, now comes into the partnership with a capital of Rs 5000, paying a premium Rs 3000 to be divided between *A* and *B* in proportion to their capital. If *B*'s capital is Rs 10000, ascertain the minimum profit that *A* may be assured of an income of Rs 6000 in the following year, excluding his share of *C*'s premium. How much will *B* and *C* receive on this basis, if they share profits equally after paying *A* ?

[*Ans.* Rs 9000 ; Rs 1500 each

***23.** The capital of a partnership is Rs 50000, contributed by *A*, *B* and *C* in the proportion of 2 : 2 : 1. Interest is arranged to be allowed on the capital at 6% p. a. before division of profits ; and charged at the same rate on drawings of partners in excess of the following monthly limits : *A*, Rs 200 ; *B*, Rs 150 ; *C* Rs 100.

The drawings were as follows :

	A	B	C
Date drawn	Rs	Rs	Rs
1st January	150	50	50
1st February	230	100	100
1st March	250	200	150
1st April	400	400	200
1st May	200	300	100
1st June	150	150	150

Distribute a profit of Rs 10000 in proportion to the respective capitals, during the period January to June, deducting drawings and interest thereon. Answer to nearest rupee.

[*Ans. A* : Rs 2621 ; *B* : Rs 2799 ; *C* : Rs 1250

*24. *A* owns ⅔-rds of a partnership with a total capital of Rs 30000. Later, *C* joins the partnership with Rs 50000, on the understanding that, besides receiving interest at 5% p. a. jointly with the other partners, he will be entitled to a half-share of the business, the proportion of *A*'s to *B*'s share being changed to 3 : 2. Three months later, *C* withdraws Rs 10000 of his capital, agreeing to give up 20% of his half-share of profits to be rateably distributed between *A* and *B*. Distribute the year's profit of Rs 29000. [*Ans. C* : Rs 12275 ; *B* : Rs 6590 ; *A* : Rs 10135

*25. A partnership between *A* and *B* is dissolved to enable the admission of *C* into a new partnership. Originally *A* and *B* shared profits in the proportion 5 : 3 ; under the new arrangement their shares are : *A*—8as, *B*—5as and *C*—3as. It is expected that the admission of *C* will result in an increase of 20% in the divisible profits. Ascertain whether the change over would be profitable to *A* and *B*.

[*Ans. A*'s share decreases by 4% ; no change in *B*'s share

§ **Government Loans and Corporation Debentures.** Governments, Corporations and public bodies sometimes take loans from the public at fixed rates of interest. The document which states the amount of the loan, the time of repayment and the rate of interest, is known as a Security. It confers on the holder the right to repayment, the right to receive the stated interest on due dates, and the right to transfer it to any other person for consideration. When such loans of public bodies are charged against definite assets, the documents are known as Debentures.

The time of repayment is ordinarily fixed in the case of a Debenture, and is usually stated as a period in case of Government Loans and Loans of public bodies. These are known as Redeemable Stocks or Terminable Loans. Some Government Loans are however in perpetuity ; and the holder of such stock is entitled only to receive on due dates the fixed interest thereon in perpetuity. These are called Irredeemable Stocks.

§ Financing of Joint Stock Companies and Apportionment of Profits. The capital of a joint stock company is subscribed by the shareholders. Persons taking the initiative in the flotation of a company are known as *promoters*, and the affairs of the company are managed by representatives of the shareholders called *directors*. In some cases, some of the promoters are prepared to stake more than the ordinary shareholder by offering to have their dividends deferred till the former can be paid a pre-determined substantial dividend. The shares they take up are known as *Deferred Shares* or *Founders' Shares*.

Some of the shareholders are prepared to take larger risks of the business, and the shares held by them are known as *Ordinary shares*. Others are not willing to wait for high problematic dividends, but are satisfied with a reasonable guaranteed return on their money out of profits, before the ordinary shareholders can be paid at all. These shares are known are *Preference shares*, and usually this preference also extends in favour of the holders in the matter of distribution of assets in case the company is wound up. While these dividends can be paid only *out of profits*, the limit set by the guarantee of return cannot be exceeded even if profits are very much in excess. All such profits go to the ordinary shareholders, up to a certain limit, usually high, before the deferred shareholders can claim a share.

The total capital with which a company in registered is known as the *Nominal* or the *Authorised capital.* That portion of the nominal capital which is issued for subscription is known as the *Issued capital.* The amount represented by the applications for shares is the *Subscribed capital*; and the amount actually paid up represents the *Paid-up capital.* The initial expenses incurred in floating the company are known as *Preliminary Expenses*, and the balance of paid-up capital after providing for them is the initial *Working capital.*

§ In all cases the proprietors' interest in a Joint Stock Company is represented by the *paid-up value* of the shares held by each. A shareholder's financial liability is limited to the face value of the shares held by him. But he has usually an unrestricted right to share, along with other shareholders, in the profits of the company. A share confers on its holder the right of transferring it to some other person for consideration, subject, if necessary, to the payment of unpaid calls.

The right to receive a share of profits may however vary in character and extent. A Preference Shareholder is guaranteed a fixed rate of dividend, provided there is profit adequate for the purpose. His right, however, is extinguished at the end of every accounting year, if profits of the year are insufficient. The holder of a *Cumulative Preference Share* is in a somewhat better position; for his right to receive his fixed rate of interest is carried over to the following financial years; and till all arrears of their claims have been met, the other Preference Shareholders and others are not entitled to any part of the profits. The Ordinary Shareholder has no such extraordinary rights; and as he takes the largest risk in connection with the business of the company, his right to share in the profit is usually unrestricted; and when a 5% Preference Shareholder gets his fixed 5%, an Ordinary Shareholder may get as much 10%, 15%, 20%, 60% or even more in a successful year. *A Participating Preference Share* entitles the holder to share with the deferred shareholders profits in excess of a minimum percentage allowed to the latter, after payment to ordinary and preference shareholders.

§ Under certain conditions, a company may issue new shares, either at par or at a premium, and even at a discount, for securing increased finance ; or issue *bonus shares* to the existing shareholders for capitalizing accumulated profits. Sometimes a company may choose to reduce its nominal capital by cancelling the unissued shares or the uncalled capital ; or by cancelling a portion of the paid-up capital to write off heavy losses. The procedure in those matters is somewhat involved, and the interest of the creditors is sought to be secured by requiring that the permission of the Court be obtained beforehand.

§ The premium received at the issue of new shares is a capital income, and is usually transferred to Reserves. Bonus paid in cash goes to deplete the Reserves created out of undistributed profits in the past. The issue of bonus shares, however, increases the capital while reducing the Reserves. It may be noted that any increase in the capital tends to depress the rate of dividend while a reduction of capital has the opposite effect.

§ Money is also raised by mortgages on the stock-in-trade, plant and machinery, and even on goodwill or anticipated profits. When considerable additional funds are required by Joint Stock Companies, they are raised on mortgage of assets by means of Debentures issued in small units to the public who take them up in return for money advanced. Debenture-holders are thus secured creditors and get a fixed periodical return on their money, which is known as interest. Interest on debentures has to be paid in time, irrespective of the amount of profit or loss that may be sustained by the business ; and profits cannot become divisible among the shareholders before the interest due on debentures has been paid in full.

Illustrations

1. A company is registered with a capital of Rs 2 00 000 divided into 20 000 shares of which 15 875 shares are issued. Rs 2 8as is paid on each share applied for, and Rs 5 is to be paid

on each on allotment. The total amount realised by the issue of shares is Rs 119375. Find the number of shares applied for.

Total sum realised	Rs 1 19 375
Allotment money on 15 875 shares @ Rs 5 each	79 375
Application money @ Rs 2 8as per share	Rs 40 000
∴ No. of shares applied for =	16 000

2. The share capital of a company was made up as follows :

1. 5% Preference shares 8 750 of Rs 100 each.

   Payable : Rs 25 with application, Rs 25 on allotment and 2 calls of Rs 25 each.

2. Ordinary shares 20 000 of Rs 10 each.

   Payable : Rs 2 8as with application, Rs 2 8as on allotment and 2 calls of Rs 2 8as each.

All the Preference shares were taken up, and 19 200 of the ordinary shares were applied for and allotted.

During the year only one call was made on the Preference shares, and 80% was duly received ; and 90% of the ordinary shares were fully paid up, as called. What dividend can be declared on paid up capital out of profits for the year amounting to Rs 49 000 ?

Available Profit		Rs 49 000
Preference Share Capital—		
8 750 shares application & allotment	Rs 4 37 500	
Call 80% of shares @ Rs 25 each	1 75 000	
	Rs 6 12 500	
Dividend @ 5%		Rs 30 625
Ordinary Share Capital—		
19 200 shares application & allotment	Rs 96 000	
Call 90% shares @ Rs 5 each	86 400	
	Rs 1 82 400	

Balance available for dividend on ordinary share Capital Rs 18 375

∴ Maximum dividend permissible = 10%.

3. Calculate the dividend that may be paid on each share of Rs 25 (Rs 20 paid up) out of profits amounting to Rs 61 865 10as. The share capital of the company is Rs 20 00 000.

Share capital	Rs 20 00 000
Do. paid up	Rs 16 00 000
Profit	Rs 61 865 10as

$$\therefore \text{ Dividend } = \text{Rs } \frac{61\,865{\cdot}625}{1\,600\,000} \times 20$$

$$= \text{Rs } \frac{{\cdot}61865625 \times 5}{4}$$

$$= \text{Rs } \frac{3{\cdot}0933}{4} = 12\text{as} +$$

$\therefore$ Dividend permissible = 12as on each share.

4. It would be possible to pay a dividend of 12½% instead of 9%, if a sum of Rs 14 770 14as were not transferred to Reserve. What is the share capital ?

Possible Dividend 12½%	$\therefore$ Share capital
Div. Recommended 9 %	$= \text{Rs } \frac{14\,770\ 14\text{as} \times 100}{3\frac{1}{2}}$
Difference 3½%	
Corresponding amount	= Rs 2 110 2as × 200
Rs 14 770 14as	= Rs 4 22 025.

5. A company with a share capital of Rs 5 00 000 made a profit of Rs 57 625 10as in a year. The directors recommend that at least 25% of the profits be transferred to Reserves. What dividend could possibly be declared ?

If the shareholders desired that the dividend recommended be declared free of income-tax, how would the Reserves be affected ? Income-tax is to be paid @ 26p in the rupee on the profits, plus a surcharge of 25%.

Profit	Rs 57 625 10as
Reserve @ 25 %	Rs 14 406 0 to nearest rupee
Available for distribution	Rs 43 219 10as

$$\text{Proposed Div. } \% = \frac{\text{Rs } 43\,219\ 10\text{as}}{\text{Rs } 5\,00\,000} \times 100\% = 8{\cdot}6\%$$

Dividend recommended by Directors = 8½%.

*(*b*) If a dividend is to be paid @ 8½% free of Income tax, the net profits allocated for the purpose should be = Rs 42 500.

Gross Profit amounts to	Rs 57 625 10as 0p	
Income-Tax @ 26 p + surcharge	9 754 5 4	
	Rs 47 871 4as 8p	
less Dividend (net) payable	42 500 0 0	
Net Amount available for transfer to Reserve		Rs 5 371 4as 8p
Reserve proposed by Directors	Rs 14 406 0a 0p	
less Income-Tax & Surcharge	2 438 8 3	
Net amount		Rs 11 967 7as 9p
Difference in Reserve		Rs 6 596 3as 1p

EXAMPLES 19

1. A company with a fully paid up-capital of Rs 100 000 earns profits of which Rs 21 531 4as is available for distribution amongst the shareholders. Find the rate of dividend. [*Ans.* 3as 5p in Re 1

2. The profits in a year of a joint stock company amount to Rs 83 564 8as. It is decided to transfer a sum of Rs 28 364 8as to Reserves. Find the dividend on a paid up capital of Rs 13 80 000. [*Ans.* 4%

3. The sudden failure of a heavy debtor of a company involved it in a loss of Rs 72 650 10as against profits amounting to Rs 1 13 512 in a year. It was decided to provide for Rs 50 000 of the loss before division of profits, and to carry forward the balance to the following year. Find the rate of dividend that can be paid on a paid up capital of Rs 15 87 800. [*Ans.* 4%

4. A company with a fully paid up share capital of Rs 12 lakhs earns profits in a year amounting to Rs 23 864 12as. Find the dividend that can be paid to the shareholders free of income tax at 26 pies in the rupee. [*Ans.* 1½% +

5. A company is authorized to issue shares as follows :

10 000	Ordinary shares of	Rs 10 each
1 000	6% Preference shares of	Rs 100 „
2 000	Founders' shares of	Rs 25 „

The Founders' shares are paid for in full ; 6000 ordinary shares are underwritten at a commission of 5%. 3715 ordinary shares and 785 preference shares are also allotted. Rs 2 8as is paid on an ordinary share with application, and Rs 2 8as on allotment except on 510 shares. Rs 25 is paid on each preference share with application, and another Rs 25 on allotment, on all shares applied for. If the preliminary expenses amount to Rs 28 836, write down the nominal, the issued, the subscribed, the paid-up and the working capital at start.

[*Ans.* Nominal and Issued	Rs 2 50 000
Subscribed	„ 2 25 650
Paid-up	„ 1 36 550
Working	„ 1 04 714

6. In the above example, during the year the outstanding calls were made and were fully paid up. If the divisible profits amounted to Rs 51 875, find what dividend may be paid to the deferred (Founders') shareholders, if at all, after the ordinary shareholders have been paid a dividend of 12½% [*Ans.* 70%.

7. After the first five years of working a cotton mill, profits amounting to Rs 36 812 8as became available for distribution for the first time in 1937 amongst the following :

6% Preference Stock	Rs 20 000
5% Cumulative Preference Stock	Rs 40 000
Ordinary Stock	Rs 40 000
Secret Reserve Fund @ 10% on	Rs 2 08 125

What is the dividend on the ordinary stock ?—and how much per cent is the total profit of the Capital ? [*Ans.* 12% ; 36·8%

8. A company with a fully paid up capital of Rs 5 00 000 decides to reduce its capital by 25%. In the following year a surplus of Rs 12 625 is available for distribution amongst the shareholders. Calculate the rate at which dividend can be paid

on the reduced capital. What return does it represent to an original shareholder on his investment? [*Ans.* $3\frac{1}{4}$% ; $2\frac{1}{4}$% app.

9. A person buys, at Rs 6 4as per share, 900 shares of Rs 10 each, of which Rs 5 only is paid up. He pays the next call of Rs 2 8as per share, and sells up the entire holding in two lots of 500 shares at Rs 9 2as and 400 shares at Rs 9 4as each. If he pays brokerage at 4as per share each time, find his profit or loss on the transaction. [*Ans.* Loss Rs 62 8as

***10.** A person applies for 100 shares in a company of Rs 100 each, paying a deposit of 20% of the value of the shares applied for. He is allotted 90 shares and is informed that the balance of his money would be adjusted towards payment of the allotment money at the rate of Rs 40 per share. He paid up the allotment money and the first call at Rs 20 per share, but has to borrow Rs 3 200 at 12% p.a. for 3 months to meet the calls. If he sells out 50 shares after 3 months at a premium of Rs 11 per share, and repays the loan with interest, find his profit, valuing his holding at par. [*Ans.* Rs 454

***11.** The Indian Forest Products Ltd. was registered with a capital of Rs 50 00 000, and issued Rs 20 00 000 divided into 40 000 shares of which 5 000 were subscribed by the promoters. The promoters paid their share money in full ; and the public applied for 18 000 shares, paying Rs 5 per share with application. When the allotment money of Rs 15 per share was realised, it was seen that 39 applications for 480 shares had not been followed up ; and they were forfeited. The first call of Rs 10 per share was met in full ; but the next call of Rs 10 per share was paid on 16 843 shares only. Write down the Nominal, the Issued, the Subscribed, the Called-up and the Paid-up Capital.

Determine the rate of dividend that may be declared with profits amounting to Rs 28 618 available for the purpose. [*Ans.* 3%

***12.** The Capital of a Company is made up as follows :

20 000	Ordinary shares of Rs 100 each fully paid up, the maximum dividend payable on which is 12%.
5 000	Deferred shares of Rs 100 each also fully paid up.
10 000	Preference shares of Rs 100 each carrying 6% p.a.

It now issues 1000 new Ordinary shares of Rs 100 each issued at a premium of Rs 20, which are immediately subscribed and fully paid up. Distribute a profit of Rs 483 700, setting aside a sum of at least Rs 46 658 for Reserves.

[*Ans.* 25% on Deferred Shares

*13. The share capital of a company is made up as follows :—

15 000 Ordinary shares of Rs 100 each ; Rs 10 lakh 6% Preference shares of Rs 100 each ; 5000 Cumulative Preference (5%) shares of Rs 100 each (Rs 50 paid up). The Court sanctions a reduction of capital on the following basis :—

(i) The uncalled capital is to be cancelled.

(ii) Preference shares to be converted to Rs 50 fully paid up shares.

(iii) Ordinary shareholders to get a refund of Rs 60 on each share in cash.

The value of each share is to be correspondingly reduced in respect of Preference stocks ; and the ordinary share is to be changed into Rs 100 shares (Rs 50 paid up).

Divide the next year's profit of Rs 80, 000.

[*Ans.* Dividend on ordinary stock 5%

*14. The reserve fund of a company has accumulated to Rs 1,30,000, and is to be partly distributed in the following manner :

Rs 20,000 in the shape of bonus to Rs 1,00,000 deferred stock.
Rs 20,000 in paying up the final call on Rs 1,00,000 ordinary stock.
Rs 20,000 capitalized by the issue of a Rs 100 fully paid up Preference Share to every holder of 5 Rs 100 Preference shares of total value Rs 1,00,000

What are the new stocks of the three kinds ? Discuss the effect of this distribution on future dividends.

*15. A Motor-car Company offers for subscription :—

1,00,000 Ordinary shares of Re 1 each, and 1,00,000 7% Preference shares of Re 1 each. The Company, although it anticipates profit of 3as per mile run per day, is content to estimate a profit of 2as per mile per day with runs of 100 miles per day for each of 10 motor cars and an average of 300 days to the year. Calculate the net estimated profits. At the end of the year the Company finds that its estimates for administrative expenses and profit of 3as per mile run are correct. What dividend did it pay on the Ordinary shares, if Rs 20,000 was placed in the Reserve fund ?—P. S. C.

***16.** Study the division of profits and the growth of the Reserve and changes in the rates of dividend of a Joint Stock Company with a fully paid up Share Capital of 50 000 ordinary shares of Rs 10 each, of Rs 20 000 6% A. Preference Shares, and of Rs 30 000 6% B. Preference Shares (Cumulative). Undistributed balances are also to be transferred to Reserve. The profits were as follows :

	Profits	Transfer to Reserve		Profits	Transfer to Reserve
	Rs	Rs		Rs	Rs
1927	3 625	3 000	1930	38 635	5 000
1928	18 528	4 000	1931	44 486	6 000
1929	28 428	4 500	1932	58 648	8 000

Winding up and Bankruptcy. A person, firm or company, unable to pay its debts, is said to go bankrupt ; when its affairs are wound up. It sometimes happens that the affairs of a business undertaking are voluntarily wound up, *e.g.* at the death of a partner, or with a view to amalgamation. In all such cases, a person is placed in charge of this work, who is known as a Receiver or a Liquidator. His duty is twofold ; he acts as a trustee of the creditors in realising assets and in distributing the proceeds equitably among them in accordance with their claims and legal rights, and on the other hand, he acts on behalf of the proprietors in their best interests.

§ A Joint Stock Company may be wound up voluntarily ; when, for example, it is unable to pay its dues, or, when reconstruction or amalgamation is in view ; or, it may be wound up by order of the Court on the application of creditors who cannot be paid.

The assets are realised at the best prices, these including all properties, movable and otherwise, goodwill etc. Moneys due from debtors are realised, although a portion thereof usually turns out to be bad ; and lastly, all balances of share money due from the shareholders are called up. A liquidator is given extensive powers to do these acts.

§ When the assets are less than the liabilities, the net proceeds thereof are distributed amongst the creditors, after meeting the expenses of liquidation, rateably with the amounts due to them. When the assets exceed the liabilities, the creditors are paid in full, and the balances handed over to the proprietors. The surplus is divided amongst partners in the manner provided for in the Partnership Deed. The shareholders of a Joint Stock Company have usually varying degrees of preference in the matter ; *e.g.* preference shareholders have usually a prior claim to ordinary shareholders.

§ In the case of winding up or bankruptcy, a distinction is made between different classes of creditors. A creditor, who holds a lien against any asset under a registered deed, releases the property only after satisfying himself. A creditor whose dues represent the result of the risk he undertook in entering into business relationship with the bankrupt is not entitled in equity to the same privileges that are enjoyed by a creditor to whom money is due in consideration of services rendered, without the implications of business risks. Thus a merchant selling goods to a bankrupt on credit with a view to making a profit on the transaction, does so at some risk ; but an employee, who puts in hard labour with a view to earning his bread, takes on himself no risks of the business.

The law, therefore, lays down a list of persons who are to be regarded as Preferential Creditors, who have to be paid in full, if possible, before the ordinary creditors can be paid rateably. If the assets realise an insufficient amount, the law provides for some order of preference, even amongst the preferential creditors ; those having the same position in the order having a right to an equitable distribution in case of need.

Government dues are always preferential, after the liquidator's expenses have been met in full. Protection is given to public utility companies, municipalities etc. by way of placing their names in the list of preferential creditors. It is obvious why all expenses incurred by the receiver or the liquidator, consequent on the winding up, are a first charge on the realisation of the assets.

The following gives an idea of the sequence of priority of creditors, after Liquidation expenses have been met in full.

Class I. (*a*) Revenue, Rates and Taxes etc. due to Government or a Local Authority ; *e.g.* Municipality, District Board etc.
(*b*) Salaries and wages due to employees, up to the last 4 months, not exceeding Rs 300 and Rs 100, respectively, in any one case ; also rent for 1 month.
(*c*) Workmen's Compensation.
(*d*) Employees' Provident Fund etc.

Class II. Secured Creditors, as Debenture-holders, etc.

Class III. Ordinary Creditors.

It will be noticed that the effect of paying out liquidation expenses is to reduce assets only ; whereas the effect of paying preferential claims in full is to reduce liabilities to the same extent as the assets.

The *dividend* is the rate at which assets available are finally distributed amongst the ordinary creditors. It is also used to describe the amount received by a creditor from the assets of a bankrupt. The rate is usually declared as a money-rate.

Illustrations

1. A person goes bankrupt for Rs 3814 12as and has assets of the value of Rs 1586 10as. If Rs 685 of the liabilities be preferential, find the dividend that may be paid. How much will a creditor receive out of a claim for Rs 538 ?

Assets	Rs 1586 10as	Liabilities	Rs 3814 12as
Preferential claim	685	Preferential claim	685
Divisible Assets	Rs 901 10as	Outstanding Liab.	Rs 3129 12as

$$\therefore \quad \text{Dividend} = \frac{\text{Rs } 901\text{·}625}{\text{Rs } 3129\text{·}75} \times 16\text{as in Re } 1$$

$$= \frac{14426}{3129\text{·}75} \text{ as/Re } 1$$

$$= 4\text{as } 7\text{p in Re } 1.$$

Creditor for Rs 538 gets :

$@\frac{4\text{as}}{\text{Re } 1}$	Rs 134 8as
$@\frac{6\text{p}}{\text{Re } 1}$	Re 16 13as
$@\frac{1\text{p}}{\text{Re } 1}$	Rs 2 13as
	Rs 154 2as

2. A receiver is appointed to take charge of a bankrupt's estate on a remuneration of 5% of the gross realisation *plus* Rs 25 per month for office expenses. The assets altogether realised Rs 15862 12as, from which the auctioneers deducted their charges to the extent of Rs 738 4as. If the liabilities amount to Rs 21131 5as 9p out of which claims for Rs 5131 5as 9p rank as preferential, determine the dividend that might be paid, taking into account the payment of office expenses for 8 months.

Assets	Rs 15862 12as	
less Auctioneers' charges ...	738 4as	
	Rs 15124 8as	
less Receiver's charges		
@ 5% on Rs 15862 12as	Rs 793 2as 3p	
Office expenses		
@ Rs 25 p.m. for 8 months	Rs 200 0a 0p	
Assets available for distribution	Rs 14131 5as 9p	
Preferential claims	5131 5 9	
Net assets for distribution		Rs 9000
Liabilities	Rs 21131 5as 9p	
Preferential claims	Rs 5131 5as 9p	
Outstanding Liabilities		Rs 16000

$$\therefore \quad \text{Dividend} = \frac{\text{Rs } 9000}{\text{Rs } 16000}$$

$$= \underline{9\text{as in Re } 1.}$$

3. A person goes bankrupt for Rs 59704. Among his assets are included Bills Receiveable to the extent of Rs 42680 of which 10% is not realised, the rest of the assets realising Rs 5716. If the receiver's expenses and charges amount to Rs 6813, find the loss sustained by a creditor who puts in a claim for Rs 6350.

Assets :

Bills Receivable Rs 42 680		
less 10% bad	4 268	Rs 38 412
Other assets		5 716
		Rs 44 128
Receiver's charges & expenses		6 813
Assets available for distribution		Rs 37 315
Liabilities		59 704

$$\therefore \quad \text{Dividend} = \frac{\overset{5}{\cancel{37315}}}{\underset{7463}{\cancel{59704}}} \times \overset{2}{\cancel{16}} \text{ as in Re 1}$$

$$= 10\text{as/Re } 1$$

$\therefore$ Loss on a claim = 6as/Re 1

Loss on a claim for Rs 6 350 @ 4as/Re 1	Rs 1587	8as
,, ,, ,, ,, @ 2as/Re 1	Rs 793	12as
Total loss on claim...	Rs 2381	4as

EXAMPLES 20

1. Calculate dividends on the following ordinary claims obtaining rates of dividend to the nearest hundredth of a pie :

	Assets	Liabilities	A Claim
(i)	Rs 35000	Rs 58615	Rs 6315
(ii)	Rs 18615 8as	Rs 43603 10as	Rs 7086
(iii)	Rs 42308 6as 9p	Rs 83815 4as 9p	Rs 5783 8as
(iv)	Rs 13565 6as	Rs 64303 10as 3p	Rs 4614 5as
(v)	Rs 64817 3as	Rs 165602 13as	Rs 9095

[*Ans.* 9as 6·65p ; 6as 9·97p ; 8as 0·92p ; 3as 4·50p ; 6as 3·15p in the rupee.

[Rs 3770 15as ; Rs 3025 3as ; Rs 2919 7as ; Rs 973 5as ; Rs 3559 13as.

2. A dividend was declared at 3as 9·5 pies in the rupee by the receiver of a bankrupt estate. Assuming that the liabilities amounted to Rs 51365 5as, what must have been the assets if the receiver's expenses amounted to Rs 2615 9as ?

[*Ans.* Rs 14788 1a 3p

3. Write up a Table of Nine Values for the dividend 1a 3·8p in the rupee and ascertain the payment that will have to be made to the creditors for the amounts given below :

Rs 685 ; Rs 318 12as ; Rs 5784 6as ; Rs 10876 5as 6p ; Rs 3008 2as 9p.

[*Ans.* Rs 56 6as ; Rs 26 3as 9p ; Rs 476 ; Rs 895 0a 6p ; Rs 247 8as 9p

4. A first dividend of 5as 4p in the rupee was declared by the receiver of a bankrupt's estate, the claims against which amounted to Rs 15180. The receiver's charges including his office expenses etc. amount to Rs 1175. If the assets realised Rs 7500, at what rate will the second and final dividend be paid ?

[*Ans.* 1a 4p in Re 1

5. A person goes bankrupt for Rs 42360. His assets realise Rs 15 840. If the receiver's expenses be Rs 2602 8as, find the dividend. [*Ans.* 32%

6. The assets of a bankrupt realised Rs 52865. The claims against the estate amounted to Rs 180613 12as, of which Rs 16385 12as was directed by the Court to be paid in full. At what rate in the rupee will the other claims be paid ?

[*Ans.* 3as 6·648p

7. The liabilities of a bankrupt were Rs 136000 and his assets were estimated at Rs 97500. The creditors received a first dividend of 8as in the rupee. On being wound up the assets realised only 85% of the estimated amount and the legal expenses absorbed 12½% of the sum realized. What additional amount n the rupee should the creditors receive ? What total sum should a creditor for Rs 27500 receive, assuming that he has no preferential claim ? —I. Com.

[*Ans.* 6⅜ pies in Re 1 ; Rs 14663 1a 6p

8. If a bankrupt pays 15s in the £ on ⅔-rds of his debts and 14s in the £ on the rest, he has £2 too little, but if he pays 14s in the £ on ⅔-rds of his debts and 15s in the £ on the rest, he would have £6 left; find his assets. —I. Com. [*Ans.* £480

9. The debts of a bankrupt amount to £2134 10s 6d. His assets consist of property worth £916 15s 4d and of a bill for £513 due 4 months hence, simple interest being reckoned at 4% p. a. How much in the £ (to the nearest penny) can he pay to his creditors? —I. Com. [*Ans.* 13s 4d/£1

10. A debtor can pay 17s 6d in the £; but if the creditors would take 20% off their dues, he could pay them and have £45 left. What is the amount of his debts, and what is the value of his assets? —I. Com. [*Ans.* £600; £525

11. The assets of a bankrupt realised Rs 11865. Claims were submitted and accepted for Rs 81636. Calculate the dividend correct to the tenth of a pie. [*Ans.* 2as 3·9p in Re 1

12. A bankrupt fails for Rs 143606. His assets are estimated at Rs 98972. A first dividend at the rate of 23 pies in the rupee is paid. On the estate being wound up the assets realize 90% of their estimated value and the expenses of realization absorb 15% of the realized value. How much in the rupee can be paid as final dividend? —I. Com.

[*Ans.* 6as 6·22p/Re 1

13. A bankrupt has book debts equal in amount to his liabilities, but on £3000 of such debts he can recover only 9s 8d in the £, on £1200 only 5s 7d in the £; the expenses of the bankruptcy amount to £1053 15s; he can only pay 10s 9d in the £. Find the amount of his debts. —I. Com. [*Ans.* £7500

14. A bankrupt's debts amount to £3675, and the dividend is reduced from 3s to 2s 6d by the admission of a claim to preferential rank. What was the amount of this claim?

—I. Com. [*Ans.* £105

*15. The books of a firm showed that the assets were represented by the following:

Cash	...	... Rs 10615 13as 8p
Stock	...	... Rs 18008 5as
Bills Receivable		... Rs 7838 10as

The liabilities were ascertained to be as under:

Salary to employees (2 mos.)		Rs 4800
Municipal Tax (3 quarters)		315
Electric Supply Charges	...	56
Secured Creditors	..	18918
Sundry Creditors	...	67514

During the progress of the liquidation the following expenses were incurred by the liquidator:

Legal Expenses	... Rs 648 12as
Office Expenses	... Rs 1530

The liquidator's fees amounted to Rs 4000. Find the dividend that could be declared, if the outstanding bills were realised in full. [*Ans.* 1a 5·6p in Re 1

*16. The realized value of the assets of a company in liquidation amounts to Rs 2 00 000. The creditors, amounting to Rs 4 12 000 include —(i) Cesses and Rates payable to the Calcutta Corporation Rs 2000; (ii) Salary for 3 months to 5 clerks at the rate of Rs 550 each per month; (iii) Wages to 10 foremen for the last 3 months at the rate of Rs 300 each per month. Find the amount received by (i) the Calcutta Corporation; (ii) each clerk; (iii) each foreman; (iv) each of the other creditors. Assume that salaries for service rendered to the company within the 2 months next preceding are preferential up to Rs 1000 in each case; and wages in similar cases up to Rs 500 in each case. —I. Com.

[*Ans.* Rs 2000 ; Rs 1305 8as ; Rs 688 ; 47%

*17. In course of the liquidation of a firm's business, the liquidator realised :

Value of stock	Rs 383684 less brokerage	@ 5%
Sale of kilns etc.	56300 „ „	@ 10%
Furniture & other Movables	10545 „ auctioneer's charge	@ 3%

His list showed that preferential claims amounted to Rs 175000 ; and sundry creditors claimed Rs 303545. His charges and office expenses, legal expenses and incidental charges amount to Rs 15535. If he had already declared an *interim* dividend of 1a in the rupee, what would be the second and final dividend ? [*Ans.* 11as 4·56p in Re 1

18. A dividend at the rate of 5as 6·3p in every rupee was declared in favour of the creditors of a bankrupt estate. Find how much will be received by creditors whose claims against the estate were accepted for (i) Rs 3500 ; (ii) Rs 5684 6as ; (iii) Rs 10681 3as ; (iv) Rs 7321.

[*Ans.* (i) Rs 1208 9as 6p ; (ii) Rs 1962 14as 3p ;
(iii) Rs 3688 5as 6p ; (iv) Rs 2528 0a 6p.

*19. An application of a creditor to treat his claim of Rs 81584 as preferential, was decreed by the Court to rank as preferential up to Rs 41589. If the assets available for distribution be Rs 78614, and the claims totalled Rs 250000, how would the dividend be affected ?

[*Ans.* Dividend reduced by 2as 2·2658p in Rupee 1.

20. An *interim* dividend was paid to the creditors of a bankrupt at the rate of 4as in the rupee. On realisation of all assets, and after payment of the above dividend and the liquidator's expenses, it was found that a further sum of Rs 12185 10as was still available for distribution amongst the creditors whose claims amounted to Rs 87350. Find the final dividend ; and ascertain the loss sustained by a creditor for Rs 15875.

[*Ans.* 2as 2·78p in Re 1 ; Rs 9692 0a 9p

21. If the receiver could finish the work of a liquidation of an estate within 6 months' time, the creditors could get 10as 6p in

the rupee. But the liquidator having taken 6 months more to finish his work, the increased expenses reduced the available assets which could in the end pay only 9as in the rupee. What were the liquidator's monthly charges, if the liabilities amounted to Rs 50000 ? [*Ans.* Rs 781 4as per month

22. The admission of a claim for Rs 3854 6as to rank as preferential resulted in the declaration of a dividend at the rate of 4as in the rupee. If the liabilities of the bankrupt amounted to Rs 44074 6as, what was the value of his assets ?

[*Ans.* Rs 13909 6as

23. The admission of a claim to be treated as preferential reduced the dividend from 3as 6p in the rupee to 2as 9p in the rupee. If the liabilities amounted to Rs 64000 and the liquidator's charges were agreed at Rs 4000, what was the value of the assets, and the amount of the preferential claim ?

[*Ans.* Rs 18000 ; Rs 3622 10as 3p

*24. The following schedule of fees was sanctioned for a receiver for winding up a business :

Commission on Sale of Stock-in-tade, Plant, machinery etc.	@ 10%
„ on realisation of outstanding bills	@ 5%
„ on realisation of other assets	@ 4%
Charges *re* payment to creditors on actual disbursements	@ 1%

Office expenses for 6 months @ Rs 150 per month.

In course of liquidation legal expenses etc. amounted to Rs 5600. Ascertain the dividend payable on claims amounting to Rs 65000, if the assets were as under :

Cash in hand and at Bank ...	Rs 8500
Plant, Machinery, Stock-in-trade etc.	43650
Motor Lorries, Furniture etc. ...	8315
Bills Receivable	10316

[*Ans.* 14as 4·74p in Rupee

***25.** The following figures of the assets and liabilities of a firm are ascertained from the books :

Assets :		*Liabilities* :	
Cash	Rs 6835	Wages & salaries	Rs 13814
Land & Bldgs.	15348	Rates & taxes	318
Furniture & Fixtures	3865	Telephone charges	23
Stock-in-trade	2100	Secured Creditors	5210
Plant & Machinery	18615	Bills Payable	37060

Assuming that the assets were realised at their book values less collection and auction charges (except on cash) amounting to 10% thereof, find the dividend that might be declared after setting aside a sum of Rs 4875 for liduidation expenses.

[*Ans.* 8as in Rupee

***26.** In a partnership business *A*'s share is 4 times that of *B*. On the death of *A* the business is wound up by a receiver who ascertains the position of the firm as under :

Assets		*Liabilities*	
Stock-in-trade	Rs 44500	Mortgage ...	Rs 15000
Furniture &c.	„ 2000	Interest on above	„ 1350
Sundry Debtors	„ 21600	Sundry Creditors	„ 21800

On winding up Rs 2600 of the firm's dues become unrealisable ; and creditors to the extent of Rs 10000 were induced to accept 75% of their dues in full settlement. The stock-in-trade realised 20% less than the book value ; furniture &c. realised Rs 2400 less 10% auction charges. Distribute the balance of the assets amongst the heirs to *A*, and *B*, after meeting the Receiver's expenses amounting to Rs 1560.

[*Ans.* *A*'s heirs...Rs 15640 ; *B*···Rs 3910

§ **Discounting of Bills.** When goods are sold abroad on D/P terms, the documents of title to the goods—the Bill of Lading, Marine Insurance Policy, along with the Invoice—are handed over to the importer only on payment in full of the value of the Bill of Exchange drawn on him. When, however, they are handed over to him on his accepting the Bill—on D/A terms, the exporter has to wait for a reasonable time before he expects payment. This may be anything up to 6 months, depending on the custom of the trade. Meantime, the exporter finances himself by discounting the Bill with a Bill-broker or a Banker. The exporter is naturally prepared to make some sacrifice to get payment on his bill ; on the other hand, the Bill-broker, who is prevented from investing his funds to the extent of his payment to the exporter against the Bill during its tenure expects some remuneration for the service that he thus renders to the seller. This remuneration is called the *discount*, and represents the difference between the value of the Bill and the money paid against it, which is known as the *present value* of the Bill.

In theory, the Bill-broker will see how much every Rs 100 that he advances would amount at the market rate of interest at the end of the remaining tenure of the Bill. Thus, if the Bill has to run another 3 months (plus 3 *days of grace**) and the market rate of interest is 4% p.a., his Rs 100 will amount to Rs 101 at the end of the period. He will, therefore, be prepared to pay Rs 100 on a Bill for Rs 101 maturing in 3 months.

This difference of Re 1 is called the Theoretical or True Discount (T. D.), and Rs 100 the True Present Value (P. V.) of the Bill for Rs 101 (B. V.). The T. D. is thus equal to the interest on the True P. V., and the Bill Value corresponds to the amount of the P. V.

Illustration

A Bill drawn on 1st January 1937 for Rs 50274 to mature 3 months after date is discounted on the 13th February 1937. What is the true discount on the Bill, if the rate of discount be 4% ?

Bill drawn	1. 1. 1937
Tenure & Grace	3. 3.
Bill matures	4. 4. 1937
Bill discounted	13. 2. 1937
Remaining tenure	50 days

* In America no days of grace are allowed, and the year is taken to be of 360 days @ 30 days for each month.

Now, if P. V. = 100
Discount (= Interest on P. V.) ·548 → ?
and B. V. = 100·548 → Rs 50274.
∴ Discount = Rs 274.

§ The Banker or the Bill-broker, however, does not make his calculation on this complicated basis of true discount. He deducts discount, which is known as the Ordinary or Banker's Discount, which is equal in value to the interest that would be earned by the amount of the Bill (B. V.) during the remaining tenure of the Bill (plus 3 days of grace) at the rate of discount.

Banker's Discount (B. D.) is thus equal to the interest on B. V. and the Banker's Present Value is the difference between B. V. and B. D.

B. D. = interest on B. V.
= interest on (True P. V. + T. D.)
= interest on True P. V. + interest on T. D.
= T. D. + interest on T. D.
∴ B. D. − T. D. = interest on T. D.

That is, the Banker really charges more than the theoretical discount.

Unless definitely stated otherwise, discount is always to be computed as Banker's Discount.

Illustrations

1. Calculate Banker's Discount in the above example.

B. V. = Rs 50274
B. D. (= Intt. for 50 d/s at 4%) = Rs 275 8as.

2. The difference between the true and the banker's discount on a bill is 1 shilling, the tenure being 1 month and the rate 6%. Find the value of the bill and the banker's present value.

From the formula 1s = difference between B. D. and T. D.
= interest on T. D. for 1 mo. @ 6% p.a.
= interest on T. D. @ ½%

∴ T. D. = £10

∴ True P. V. = £2000

B. V. = £2010

Also, B. D. = £10 1s

∴ Banker's P. V. = £1999 19s

3. Calculate discount on a bill for £300 accepted on March 16 at 2 months after sight, discounted on April 14 @ 2½% p.a.

Bill accepted	16. 3
Tenure & Grace	3. 2
Bill due	19. 5
Bill discounted	14. 4
Intervening period	35 days

Required discount = Intt. on £300 @ 2½% p.a. for 35 days.

Applying the *3rd*, *10th* and *10th Rule*, discount = 14s 5d correct to the nearest penny.

4. A bill is due to be paid after 73 days. The difference between the true and banker's discount on the bill works out at 2 annas only at 2½%. Find the value of the bill.

2½% p.a. for 73 days is equivalent to a rate ½%.

∴ Difference between T. D. and B. D. @ ½% = 2as

i.e., Interest on T. D. at this rate = 2as

∴ T. D. = 2as × 200 = Rs 25 = Intt. on true P. V.

∴ True P. V. = 200 × Rs 25 = Rs 5000

∴ Bill value = True P. V. + T. D. = Rs 5025.

5. The true discount on a bill payable after 57 days at 4% is Rs 25. Find the value of the bill.

If P. V. of Bill be 100

T. D. ... ·62466 → Rs 25

and B V.... 100·62466 → ?

$$? = \text{Rs } \frac{100\cdot62466 \times 25}{\cdot62466} = \text{Rs } 4027 \text{ app.}$$

Note that the T. D. on 100 is obtained by the 3rd, 10th and 10th Rule.

EXAMPLES 21

Find Banker's Discount on the following Bills :

	Bill	Date accepted	Tenure	Rate of Disct.	Date Discounted
1.	Rs 6485	5. 9. 35.	3 mos.	4%	3. 10. 35.
					[*Ans.* Rs 46 14 as 6p
2.	Rs 3463 10as	4. 6. 35.		3%	8. 8. 35.
					[*Ans.* Rs 17 1a 3p
3.	Rs 8348 12as	2. 2. 35.		2½%	21. 2. 35.
					[*Ans.* Rs 24 9as 6p
4.	Rs 16416 4as	3. 3. 36.		3%	6. 4. 36.
					[*Ans.* Rs 82 5as
5.	£3075 10s	15. 6. 36.		4%	5. 7. 36.
					[*Ans.* £25 5s 6¾d
6.	Rs 8346 6as	3. 12. 34.		4%	7. 1. 35.
					[*Ans.* Rs 81 6as 9p
7.	£9835 3s	9. 10. 35.		3%	2. 1. 36.
					[*Ans.* £8 1s 8d
8.	Rs 15346 3as	15. 10. 36.	4	5%	19. 11. 36.
					[*Ans.* Rs 191 4as 9p
9.	Rs 2528 2as	4. 2. 37.	3	4%	23. 2. 37.
					[*Ans.* Rs 20 3as 6p
10.	Rs 6469 10as	6. 1. 37.	2	4%	2. 2. 37.
					[*Ans.* Rs 24 13as

11. A bill for £531 6s 6d was accepted on the 26th January 1937 at three months after sight. What will be the present value of the bill on the 8th February following, the rate of discount being 5½% ? [*Ans.* £524 18s 4¾d

12. A bill for £635 11s 6d maturing on the 28th May 1937, is discounted on the 3rd March 1937. What will be the banker's charge at 3½% ? [*Ans.* £5 4s 10d

13. Calculate the difference between the true and the banker's discount on a bill for Rs 5535 payable in 4 months' time at 6%. [*Ans.* Rs 2 2as 9p

14. If the difference between the true and banker's discount on a sum due in 4 months at 3% is Rs 10, find the amount of the bill. —I. Com. [*Ans.* Rs 101000

15. A man agrees to pay Rs 2000 for the lease of a house dating from June 24th, and to pay for the same on April 21st, subject to a discount of 3% p. a. How much should he pay, and what is the difference between true and commercial discount ?
—I. Com. [*Ans.* Rs 1989 7as 8p ; 11p.

16. A banker discounts a bill which has 25 days to run before it is legally due at $5\frac{1}{16}$% p. a. This discount amounts to £1 0s 3d. For what sum was the bill brawn ? —I. Com.
[*Ans.* £292

17. A Bill for £713 15s was drawn on 3 April and made payable 3 months after date. It was discounted on 15 April at $4\frac{1}{4}$%. What was the discounted value of the Bill ? —I. Com.
[*Ans.* £706 18s $8\frac{1}{2}$d

18. A Bill drawn on July 10 and payable 3 months after date was discounted on July 25 at 4%, and the sum of money paid was £452 13s 10d. For what amount was this bill drawn ?
—I. Com. [*Ans.* £456 13s 11d

19. Two bills for equivalent amounts are due, one on the 13th September 1937 and the other on the 4th November 1937. The rate of discount in the country in which the first bill is payable is 5% ; and that in the second country in which the other bill is due is 4%. Which bill should be discounted on the 4th July 1937 for preference ? What will be the difference on realisation ?
[*Ans.* $\frac{3}{8}$% less on second Bill

20. An exporter draws three bills : Rs 2500 for 3 months, Rs 3000 for 6 months, and Rs 2000 for 4 months, on the 16th March 1936. He advises his bankers in the three different countries of export to discount the bills on the date of their acceptance at current rates. The bills are accordingly discounted, the first on the 31st March 1936 at $2\frac{1}{2}$% ; the second on the 13th April at 3% ; and the third on the 3rd April at $2\frac{3}{4}$%. What is the total value of the remittances that he receives on these bills ? Answer to the nearest anna. [*Ans.* Rs 7431

21. A Bill for $780·00 dated the 5th May 1936, payable in 6 months with interest at 6%, is discounted at 6% on the 3rd August 1936. Find the proceeds. (No grace is allowed in American Home Trade and a month is taken to be of 30 days). [*Ans.* $791·08

22. A buyer unable to pay the cash price of £584 2s of certain goods makes out a bill payable 3 months after date, which will realise this amount if immediately discounted at 4%. Find the value of the bill. [*Ans.* £590

23. Find the true discount on a bill for Rs 6030 drawn on the 4th October 1936 at 4 months and discounted on the 26th November following at 2½%. [*Ans.* Rs 30

24. Calculate the true present value of a bill for £504 15s 6½d drawn on the 15th April at 6 months, and discounted 93 days before maturity at 3¾%. [*Ans.* £500

25. A bill for £264 6s 9d is retired on 16th March 1936. A true discount of £1 16s 9d is allowed at 3½%. When was the bill due ? [*Ans.* 25th May 1936

SECTION EIGHT

THE STOCK EXCHANGE

§ Stocks, shares and debentures are transferable assets ; and they are dealt with on the Stock Exchange. The three important Stock Exchanges in India are located in Calcutta, Bombay and Madras. On the London Stock Exchange and in "Wall Street" (the popular name for the American Stock Exchange) as also in the different Bourses—important financial centres on the continent —extensive trading takes place in international securities.

Members of the Stock Exchange act as brokers on behalf both of the buyer and the seller, and charge brokerage at scheduled rates from both parties. The effect of brokerage is to increase the price of the stock to the buyer, or to reduce the rate at which proceeds are realised by the seller. The margin between the buyer's price and the seller's price represents profit to the broker or the jobber.

§ In most of the advanced Stock Exchanges members act merely as brokers. When they trade in securities they are known as *jobbers.* In India, however, no members are officially recognized as jobbers, and they act only on behalf of principals.

When a broker closes a transaction on behalf of a principal, he sends a Contract Note (Bought Note or Sold Note) to him, setting forth the datails of the transaction together with his commission, indicating also the time of delivery. Between the members of the Stock Exchange, *settlements* take place periodically for adjustment of sales and purchases. In foreign stock exchanges the settlement lasts three days—

(i) The *Contango Day*, when arrangements are made to carry over the transactions to the next settlement in anticipation of a more favourable market.

(ii) The *Ticket Day*, when the first seller and the last buyer are brought together for closing a transaction.

(iii) The *Settlement* or *Account Day*, when the actual transfer of securities takes place and accounts are finally adjusted.

§ It may happen that the buyer cannot immediately take up the securities bought on his behalf by his broker. The broker finances himself by borrowing against the securities at a fair rate of interest. At the same time, he charges his party, the buyer, a higher rate of interest known as the *contango* for this accommodation. On the other hand, if the seller is unable to deliver in time the securities contracted to be sold by him, the broker borrows the necessary securities from another, paying him a commission known as *backwardation*, and thus honours his contract.

§ The operators on the Stock Exchanges are either investors or speculators. While the investor's chief concern is with the safety of his investment aud the yield or return on his capital, the speculator tries to clear his profit on the fluctuations in the prices. The basic price of a security is regulated to a certain extent by the investors; but the operations of the speculators tend to produce all kind of fluctuations. The speculators ordinarily do not think of taking up or delivering securities and adjust their accounts with the brokers by realising or paying up *differences* in the prices at which they were bought and sold. A speculator who persistently goes on buying in anticipation of favourable prices at settlement, is called a *Bull* or *Tejiwalla*; and the speculator who persistently goes on selling with a view to covering at a lower price is known to be a *Bear* or *Mandiwalla*.

§ While Government Stocks, Municipal Debentures etc. are quoted at so much per 100 stock, exclusive of brokerage, shares are quoted at so much on their paid-up values. Dividends are declared either as a percentage on the paid up value of a stock or as a certain amount per share.

§ In the case of Government Stocks the rate of interest is fixed, and the proportionate dividend actually earned to the date of purchase, but not paid, is also realised from the buyer and paid to the seller. The right to receive the next dividend is sometimes transferred with a share to the next holder, who has to pay the seller the estimated amount of the dividend in addition to the price. Such sale is known as a sale *cum dividend* (c. d.). From the investor's point of view, this addition may be regarded as a kind of temporary advance to the seller, which is realised as soon as the next dividend is paid. The amount of this anticipated dividend, therefore, does not affect the amount of investment, which is represented by the price inclusive of the brokerage. When the right to receive the next dividend is reserved by the seller, he is said to sell the share *ex dividend* (x. d.).

§ *Bearer Securities* are transferable by simple delivery. Prize Bonds now issued by Government are such securities. *Stock certificates* are sometimes issued by Joint Stock Companies in lieu of shares, which are registered with the issuing authority. These represent the right to receive dividends in proportion to the holding. More frequently Stocks are certificates of loans issued by Government, Railways etc. and carry a guaranteed interest. Some holders of Government Stocks leave their Stock certificates *inscribed* with the Reserve Bank of India; and interest warrants are forwarded to their registered address regularly.

§ The prices of stocks and shares are constantly varying. The speculator on the Stock Exchange takes advantage of these variations, earning the differences in price as his profit. The investor on the other hand has two chief considerations to make before taking up a particular stock or share—the safety of the capital invested, and the return on the investment. These are also extensively reflected in the prices quoted. In the case of Government Stocks and other gilt-edged securities, which assure the holder of perfect safety of the capital invested, the rate directly depends on the prevailing money market rate. In India the 3½% Irredeemable Government Stock faithfully registers the changes in the money rates under the operation of the Laws of Demand and Supply, and is known as the Barometer of the

Indian Money Market. The prices of other Government Stocks also follow these variations, in consideration of the different rates of interest earned by them and the different periods after which they will fall due for repayment.

Shares, which are quoted on the basis of their paid-up values, register prices depending on a large number of considerations, the chief among which is the probable dividend that will be earned by them. The comparative stability and reputation of the company, of its Managing Agents and the Directorate, among others, are also mirrored in the prices.

A share is said to be *at par* when its face value and market value are the same. It is at a *discount* when the market value is below and is at a *premium* when it is above *par*, the premium and the discount representing the difference between the market and face values.

The transfer of shares, it may be noted, comes under the operation of the Indian Stamp Act, the cost of the stamp being paid by the buyer.

§ In selecting an investment, apart from the question of the safety of capital invested, the investor looks to the *yield* on the particular stock, which is a measure of the income derived from every unit of Rs 100 invested. Naturally, other things being equal, he will choose the one with the highest yield. The yield is calculated as Rate of Dividend × 100 ÷ Price of Stock. Yields are most easily compared on a graph paper on which points are plotted with the dividend and the price as co-ordinates. The inclination that the line joining the origin to each such point is a measure of the yield on that particular stock at the price.

§ In India the highest rate of Income Tax, *i.e.*, 30 pies in the rupee plus* surcharge, is deducted at source on dividends, unless they are declared free of the tax. When dividends of companies are declared free of income tax, the tax on the entire profit is deducted before paying out dividends.

§ It may be noted that Government and other interest bearing securities are not issued in units other than of multiples of Rs 100.

On the Indian Stock Exchanges, shares of different paid up values are usually dealt with in lots varying from 5 to 100. Thus shares with paid up value below Rs 50 are normally dealt in in lots of 100, smaller lots (usually not less than 25) being regarded as odd lots. For Rs 50 shares a lot is usually 50 ; for Rs 100 shares it is 25 ; and for Rs 500 shares and up, it is ordinarily 5.

Illustrations

1. A person's investments are £780 each in the 3 per cents at $79\frac{7}{8}$ and in the 4 per cents at $97\frac{3}{8}$. On receiving the first year's dividend on each he sells out of the first stock at 96 and the second at par, and invests the entire amount, less expenses of sale amounting to £5 5s, at 5% p.a. Find the change in his income. (Brokerage on purchase $\frac{1}{8}$%)

Investments :

1. £780 @ $(79\frac{7}{8}+\frac{1}{8})$ buys stock $£780 \times \frac{100}{80}$

= £975 stock

Income @ 3% £29 5s

2. £780 @ $(97\frac{3}{8}+\frac{1}{8})$ buys stock $£780 \times \frac{100}{97\frac{1}{2}}$

= £800 stock

Income @ 4% £32

Total income £61 5s

Realisations :

1. £975 stock @ 96……£936
2. £800 stock @ par…..£800

£1736

Less Expenses £5 5s

£1730 15s

New investment :

Net Realisation	£1730 15s	
Add 1 year's Interest	61 5	
Total available for investment	£1792 0s	
New income @ 5% on £1792		£89 12s
Original total income		£61 5s
Increase in income (anuual)		£28 7s

2. Determine which of the following investments is the better: Rs 25 shares, on which Rs 20 has been paid up, selling at 22½ and paying a dividend of 8%; or, Rs 100 shares, fully paid up, quoted at 177½ and paying 12½%.

Investment 1.

Paid-up value of a share	...	Rs 20	
Dividend @ 8%	...	Re 1·6	
Price	...	Rs 22·5	∴ Yield = 7·11%

Investment 2.

Value of share	...	Rs 100	
Dividend	...	Rs 12·5	
Price	...	Rs 177·5	∴ Yield = 7·04%

∴ First investment is better, the difference in return being ·7‰ on investment.

3. A person sells £7000 stock in the 4 per cents at 102⅝ and invests the proceeds in the 3 per cents at 74⅞. Calculate the change in his income, brokerage being at ⅛%.

£7000 stock realises @ 102⅝ *less* brokerage ⅛, *i.e.*
@ 102½... £7175

£7175 invested @ 74⅞ *plus* brokerage ⅛, *i.e.*, @ 75
buys £7175 × $\frac{4}{3}$, *i.e.*, £9566⅔ stock.

Original income @ 4% on £7000 stock = £280
New income @ 3% on £9566⅔ stock = £287
∴ Increase in annual income £7

4. A person invests Rs 11600 partly in 4% stock @ 88 and partly in 6% stock at 120. What is his holding in each, his total income from the investments being Rs 560?

Yield on 1st stock $= \frac{\cancel{4}}{\underset{22}{\cancel{88}}}$; on 2nd stock $= \frac{\cancel{6}}{\underset{20}{\cancel{120}}}$; and

$$\text{average yield} = \frac{560}{11600} = \frac{7}{145}.$$

Adjusting differences in yield, we get after removing fractions,

Bought Yield		Yield	Av. Yield	Diff.
$\frac{1}{22}$		290		11
	$\frac{7}{145}$		308	
$\frac{1}{20}$		319		18

The amounts invested in the two stocks are in the ratio of 11 : 18.

∴ Amount invested in stock 2 = Rs 7200 buying Rs 6000 stock 6%
and „ „ „ „ 1 = Rs 4400 „ Rs 5000 „ 4%

5. A ten-rupee share is quoting at Rs 29 12 *cum dividend.* The dividend is Re 1 8as per share for the half year. Ascertain the yield on the investment, assuming that the rate of dividend is unaltered.

Price *c.d.*	Rs 29 12as
Dividend	1 8
Investment *x. d.*	Rs 28 4as
Annual Return	3

$$\therefore \quad \text{Yield } \% = \frac{3}{28{\cdot}25} \times 100 = \underline{10{\cdot}62\% \text{ p. a.}} \text{ app.}$$

6. An investor is prepared to invest Rs 16,500 in 4% Government Stock @ 113$\frac{3}{8}$. Determine the stock that may be purchased, and his annual income therefrom. Brokerage is at one anna per centum.

Price inclusive of brokerage is Rs 113$\frac{7}{16}$ for Rs 100 stock.

∴ Value of stock that may be purchased

$$= \text{Rs } 16500 \times \frac{100}{113\frac{7}{16}} = \text{Rs } 14545\frac{5}{11} \text{ stock.}$$

For obvious reasons he can get only Rs 14500 stock, as these stocks are not issued in fractions of Rs 100. For this Rs 14500 stock he actually pays @ 113$\frac{7}{16}$ Rs 16,448 7as, and is left over with a balance out of funds available for investment.

His income on Rs 14500 stock @ 4% = $\underline{\text{Rs 580 p. a.}}$

7. Dividends on 3$\frac{1}{2}$% Government paper are paid half-yearly on the 31st March and 30th September. On Aug. 3 a purchaser buys Rs 10000 of the stock at 94$\frac{7}{8}$. The brokerage is at 1 anna% with a minimum of Re 1. Find the amount payable by the buyer; and the amount of the interest received by him on September 30 following after deduction of Income Tax @ 30p in the Rupee.

Rs 10000 stock	@ par cost	Rs 10000	
Less Discount	@ $5\frac{1}{8}$%	512	8
	@ $94\frac{7}{8}$	Rs 9487	8a
Add Brokerage	@ $\frac{1}{16}$%	6	4
Intt. from April 1 to Aug. 3 inclusive (125 days)		119	14
Total amount payable		Rs 9613	10as

Half-yearly interest @ $3\frac{3}{4}$% p. a. on Rs 10 000 stock		Rs 187 8as
Less I. Tax @ 24p...Rs 23 7as		
@ 6p... 5 14as		29 5as
Net amount		Rs 158 3as

8. An investor desires to earn a minimum income of Rs 3000 a year by investing a lakh of rupees in the following :

$3\frac{1}{2}$% Govt. Paper @ $97\frac{7}{8}$ (Bkge. $\frac{1}{8}$)
8% Preference Stock @ 207 (Bkge. 1%)
Fixed Deposit in a Bank carrying $2\frac{1}{2}$% p.a.

Distribute his investments.

Arranging the yields on the 3 investments with the average yield of 3% p.a., we obtain the proportion of money invested in three different ways by the Method of Mixture.

Yields	Av. yield		Difference	Proportion
$\frac{1}{28}$		$\frac{1}{200}$		7
$\frac{1}{26}$	$\frac{3}{100}$		$\frac{1}{200}$	13
$\frac{1}{40}$		$\frac{4}{700}$	$\frac{11}{1300}$	30

∴ Investment in $3\frac{1}{2}$% stock should be Rs $\frac{7}{50} \times 100000$,
i.e., Rs 14000

Ditto in Pref. Stock...Rs $\frac{13}{50} \times 1{,}00{,}000$, *i.e.*, Rs 26000
Ditto in Bank Deposit...Rs $\frac{3}{5} \times 1{,}00{,}000$, *i.e.*, Rs 60000

Although theoretically the solution is correct, it is unpractical. Since the lowest unit of Government stock is Rs 100, and it may at best be conceived that a single Preference share may be purchased (although practically the ordinary minimum dealt with on the Indian Stock Exchange is 25 shares, when the unit is Rs 100), the amounts indicated above cannot be absorbed in the purchase of complete units.

Thus, Rs 14000 @ 98 would buy Rs 14285$\frac{5}{7}$ stock. But as such stock is not available, he will have to take up Rs 14300 (or Rs 14200)* stock ; and spend therefor...Rs 14014

Also, Rs 26000 @ 208 would buy 125 shares of Rs 100 each.

∴ He should invest the balance of Rs 59986 by keeping it on fixed deposit with the bank.

EXAMPLES 22

1. Find the cost of (i) Rs 15500 Government 3½% Stock at 99⅝ ; (ii) Rs 8600 Port Trust Debentures at 113¾ ; (iii) Rs 18900 4% Municipal Loan at 103½.

[*Ans.* Rs 15441 14as ; Rs 9782 8as ; Rs 19561 8as

2. Calculate the income from the following investments, deducting Income Tax at 26 pies in the rupee *plus* a surcharge of 1 pie in every anna of the Tax.

(i) Rs 5 600 @ 3½%	[*Ans.* Rs 167 4as
(ii) Rs 10 100 @ 6½%	[*Ans.* Rs 560 3as
(iii) Rs 18 600 @ 6¾%	[*Ans.* Rs 1071 5as

3. Compare the yields on the following :

3½% Government Stock	@ Rs 97 8as	[*Ans.* 3·59%
3% " "	@ Rs 83 12as	[*Ans.* 3·58%
5½% Dalhousie Property Debenture	@ Rs 102.	[*Ans.* 5·39%

4. If the 3½% Irredeemable Stock is quoted at Rs 98 8as, what would be a fair price of the 3% Irredeemable Stock ? [*Ans.* 84$\frac{7}{16}$

5. A person holds Rs 15500 3½% Government Stock at Rs 99 14as. Should he transfer his holding to a Railway Stock paying dividend at 6% per annum for the last ten years, if it is quoted at Rs 133 5as ? What will be the change in his income ?

[*Ans.* Increase in annual income Rs 153 8as

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*He should not however choose to take up Rs 14200 stock, which gives a much higher yield than bank interest, as there would then be a chance of the total income getting below the minimum Rs 9000.

6. Compare yields on the following investments :

(i) 3½% Stock @ Rs 98 14as. Brokerage 1 anna %.

(ii) 5% Debenture Stock @ Rs 132 10as. Brokerage Re 1 per Debenture of Rs 100 each.

(iii) 6% Preference Shares of Rs 25 each @ Rs 35 4as. Brokerage 8as per share.

[*Ans.* 3·54% ; 3·74% : 4·20%

7. A person holds Rs 100000 stock in the 3½ per cents at 77⅛. He sells out and invests the proceeds in the 4½ per cents at 92½. If the brokerage on the first is at 2as %, and that on the second is at ¼%, find the value of the stock and the change in the person's income.

[*Ans.* Rs 83000 stock + ; Annual Income increases by Rs 235

8. A person invested Rs 43700 in the purchase of 4% Improvement Trust Loan at 115, and sells out when the price goes up to 125, investing the proceeds in a 3% new issue at 95. What is the change in his annual income ?

[*Ans.* Decreases by Rs 20.

9. A person has an annual income of Rs 212 8as from a holding in the 4¼% stock. He is offered a price of Rs 120 if he subscribes to a new issue of 3½% stock at par. Should he change ?

[*Ans.* If he changes, annual income will diminish by Rs 2 8as

10. The holder of Rs 100000 stock in the 3 per cents at Rs 91 10as (brokerage ⅛%) increases his income by Rs 1 000 a year by changing into an 8% Cumulative Preference Stock. What is the value of the new holding and the price of the stock, brokerage being 8as% ? [*Ans.* Rs 50000 stock. Price 182½

***11.** A speculator starts with Rs 50500 invested in shares quoted at Rs 25 4as (Rs 20 paid up). In a few days the shares are quoting at Rs 28 12as, when he sells out. The expenses of the sale amount to Rs 305. He re-invests the net proceeds in another share (Rs 50 fully paid up) quoting at 44, brokerage being at Re 1 per share. He then sells out making a clear profit of Rs 405 ; and with the proceeds he takes up a new Government

4% Loan at par. He sells out again as soon as the stock registers a fall of 8as%, paying brokerage at $\frac{1}{16}$%. How much does he make out of these transactions ? [*Ans.* Rs 6776

*12. An investor desires to put Rs 20000 in gilt-edged stocks, so that he may make an average of 4% at least. If he chooses to invest about Rs 5000 in 5% Port Debentures at 142½, and the balance in 3% Government Stock, what would be the maximum price he would be prepared to pay for the latter ? [*Ans.* Rs 71 13as

*13. An Insurance Company, holding 5% Tax Free Government War Loan of the value of Rs 256000, tenders its entire holding at the price of 123¾ in subscription to a new 4% issue at par. What is the value of the new stock and what is the change in the net income from the investment ? Calculate income tax at 26 pies in the rupee *plus* a surcharge of 8⅓% thereon.

[*Ans.* Stock Rs 316800. Decrease Rs 1987 a year

14. A bank buys equal stocks in the 4½ per cents at 117 and in the 5 per cents at 130 and obtains a total return of Rs 46800 from the investments in course of 12 years. What was the total investment ? What is the average yield on the investments ?

[*Ans.* Yield = $3\frac{11}{13}$%. Investment Rs 101400

15. If the four per cents are quoting at 105¾, what should be the price of the five per cents on the basis of yields on the investments ? [*Ans.* $132\frac{3}{16}$

16. A person sells out Rs 6900 stock of one kind at 117½, and Rs 5800 stock of a different kind. With the proceeds he buys Rs 10000 stock at 146 (brokerage ⅛%) after meeting expenses amounting to Rs 49 in respect of the sale. Find the price at which the second stock was sold. [*Ans.* 113

*17. A person hands over Rs 17600 stock at 88 and Rs 8300 stock at 96¼ to his brokers, and instructs them to purchase with the proceeds Rs 25000 Stock in Port Debentures at the best price of the day. The brokers send him the desired stock and a cheque for Rs 2453 3as being the balance after deduction of their charges amounting to Rs 86 1a. What was the price of the Debentures ? [*Ans.* 83¾

18. What would be the change in a person's annual income who converts £5850 in the 3 per cents at 103¾ to buy into a Debenture Stock at 117½, and sells it later at 94 to buy a new stock carrying 4% at 83? [*Ans.* Income increases by £58 10s

***19.** On retirement a person's savings amounted to £26 488. He desires to invest the money to assure himself of an annual income of at least £1258. Part of it he accordingiy invests in the 3½ per cents at par, and the balance he lends out on mortgage at 9% per annum. How is the investment distributed?

[*Ans.* Min. yield 4¾% p.a.; £6020 on Mortgage; £20468 in stock

***20.** A superannuated person is granted gratuity and provident fund savings amounting to Rs 45600. He intends to invest just as much of this amount (in units of Rs 500) in 4% Government Stock at par as would yield him an annual income of at least Rs 1400 net after payment of income tax at 6 pies in the rupee. What is the amount of this investment? What income will he derive by investing the balance in Rs 50 shares (Rs 40 paid up) at Rs 45 8as paying a dividend of 5% p. a.?

[*Ans.* Rs 36500; Rs 400 a year

21. An investor puts in equal sums of money in the three per cents, and in the 3½ per cents at 87½. If the income from the two are equal, what is the price of the 3 per cents? [*Ans.* 75

22. An investor was advised to buy certain 6% Debentures at 126, instead of a 9% Preference Share at 210. It was pointed out that he would have an income larger by Rs 225 a year thereby. What is the amount of investment? [*Ans.* Rs 47250

23. A person takes up 100 shares of Rs 100 each, on which Rs 75 only has been paid up, at 87½. A dividend of 8% is declared; but during the year the balance of the capital is called up. What is the average return on his investment? [*Ans.* 5⅓%

24. An investor desires to put in Rs 10000 partly in the 4½ per cents at 120, and partly in shares paying 4% at Rs 40 for each Rs 50 share; and expects an income of Rs 450 from his investments. How would the investment be distributed?

[*Ans.* Rs 4000 in 4½ per cents; 150 shares of Rs 50 each

25. An investor buys 200 shares of Rs 100 each in a certain company at 99¾ *cum dividend*, expecting a dividend of 5%. Dividend was actually declared shortly after at 4¾%. What return would he expect on his investment, assuming that a 4¾% rate of dividend is maintained? [*Ans.* 5%

***26.** A person hands over to his brokers Rs 8300 stock at 77¾ and Rs 9700 stock at 87½, and instructs them to buy with the proceeds Rs 15000 4% stock at the best price of the day. The brokers buy the third stock at 94⅜ and charge 95⅛ to the customer, to whom they send a cheque for the balance after deducting their charges amounting to Rs 83 7as. What is the value of the cheque, and how much did the brokers clear on the transaction? [*Ans.* Cheque for Rs 588 9as. Profit Rs 195 15as

***27.** The price of a certain 5% stock is just ⅛th as much again as the price of a 4% stock. A person invests twice as much in the latter stock as in the former. His income from the former is Rs 638 7as. What is his income from the latter? [*Ans.* Rs. 1149 3as

28. A Balance Sheet shows on the Assets side certain stocks valued at Rs 95550 at cost price 73½. The current quotation is 15% higher; and this advantage is taken to create a Secret Reserve to provide for likely losses in other investments. What is the value of this Reserve? [*Ans.* Rs 19500

29. A person buys 100 shares in a Jute Mill at 67½ (Rs 50 paid-up). As the share goes up in value, he buys another lot of 100 shares at 69 and further lots of 100 shares each at 70½ and at 72. Find the average cost of the shares held by him, and his profit on the whole transaction, if he liquidates his entire holding at 71½ to avoid having to pay up the uncalled liability thereon. [*Ans.* 69¾; Profit Rs 700

30. What stock should I sell out at 97½ to realize £1300? If this is again invested at 102 in 4¼ per cents, my income thereby increases by £8 16s 8d annually. What was the rate of interest on the former stock?— I. Com. [*Ans.* £1333⅓ stock; 3⅖%

31. *X* invests £27900 partly in 4% stock at 114⅞ and partly in 3% stock at 96⅜. If the resulting income is £873, find how much is invested in each stock, allowing a brokerage 2s 6d for each £100 stock, and £3 for stamp on the total amount invested. —I. Com. [*Ans.* £1552 10s ; £26344 10s

32. A shareholder in a mining company obtains from his dividends an income of £200 per annum after paying income tax at 4s in the £. The shares are 5s shares upon which were paid two dividends in the year of 1s and 1s 6d respectively. How many shares does he hold in the company and what money would be realised by their sale if the present market quotation is 17s 6d ? —I. Com. [*Ans.* 2000 shares ; £1750

33. How must an investor divide £1400 between a 3½% stock at 83⅜ and a 5% stock at 99⅝ so that he may obtain 4½% return for his money ? If he sells his 3½% stock at 85⅛ and invests in the 5% stock at 101⅛, what rate per cent will he receive on the original £1400 ? (Brokerage ¼% on each transaction) —I. Com. [*Ans.* 5 : 3 ; 5% app.

34. A person has £5000 which he instructs an agent to invest in 3% stock standing at 102. The agent buys in lots of £500 each and as each deal is completed the price of the stock rises ¹⁄₁₆%. The agent buys 9 lots, when he is instructed to cease buying. If his charge amounts to £10, how much money should he return to his employer ? —I. Com. [*Ans.* £388 15s

35. If a person sells out £10,000 Great Western Railway 4% Debenture stock a £133 and buys £8000 stock in 2¾% Consols at £95¾ and lends the rest of the money at mortgage at 3¾%, find the change in his income. —I. Com.

[*Ans.* Increase £31 10s p. a.

36. A man had £2100 stock 3% which he transferred to the 3½% stock at £87½, thus increasing his income by £7. What is the price of the 3% stock ? —I. Com. [*Ans.* £83⅓

37. A man sells out £6450 India 3½% stock at 64⅜ and with the proceeds bought as many Dunlop Rubber Co.'s 5% Preference £1 shares at 16s 3d as he could. What was his gain in income derived by this change in investment ? (Brokerage ⅛% for India Stock and 3d per £1 share) —I. Com. [*Ans.* Increase £25 8s p.a.

38. A man has Rs 6680 stock in the 4 per cents at 98. When the stock has fallen 2% he transfers his capital to the 3½ per cents at 83½. Find the alteration in his income to the nearest anna. —I. Com. [*Ans* Increase Re 1 10as p.a.

39. Having invested a sum of money in 2½% Consols at 60⅝, I held the stock until I had received a half year's dividend. I then sold it at 60⅛, and invested the proceeds of the sale together with the dividend, so as to produce 4¼% ; and my income was thus improved by £90 19s 6d per annum. If brokerage of ⅛th per cent be charged upon sales and purchases, what sum did I originally invest ? —I. Com. [*Ans.* £53372

40. Imperial Bank of India fully paid shares are selling at 1054½ and 4% Calcutta Port Trust Debentures (1908-38) at Rs 83 12as. An investor has Rs 9307 12as at his disposal and he purchases a total number of 30 units of the above two securities. Calculate how many securities of each class he buys.
—I. Com. [*Ans.* 7 units of Bank and 23 units of Debentures

41. In a certain year Rs 4936600 five-per cent stock was converted into four-per cent stocks, Rs 100 of the first being equivalent to Rs 105 of the second. A sum of Rs 619300 was paid off to those people unwilling to accept the reduction in the rate of interest. Find the annual saving in interest.
—I. Com. [*Ans.* Rs 65503 6as 5p

42. Having a certain sum to invest, a man puts half of it in 5% stock at 105 and the other half in 4½% stock at par, the prices in each case being inclusive of brokerage. If he had invested ⅗ of it in the first and the rest in the second, his annual return would have been £2 15s more. How much did he invest ?
—I. Com. [*Ans.* £10500

***43.** A speculator deposits Rs 10000 with his bankers and enters into an agreement with them that they will buy and sell stocks in his behalf as advised, and charge an inclusive rate of 4as per cent on all amounts. During a certain month the following transactions were completed :

(i) Sale of Rs 8800 Municipal Stock at 93⅝.

(ii) Sale of Rs 15400 Government Stock at 87½.

(iii) Investment of Rs 42100 in Debentures at 105¼.

(iv) Purchase of Rs 8000 Government Stock at 98¼.

What will be the bankers' charge for the month ; and how will the customer's account be affected ?

[*Ans.* Banker's charge Rs 179 3as

44. The 3½% stock is quoting at 82½. Find the net yield on a person's investment after providing for Income Tax at 32½ pies in the rupee. If the price goes down to 78¾, and he purchases an equal stock at this rate, find the average net yield on the investment. If an equal amount of money were invested in the second instance, what would be the average net yield ?

[*Ans.* 3·52% ; 3·61% : 3·56%

45. A person buys shares in the Indian Copper Corporation Ld. as follows :

200 shares @ Rs 2 10as each ; 300 shares @ Rs 2 2as each ; 400 shares @ Re 1 12as each ; 600 shares @ Re 1 9as each.

Find the average price at which the shares are held, and the return on his investment, if the dividend declared be 4% free of Income tax. The shares have a face value of 2sh each, and exchange is at 1s 6d. [*Ans.* Rs 1 14as – ; Rs 80

46. A speculator buys Steel Corporation shares in a rising market as follows :

100 shares @ Rs 15 12as ; 200...@ Rs 16 4as ; 400... @ Rs 17 2as ; 600...@ Rs 18 4as ; 400...@ Rs 19 2as.

He sells out the whole lot when the price touches Rs 19 10as (*c.d.*) per share. Find the average cost of his holding, and the profit made on the transaction. [*Ans.* Rs 17 13as + ; Rs 3087 8as

47. A speculator contracts to sell shares of the Indian Iron and Steel Co., Ld., as follows :

200 shares @ Rs 33 4as each ; 200...@ Rs 32 12as ; 200... @ Rs 31 8as ; 400...@ Rs 29 12as ; 600...@ Rs 28 4as.

He covers the sales by purchasing the whole lot of shares for delivery @ Rs 28 each. Find his total profit. [*Ans.* Rs 3550

SECTION NINE

LOGARITHMS

§ If $x = 10^y$, y is said to be the *logarithm* of x to the *base* 10; or, merely, the common log. of x; and x is said to be the *antilogarithm* of y.

These values of y corresponding to all possible values of x—from 0 to 99999—are shown in most of the ordinary Tables of Logarithms.

It would appear from a Table that the logarithm of $3{\cdot}73 = {\cdot}5717$, which means that $10^{{\cdot}5717} = 3{\cdot}73$. The following results follow:

$$37{\cdot}3 = 10 \times 3{\cdot}73 = 10 \times 10^{{\cdot}5717} = 10^{1{\cdot}5717}$$

$$373 = 10^2 \times 3{\cdot}73 = 10^2 \times 10^{{\cdot}5717} = 10^{2{\cdot}5717}$$

$$3730 = 10^3 \times 3{\cdot}73 = 10^3 \times 10^{{\cdot}5717} = 10^{3{\cdot}5717}$$

$$37300 = 10^4 \times 3{\cdot}73 = 10^4 \times 10^{{\cdot}5717} = 10^{4{\cdot}5717}$$

Again,

$${\cdot}373 = 10^{-1} \times 3{\cdot}73 = 10^{-1} \times 10^{{\cdot}5717} = 10^{\bar{1}{\cdot}5717}*$$

$${\cdot}0373 = 10^{-2} \times 3{\cdot}73 = 10^{-2} \times 10^{{\cdot}5717} = 10^{\bar{2}{\cdot}5717}*$$

&c. &c.

Or, in other words,

$\log 3{\cdot}73 = {\cdot}5717$; $\log 37{\cdot}3 = 1{\cdot}5717$

$\log 373 = 2{\cdot}5717$; $\log 3730 = 3{\cdot}5717$

$\log 37300 = 4{\cdot}5717$; $\log {\cdot}373 = \bar{1}{\cdot}5717$*

$\log {\cdot}0373 = \bar{2}{\cdot}5717$.* $\log {\cdot}00373 = \bar{3}{\cdot}5717$*

It is to be noted that products and quotients of 3·73 by powers of 10 have the same decimal portion in the logarithms; the integral portion being the same as the number of integral places *less* 1, or the number of decimal places at which the first significant figure occurs, with a bar on the top.

Advantage has been taken of this fact to tabulate merely the decimal portions of the logarithms, which are called the *mantissæ*,

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*The notation $\bar{1}{\cdot}5717$ means that the integral part is negative being $= -1$: but the decimal part is positive. Thus it is really $= -{\cdot}4283$. In using Tables the *mantissa is always to be positive.*

the integral portion, called the *characteristic*, being always capable of being written down as above by inspection.

Thus looking at the Tables again, $\log 4{\cdot}83 = {\cdot}6839$. Logarithms of the following numbers may be directly written down as below:

$$\log 48{\cdot}3 = 1{\cdot}6839 \qquad \log 48300 = 4{\cdot}6839$$

$$\log {\cdot}483 = \bar{1}{\cdot}6839 \qquad \log {\cdot}000483 = \bar{4}{\cdot}6839$$

§ **Lemma.** $\log (x_1 . x_2) = \log x_1 + \log x_2$.

This fundamental relationship follows from the fact, that if

$$\log x_1 = y_1, \text{ and } \log x_2 = y_2,$$

$$x_1 = 10^{y_1} \quad \text{and} \quad x_2 = 10^{y_2}$$

$$\text{and } x_1 . x_2 = 10^{y_1 + y_2}$$

$$\textit{i.e.}, \ \log (x_1 . x_2) = y_1 + y_2$$

$$\text{which } = \log x_1 + \log x_2.$$

It may be similarly demonstrated that

$$\log \left(\frac{x_1}{x_2}\right) = \log x_1 - \log x_2.$$

These relationships are easily extended to more than two numbers, involving both multiplication and division. Thus

$$\log \frac{x_1 . x_2 . x_3 \ldots}{z_1 . z_2 . z_3 \ldots} = (\log x_1 + \log x_2 + \log x_3 + \ldots)$$

$$- (\log z_1 + \log z_2 + \log z_3 + \cdots).$$

This property of logarithms is used in the performance of multiplication and division.

Illustrations

1. To find the value of $41{\cdot}2 \times 4{\cdot}86$

$$\text{Now,} \quad \log 41{\cdot}2 = 1{\cdot}6149$$

$$\text{and} \quad \underline{\log 4{\cdot}86 = \ {\cdot}6866}$$

$$\therefore \log (41{\cdot}2 \times 4{\cdot}86) = 2{\cdot}3015 = \log N, \text{ say.}$$

The product $41{\cdot}2 \times 4{\cdot}86$ is then the number N of which the logarithm is $2{\cdot}3015$.

Looking up the Tables, it is noted that the mantissa ·3016 is the logarithm of 2·005.

$\therefore$ $N = 2{\cdot}005 \times 10^2$, the characteristic of the log of N being $= 2$.

$$\therefore \quad 41{\cdot}2 \times 4{\cdot}86 = \underline{200{\cdot}5 \text{ nearly.}}$$

2. Find $\cdot 00346 \div \cdot 0531$

Now, $\log \cdot 00346 = \bar{3}\cdot 5391$

$\log \cdot 0531 = \underline{\bar{2}\cdot 7251}$ [$\because \bar{2} + 1(\textit{carrying}) = \bar{1}$; $\bar{3} - \bar{1} = \bar{2}$]

Diffce. $= \bar{2}\cdot 8140 = \log N$, say,

where $N = \dfrac{\cdot 00346}{\cdot 0531}$

From the Table $N = \text{Antilog } \bar{2}\cdot 8140 = \underline{\cdot 0652}$ app.

§ **Lemma.** If $x = 10^y$, $x^m = (10^y)^m = 10^{my}$, and $\log (x^m) = my = m \log x$.

This relationship is true for all values of m, and it is used in finding powers and roots of numbers.

Illustrations

1. Find $(10\cdot 2)^6$.

Now, $\log (10\cdot 2)^6 = 6 \log 10\cdot 2$.

$$\log 10\cdot 2 = 1\cdot 0086$$
$$\underline{\quad 6}$$

$\therefore \log (10\cdot 2)^6 = 6\cdot 0516 = \log N$, say.

From the Table $\log 1\cdot 126 = \cdot 0516$.

$\therefore \log (1\cdot 126 \times 10^6) = 6\cdot 0516$

$\therefore N = 1\cdot 126 \times 10^6 = \underline{1126 \times 10^3}$ nearly.

2. Find the 5th root of ·243.

Now $\log \sqrt[5]{\cdot 234} = \log (\cdot 243)^{\frac{1}{5}} = \frac{1}{5} \log \cdot 243$.

From the Tables, $\log \cdot 243 = \bar{1}\cdot 3856$

$\therefore \log \sqrt[5]{\cdot 243} = -\cdot 2 + \cdot 0771 = (-1 + \cdot 8) + \cdot 0771 = \bar{1}\cdot 8771$.

Looking up the Tables for the antilog. of ·8771, it is seen that $\log 7\cdot 535 = \cdot 8771$.

$\therefore \bar{1}\cdot 8771$ is the log. of ·7535.

$\therefore \sqrt[5]{\cdot 243} = \underline{\cdot 7535}$ app.

With the help of more accurate Tables, the above results could be obtained to much greater degrees of accuracy.

§ **To Read the Tables.** The mantissæ of all numbers shown to the extreme left of the log Tables are read out in the following column. Thus log 3·70 = ·56820, say. The succeeding columns show the logarithms of numbers from 3·71 to 3·79 under the heads 1, 2, 3...9 respectively. The columns following that under the head 9 show differences with the help of which extended places of decimals may be worked.

In the Antilog Tables, the mantissæ of the logarithms of numbers are shown to the extreme left, the antilogarithms being tabulated as above. The figures of the antilog are properly pointed *at the last stage* in reference to the characteristic.

Illustrations

1. To find the logarithm of 3·7366.

log 3·73 = ·57171
Add Mean Diffce. for 6 = 70
" " " " 6 = 7|0
log 3·7366 = ·57248

2. To find the Antilogarithm of 2·04947.

Ignoring the characteristic, Antilog 049 = 11194
Add Mean Diffce. for 4 = 10
" " " 7 = 1|8
Antilog 04947 = 11206

∴ Antilog 2·04947 = 112·06, for a characterstic 2 indicates that there should be 3 integral places.

3. Find the value of $\dfrac{\sqrt[3]{48{\cdot}2} \times \sqrt{16{\cdot}3}}{\sqrt[5]{10{\cdot}3} \times \sqrt{98{\cdot}6}}$ (= N, say)

$$\log N = \tfrac{1}{3} \log 48{\cdot}2 + \tfrac{1}{2} \log 16{\cdot}3 - \tfrac{1}{5} \log 10{\cdot}3 - \tfrac{1}{2} \log 98{\cdot}6$$

Now, log 48·2 = 1·6812
and ⅓rd above = ·5604
log 16·3 = 1·2122
and ½ above = ·6061
Sum 1·1665

Also, log 10·3 = 1·0128
and ⅕th above ·2026
log 98·6 = 1·9939
and ½ above ·9969
Balance $\bar{1}{\cdot}9670$

N = Antilog. of above = ·9268 app.

§ It should be carefully noted that the mantissa has always to be used in a positive form. The following method may be used in transforming the logarithm of a number into a number with a positive mantissa.

Thus $-\cdot 2+\cdot 0771$ would simplify into $-\cdot 1229$, a value which cannot be used in reference to a Table of Logarithms. It is, therefore transformed, as below :

$$-\cdot 2+\cdot 0771=-(\cdot 2+\cdot 8)+\cdot 8+\cdot 0771=-1+\cdot 8771,\ \textit{i.e.}=\bar{1}\cdot 8771.$$

Also, $\frac{1}{6}$ of $\bar{2}.63188=-\cdot\dot{3}+\cdot 10531$

$$=-(\cdot\dot{3}+\cdot\dot{6})+\cdot 66667+\cdot 10531=\bar{1}\cdot 77198.$$

Illustration

Solve : $(1+x)^{-10}=\cdot 9584$.

$$-10\log(1+x)=\log\cdot 9584=\bar{1}\cdot 98155$$

$$\therefore\quad \log(1+x)=(-\tfrac{1}{10}\text{ of }-1)+(-\tfrac{1}{10}\text{ of }\cdot 98155)=\cdot 1-\cdot 098155$$

$$=\cdot 001845=\log 1\cdot 004$$

$$\therefore\quad 1+x=1\cdot 004$$

$$\text{and } x=\underline{\cdot 004}$$

§ The following arithmetical operations involving negative characteristics may be noted.

(i) Any negative number may be changed into a number with a negative characteristic and a positive mantissa.

Thus, $-4\cdot 6839=-5+(5-4\cdot 6839)=\bar{5}\cdot 3161$.

(ii) $\bar{3}\cdot 6839\times 5=5\times -3+5\times\cdot 6839=-15+3\cdot 4195=\bar{12}\cdot 4195$.

EXAMPLES 23

Find the value of the following with the help of a 4-figure Table of Logarithms :

1. $34\cdot 68\times 90\cdot 01$ [*Ans.* 3122 2. $\cdot 8176\times 13\cdot 64$ [*Ans.* 11·15

3. $63\cdot 18\times\cdot 8502$ [*Ans.* 53·72 4. $98\cdot 68\div 3\cdot 251$ [*Ans.* 30·35

5. $\cdot 6418\div\cdot 0169$ [*Ans.* 37·97 6. $\cdot 001834\div\cdot 04316$ [*Ans.* ·04249

7. $5785\div 28\cdot 64$ [*Ans.* 201·9 8. $\sqrt[3]{316\cdot 6}$. [*Ans.* 6·816

9. $\sqrt[5]{.08146}$ [*Ans.* .6056. **10.** $\dfrac{18.64 \times 46.56}{.9623 \times 17.18}$ [*Ans.* 52.50

11. $\dfrac{241.6 \times .3814 \times 6.842}{.4618 \times 38.25 \times 73.65}$ [*Ans.* .4846

12. $\dfrac{.001831 \times .1972 \times .5374}{.6408 \times .0896 \times .1836}$ [*Ans.* .01841

13. $\dfrac{\sqrt[3]{.4165} \times \sqrt[5]{.8264}}{\sqrt[3]{.6438} \times \sqrt[6]{.0186}}$ [*Ans.* 1.370

14. $\dfrac{\sqrt[10]{962.4 \times .6418}}{\sqrt[6]{.5431} \times \sqrt[4]{6834}}$ [*Ans.* .2315

15. Evaluate $\dfrac{483491}{678463}$; and estimate its altered value when the numerator increases by 6%, and the denominator decreases by 15% of their respective values. Answer to 4 decimal places. [*Ans.* .7126 ; .8887

16. A merchant, failing for Rs 5381 7as, pays a dividend of 3as 8.63p in the rupee. The book value of the assets was Rs 1309. Find the loss on realization of the assets. [*Ans.* Rs 58 app.

17. Find the value of a bill for Rs 1737 8as in francs, given that Rs 100 = 251.80 fr. [*Ans.* 4375.03 fr.

18. Express 1.50 fr. per metre in pence per yard, being given £1 = 25.00 fr. and 1 metre = 39.37 ins. [*Ans.* 13.17d per yd.

19. A bankrupt has assets Rs 11550 and liabilities Rs 173250. How much does a creditor for Rs 8326 4as receive? [*Ans.* Rs 555 1a 3p

20. The area of a triangle is equal to $\sqrt{s(s-a)(s-b)(s-c)}$, where $2s = a+b+c$. Find the area of a triangular plot of land, the sides measuring 37, 45 and 54 chains. [*Ans.* 82.39 acres

***21.** Find the weight in tons of a liquid (sp. gr. 1.25) contained in a right cylindrical vessel with a base of 7½ ft diameter and a height of 23½ ft, given that 1 cubic foot of water weighs 62.5 lbs. [*Ans.* 34.26 tons

***22.** The amount of P at the rate of interest i per unit (compound) for n years $= P(1+i)^n$.

(*a*) Find the amount of Rs 4300 at 4% p. a. compound for 5 years. [*Ans.* Rs 5231 10as

(*b*) In how many years will Rs 9000 earn compound interest at 5% p. a. of Rs 1418 10as. [*Ans.* 3 years

*23. A General Average Ratio equals 6·972027 per cent. Ascertain the amounts due from the owner of the steamer worth Rs 185000 ; and from shippers whose contributing values are assessed at Rs 11700 2as 5p ;Rs 26913 4as 6p ; Rs 11047 4as 8p ; Rs 4732 ; Rs 11198 6as 4p.

[*Ans.* Rs 12898 4as ; Rs 815 12as ; Rs 1876 6as ; Rs 770 4as ; Rs 330 ; Rs 780 12as

*24. The income tax on an annual income of Rs 8917 comes to Rs 470 1a. Find the average rate of income tax and the amount of the rebate on Rs 1453 of the income *pro rata.*

[*Ans.* 10·12p ; Rs 76 9as

*25. The taxable profit of a partnership business is Rs 11357 in a year, the income tax on which is Rs 724 5as. Find the average rate of the tax to the nearest 100-th of a pie. Also, ascertain the rebate on premiums for the insurance of the lives of the partners, amounting to Rs 1875 annually.

[*Ans.* 12·25p ; Rs 119 10as

*26. In a Departmental Store, the sales efficiency of a department is measured by the ratio of establishment charges for the department to a fixed percentage of the turnover of the same department, multiplied by 100. Compare the efficiency of the 3 departments A, B & C from the following :

Department	Sales Rs	Percentage	Establishment charges Rs
A Hosiery	15731	5	635
B Crockery	8976	7½	567
C Leather Goods	9210	10	762

[*Ans.* *A* : 80·7 ; *B* : 84·2 ; *C* : 82·7

*27. Solve :

(i) $(1+x)^{26} = 1{\cdot}625$ [*Ans.* ·019

(ii) $(1-x)^{10} = 0{\cdot}945$ [*Ans.* ·019

(iii) $1563\,(1+i)^8 = 1624$ [*Ans.* ·005

(iv) $(1+i)^3 = 1586\,(1+i)^{-10}$ [*Ans.* ·036

(v) $(1{\cdot}053)^n = 1{\cdot}865$ [*Ans.* 12·06

(vi) $(1{\cdot}025)^n \times (1{\cdot}024)^{n-4} = 1{\cdot}684$ [*Ans.* 12·77

(vii) $(1+i)^{15} = \dfrac{2{\cdot}654 \times 1{\cdot}838}{2{\cdot}083}$ [*Ans.* ·056

(viii) $4{\cdot}8 \times (1{\cdot}045)^n = \dfrac{23{\cdot}24 \times 18{\cdot}63}{28{\cdot}53}$ [*Ans.* $n = 59{\cdot}14$

(ix) $$\left.\begin{array}{c} (1+i)^{10} \times (1+i')^6 = 1{\cdot}016 \times \dfrac{(1+i')^{10}}{(1+i)^6} \\ 456(1+i)^{13} = 498 \end{array}\right\}$$

[*Ans.* $i = {\cdot}007$; $i' = {\cdot}023$

(x) $5^{5-3x} + 4^{\frac{x}{2}+3} = 5^{7-3x} - 2^{x+5}$ [*Ans.* 1·2064

[**Hint.** $5^{7-3x} - 5^{5-3x} = 2^{2(\frac{x}{2}+3)} + 2^{x+5}$ or, $5^{5-3x}(5^2-1) = 2^{x+5}(2+1)$ *i.e.* $8.5^{5-3x} = 2^{x+5}$. Take logs. and proceed.]

***28.** If the deflection *d* of a beam of radius *a* and length *l*, due to a load *W* is measured, Young's Modulus for the material of which the beam is made, can be found from the formula $E = \dfrac{4Wl^3}{3\pi da^4}$. If in a certain case the deflection was 4·2 and $W = 14{\cdot}8$, $l = 17{\cdot}56$, $a = {\cdot}39$, find the value of E, taking $\pi = 3{\cdot}142$.

[*Ans.* $3{\cdot}5 \times 10^5$

***29.** If the population of a town increases every year by 1·8% of the population at the beginning of that year, in how many years will the total increase of population be 30% ?

—I. Com. [*Ans.* $14\frac{3}{4}$ yrs. app.

SECTION TEN

COMPOUND INTEREST AND ANNUITIES

§ **Compound Interest.** If i be the rate of interest per unit per period, a principal 1 accumulates at compound interest in the following manner :

$$\begin{array}{lcll}
P_1 & \ldots & 1 & \\
I_1 & \ldots & i & \\
\hline
P_2 & \ldots & (1+i) & \\
I_2 & \ldots & i.(1+i) & \\
\hline
P_3 & \ldots & (1+i)^2 & \because (1+i)+i(1+i)=(1+i)(1+i) \\
I_3 & \ldots & i.(1+i)^2 & \\
P_4 & \ldots & (1+i)^3 & \because (1+i)^2+i(1+i)^2=(1+i)^2(1+i)
\end{array}$$

and so on.

And, the amount at the end of n periods $=(1+i)^n$. If the principal be P, the amount at the end of n periods $A=P(1+i)^n$.

§ This formula may be used logarithmically as $\log A=\log P+n\log(1+i)$. It follows that a sum doubles itself in $\frac{70\frac{1}{2}}{\text{rate}\%}$ years, or more approximately $\left(\frac{\cdot 693}{i}+\cdot 35\right)$ years when i is small.

P may also be looked upon as the present value of A due at the end of n periods, being equal to $\frac{A}{(1+i)^n}$, or in the customary notation $P=A.v^n$, v^n being the present value of 1 due at the end of n years.

§ If the interest is j per unit per annum *nominal*, payable p times a year, $\frac{j}{p}$ is converted into principal p times a year on an initial principal 1. In a year, therefore, 1 accumulates to $\left(1+\frac{j}{p}\right)^p$; and the rate j per unit per annum *convertible* p times a year produces $\left(1+\frac{j}{p}\right)^p-1=i$, interest on 1 in a year. This is known as an *effective rate of interest* corresponding to j. With this frequency of conversion a principal P accumulates to $A = P.\left(1 + \frac{j}{p}\right)^{np}$ in n years.

§ An *annuity* is a series of payments, ordinarily of a fixed amount payable regularly at the end of a fixed period, usually a year, for a fixed number of years. The annual payment may in some instances be arranged to be paid in equal instalments. Annuities are divided into two classes—Annuities Certain and Life Annuities. In an annuity certain payments are to be made unconditionally; in a life annuity, payments commence at the happening of a contingent event,—usually the death of a person, or of his reaching a specified age. The calculation of life annuities is based on that of annuities certain, and the probable number of years for which the payment has to be arranged. The ascertainment of the most probable period is dependent on actuarial calculatious based on mortality experience.

§ **Annuities Certain.** When the first payment under an annuity is to be paid at the end of a year, the annuity is said to be *immediate*. It has been seen that the present value of

1 due at the end of 1 year is v
1 » » » » » 2 years is v^2
1 » » » » » 3 years is v^3
1 » » » » » 4 years is v^4

1 » » » » » n years is v^n

Hence, the *present value* of an *immediate annuity* of 1 for n years

$$= v + v^2 + v^3 + \cdots\cdots + v^n = v \cdot \frac{1-v^n}{1-v} = \frac{1}{1+i} \cdot \frac{1-v^n}{1-\frac{1}{1+i}} = \frac{1-v^n}{i}.$$

The usual symbol for the present value of an immediate annuity, the first payment of which is due at the end of a year from the date of computation, is $a_{\overline{n}|} = \frac{1-v^n}{i}$. When the annuity is perpetual, or virtually so, it is said to be a *perpetuity*, and $= \underset{n=\infty}{\text{Limit}}\left[\frac{1-v^n}{i}\right] = \frac{1}{i}$. If the first payment be due $(t+1)$ years after the date of computation, the present value of such a *deferred annuity* $_{t|}a_{\overline{n}|} = v^t.a_{\overline{n}|}$, and is also $= a_{\overline{n+t}|} - a_{\overline{t}|}$.

In an *annuity-due*, the first payment is made at the beginning of the first period and the last at the beginning of the last period. The present value of such an annuity $\mathbf{a}_{\overline{n}|} = 1 + a_{\overline{n-1}|} = (1+i).\,a_{\overline{n}|}$, which may also be obtained directly.

§ If 1 is set aside at the end of every year for n years, at the end of n years—

the first payment will amount to $(1+i)^{n-1}$
the second " " $(1+i)^{n-2}$
the third " " $(1+i)^{n-3}$

the nth " " 1

The *amount* of an annuity, usually symbolised by

$$s_{\overline{n}|} = (1+i)^{n-1} + (1+i)^{n-2} + (1+i)^{n-3} + \cdots\cdots + 1$$
$$= \frac{(1+i)^n - 1}{(1+i) - 1} = \frac{(1+i)^n - 1}{i}.$$

If the annuity be deferred t years, that is the first payment be made after $t+1$ years, the present value of the amount of such an annuity $= v^{n+t}.s_{\overline{n}|}$.

§ It may be easily demonstrated, both from theoretical considerations as well as from the above formulæ, that

$$a_{\overline{n}|}.(1+i)^n = s_{\overline{n}|}\ ;\ a_{\overline{n}|} = v^n.s_{\overline{n}|}\ ;\ \text{and}\ \frac{1}{a_{\overline{n}|}} = i + \frac{1}{s_{\overline{n}|}}.$$

§ If interest j per unit per annum is converted into principal p times a year, the above formulæ are changed as follows :

The present value of an immediate annuity payable p times a year, written as $a^{p}_{\overline{n}|} = \frac{1}{p}.a_{\overline{np}|}$, $a_{\overline{np}|}$ being calculated at rate $\frac{i}{p}$ per period.

Also, the amount of an immediate annuity on this basis, symbolized by $s^{p}_{\overline{n}|} = \frac{1}{p} \cdot s_{\overline{np}|}$, $s_{\overline{np}|}$ being calculated at rate $\frac{j}{p}$ per period.

Short Methods

§ 1. The direct method of calculating the compound amount of a principal is the safest in the absence of Tables. It should be observed that the compound *interest* on a principal cannot be obtained but by obtaining the difference between the amount and the principal.

The actual calculation should in that case be made with the help of interest tables, or by the method of aliquot parts.

2. A method is to start with 1 or 100 as the principal, obtaining values of amount and interest corresponding thereto; and then to obtain the actual values by the method of proportion.

Illustrations

1. Calculate compound interest on Rs 1535 for 3¼ years @ 4% per annum.

*First method** :

	Rs
Principal	1535
Interest for 1st year ...	61.4
Principal for 2nd	1596'4
Interest	63'856
Principal for 3rd	1660'256
Interest	66'4102
Principal for last quarter ...	1726'6662
Interest for last quarter ...	17'2667
Amount at end of 3¼ years ...	1743'9329
Original Principal ...	1535
Compound Interest	Rs 208'9329
i.e.,	Rs 208 15as

---

*The work should ordinarily be done correct to 4 decimal places.

[N. B. In some cases it may be found convenient to work with annas and pies instead of decimals. This is also the bazar method in India. Any discrepancy that may be observed between the results obtained by the bazar method and the decimal method may be safely ascribed to to the former.]

Second method :

Amount $A = \text{Rs } 1535\,(1 + {\cdot}04)^{3\frac{1}{4}}$

Taking log of both sides,

$$\log A = \log 1535 + 3\tfrac{1}{4} \log 1{\cdot}04$$

$$\log 1535 = 3{\cdot}18609 \qquad \log 1{\cdot}04 = {\cdot}01703$$

$$\therefore\ 3\tfrac{1}{4} \times \log 1{\cdot}04 = 0{\cdot}05535$$

$$\log A = 3{\cdot}24144$$

Hence $A = \text{Rs } 1743{\cdot}9+ = \text{Rs } 1743 \text{ 15as}$

Principal 1535

∴ Compound interest Rs 208 15as

2. A loan of Rs 10000 is to be repaid by 30 equal annual instalments. Find the amount of each instalment to cover principal and compound interest at 4% per annum.

Evidently, the amount of the principal at the end of 30 years at 4% p. a. compound should be equal to the sum of 30 annual payments at the same rate. Then, if the value of the annual payment be Rupees p, Rs $10000 \times (1 + {\cdot}04)^{30} = p.\, s_{\overline{30}|}$

$$= p.\frac{(1 + {\cdot}04)^{30} - 1}{{\cdot}04}\ ,$$

$$\therefore\ p = \text{Rs}\frac{400 \times (1{\cdot}04)^{30}}{(1{\cdot}04)^{30} - 1}\ .$$

This may be evaluated with the help of Tables or with the help of logarithms, and found to be

$$= \text{Rs}\frac{400 \times 3{\cdot}24340}{2{\cdot}2434} = \text{Rs}\frac{1297{\cdot}36}{2{\cdot}2434}$$

$$= \underline{\text{Rs } 579.}$$

Or, directly, $10000 = p.\, a_{\overline{30}|}$: whence $p = \text{Rs } 579$.

3. A company sets aside a sum of Rs 10000 annually accumulate at 4% p. a. compound for 10 years to repay a

Debenture issue. Find the surplus after paying off the Debenture Stock amounting to Rs 120000.

The value of the Stock is nearly equal to the amount of an annuity of Rs 10000 for 10 years

$= \text{Rs } 10000 . s_{\overline{10}|}$

$= \text{Rs } 10000 \times 12{\cdot}0061$, from the Tables

$= \text{Rs } 120061$

$\therefore$ Surplus $= \underline{\text{Rs } 61}$.

4. Find the *effective rate* corresponding to 6% p. a. convertible quarterly.

In 1 year 1 will accumulate to

$$\left(1+\frac{{\cdot}06}{4}\right)^4 \text{ i.e. to } (1{\cdot}015)^4$$

Now, $\log 1{\cdot}015 = {\cdot}00647$; and $4 \log 1{\cdot}015 = {\cdot}02588 = \log 1{\cdot}0614$

$\therefore$ Compound interest on 1 in 1 year $= 1{\cdot}0614 - 1 = {\cdot}0614$

$\therefore$ Effective rate of interest $= 6{\cdot}14\%$ p. a.

Aliter. From the Tables $(1 + {\cdot}015)^4 = 1{\cdot}0614$. Hence the result.

5. Rs 1000 is deposited in a Bank for 10 years. If interest is at 4% p. a. convertible semi-annually, find the amount at the end of the period.

The effective rate of interest $= (1{\cdot}02)^2 - 1 = {\cdot}0404$ per unit p. a.

The amount at end of 10 years $= \text{Rs } 1000 \times (1 + {\cdot}0404)^{10}$

$\log 1{\cdot}0404 = {\cdot}017199$

$10 \log 1{\cdot}0404 = {\cdot}17199 = \log 1{\cdot}4832$

$\therefore$ $(1{\cdot}0404)^{10} = 1{\cdot}4832$, and required amount

$= \underline{\text{Rs } 1483}$ to the nearest rupee.

6. A loan of Rs 1000 is to be paid in 5 equal annual payments interest being at 6% p. a. compound. Analyse the payments into those on account of interest and on account of amortization of the principal.

If the annual payment be R,

$$\text{Rs } 1000 = R.\ a_{\overline{5}|} = R.\ 4{\cdot}2124$$

$$\therefore\quad R = \text{Rs } \frac{1000}{4{\cdot}2124} = \text{Rs } 237{\cdot}39$$

$$= \text{Rs 237 6as to the nearest anna.}$$

AMORTIZATION TABLE

End of year	R	Intt. due	Amortization	Principal earning intt.
1st	Rs 237 6as	Rs 60 0a	Rs 177 6as	Rs 1000
2nd	Rs 237 6as	Rs 49 6as	Rs 188 0a	Rs 822 10as
3rd	Rs 237 6as	Rs 38 1a	Rs 199 5as	Rs 634 10as
4th	Rs 237 6as	Rs 26 2a	Rs 211 4as	Rs 435 5as
5th	Rs 237 6as	Rs 13 7as	Rs 223 15as	Rs 224 1a
Total	Rs 1186 14as	Rs 187 0a	Rs 999 14as	*Balance* 2as

In practice, the outstanding 2as will be adjusted in the last payment which will be Rs 237 8as. This difference, it should be noted, is merely the result of approximation at the different stages to the nearest anna.

EXAMPLES 24

[*Answer to nearest 3-pie or farthing*]

Calculate compound interest on the following

1. Rs 3500 at 9% per annum for 3 yrs. [*Ans*. Rs 1032 9as 6p
2. Rs 15850 at 4% " " 2 " [*Ans*. Rs 1293 5as 9p
3. Rs 23846 at 5% " " 3 " [*Ans*. Rs 3758 11as 6p
4. Rs 56615 at 3% " " 4 " [*Ans*. Rs 7105 11as
5. Rs 28411 at 4% " " 3½ " [*Ans*. Rs 4186 11as
6. Rs 61185 at 6% " " 2¾ " [*Ans*. Rs 10656 1a 9p
7. £1548 at 2% " " 4 " [*Ans*. £127 12as 1¼d
8. £9648 10s at 5% " " 3 " [*Ans*. £1520 16s 10¾d
9. £1468 3s 6d at 2½% " " 4½ " [*Ans*. £162 10s 10¾d
10. £5864 9s 11½d at 3% " 2½ " [*Ans*. £450 9s 5½d
11. Rs 5058 10as at 4% " " 2 " [*Ans*. Rs 412 12as 6p
12. Rs 6838 5as 9p at 4¾% " 3 " [*Ans*. 1021 7as 9p

13. Rs 10848 3as 5p at 3½% „ 2 „ [*Ans.* Rs 772 10as 6p

14. Rs 24136 9as 10p at 1¾% „ 3½ „ [*Ans.* Rs 1511 15as 3p

15. £9208 3s 5¾d at 3¾% „ 2¾ „ [*Ans.* £982 6s 7d

16. £3184 18s 2½d at 2¼% „ 3¼ „ [*Ans.* £239 0s 1½d

17. Rs 9568 4as 10p at 4% p. a. for 2 years with half-yearly rests. [*Ans.* Rs 788 11as 9p

18. £5165 10s 10½d at 3% p. a. for 2½ years convertible half-yearly. [*Ans.* £399 4s 3¼d

19. What sum will amount to Rs 5730 8as 6p at 2% p. a. compound in 3 years ? [*Ans.* Rs 5400

20. What sum will earn compound interest of £666 14s 6d in 2½ years at 4% p. a. ? [*Ans.* £6458 10s 2½d

21. Find the difference between simple and compound interest at 2½% p. a. on £438 5s for 4 years. [*Ans.* £1 13s 6d

22. The interest accrued on a certain sum in a Savings Bank at the end of a year is Rs 200, and the total addition to the principal at the end of the second year, at the same rate, is Rs 408. What is the rate of interest ? What was the original principal ? [*Ans.* 4% p. a. ; Rs 5000

23. What sum at compound interest will amount to £650 at the end of the first year ; and £676 at the end of the second year ? —I. Com. [*Ans.* £625

24. Equal sums are invested by two persons at 5% per annum. The first person accumulates his interest as accrued ; but the second person is permitted to withdraw the interest half-yearly. How much more would the first person receive as interest in 5 years' time ? [*Ans.* 10·5%

25. Find the number of years and fraction of a year in which a sum of money will treble itself at compound interest 4% p. a. —I. Com. [*Ans.* 28 years app.

26. A man obtains a loan of Rs 6000 and agrees to repay it in 5 annual instalments the first payment to be made at the end of a year. What should each instalment be ? —I. Com. [*Ans.* Rs 1386 app.

*§ **Freehold and Leasehold Estates.** The proprietor of a freehold estate enjoys a rent in perpetuity. The value of a freehold carrying an annual rent $R = \frac{R}{i}$. A freehold, as other perpetuities, is frequently considered to be worth a *number of years' purchase*, the value of the freehold being equated to the total return by way of rent during those years. That is $n.i = 1$; and the number of years' purchase n is thus $= \frac{1}{i}$.

On the expiry of the time stipulated in a lease, the estate reverts to the original proprietor. If the estate is worth A, and is to revert after n years, the value of the *reversion* $= A.v^n$. If the value of the estate is expressed in terms of the annual rent, R, $A = \frac{R}{i}$, and the value of the reversion $= R.\frac{v^n}{i}$.

When a leasehold is transferred, account has to be taken of the present value of the *perpetuity* of the rent, in terms of the lease, and the present value of the *reversion*. Thus, if the *freehold* be worth A and the price of the lease be L,

$$\begin{aligned} A &= L + \text{P. V. of Reversion} + \text{P.V. of an annuity of the Rent } R' \text{ under lease} \\ &= L + A.v^n + R'.a_{\overline{n}|} \end{aligned}$$

$$\therefore \quad L = A(1 - v^n) - R'.\, a_{\overline{n}|} = a_{\overline{n}|}(Ai - R') = (R - R').a_{\overline{n}|}.$$

This realationship may also be directly written down.

Illustrations

1. A leasehold carries a rent of Rs 283 12as and has still 10 years to run. What is the price of the lease, reckoning interest at 5% p. a. compound ?

Required price = Present value of an annuity of Rs 283·75 for 10 years = Rs 283·75$a_{\overline{10}|}$ @ 5% = Rs 283·75 × 7·7217 from Tables = Rs 2191·33 = Rs 2191 to nearest rupee.

2. An estate worth Rs 2,00,000 is leased out for 99 years. Find the present value of the reversion to the owner, reckoning compound interest at 5% p. a.

$$\begin{aligned}\text{The present value} &= \text{Rs } 2{,}00{,}000 \times v^{99}\\ &= \text{Rs } 2{,}00{,}000 \times \cdot 007963\\ &= \underline{\text{Rs } 1593} \text{ to nearest rupee.}\end{aligned}$$

3. A freehold with an annual income of Rs 2000 is to revert to the owner after 10 years. Find the present value of the reversion at 5% p.a. compound.

The P. V. required = a P. V. of a perpetuity of Rs 2000 deferred 10 years $= v^{10} . \frac{\text{Rs } 2000}{i} = \text{Rs } \cdot 6139 \times \frac{2000}{\cdot 05}$

$$= \text{Rs } 40000 \times \cdot 6139 = \underline{\text{Rs } 24556}$$

4. Find the value of the leasehold in illustration 2 above. The income from the freehold in perpetuity may be taken at 4% p.a., and the annuity is to be calculated at 5% p.a. compound, the lessee paying a ground rent of Rs 6000 per annum.

Here $R = 4\%$ of Rs 2,00,000 = Rs 8000,

$R' =$ Rs 6000

$$\begin{aligned}\therefore \quad \text{Value of lease} &= \text{Rs } (8000 - 6000) . a_{\overline{99}|}\\ &= \text{Rs. } 2000 \times 19 \cdot 84030\\ &= \underline{\text{Rs } 39681} \text{ to nearest rupee.}\end{aligned}$$

5. A leasehold is purchased for Rs 50000. It has a tenure of 80 years and annual ground rent of Rs 1000. What additional sum must be paid to convert it into a freehold on the basis of 4% per annum compound interest ?

At 4% p. a. a freehold of value A should have an income $A \times \cdot 04$ p.a. Value of lease Rs 50000 = P.V. of an annuity of

$$(A \times \cdot 04 - 1000) \text{ for 80 years.}$$

$$\text{Or, } 50\,000 = (A \times \cdot 04 - 1000) \times 23 \cdot 915$$

$$\text{Or, } A \times \cdot 04 = \text{Rs}\left(\frac{50\,000}{23 \cdot 915} + 1000\right)$$

$$\therefore \quad A = \text{Rs } 77270$$

and additional sum = $\underline{\text{Rs } 27270.}$

§ **Endowment Fund.** A fund to pay 1 per annum held at i per unit per annum has the same value as the present value of a perpetuity, and it $= \frac{1}{i}$.

A fund to assure a *rent* of R per annum $= \frac{R}{i}$.

§ **Sinking Fund.** When a sum is set aside annually to accumulate at compound interest to liquidate a loan on a given date, the fund thus created is known as a Sinking Fund. If L be the amount of the loan to be paid off at the end of n years by the accumulation of the annual sum P,

$$L = P.s_{\overline{n}|}, \text{ and } P = \frac{L}{s_{\overline{n}|}}, \text{ or } = \frac{Li}{(1+i)^n - 1}.$$

It has been assumed that the interest on the loan is met from a different source. If, however, it is proposed to pay back the principal of the loan with interest at i' per unit per annum compound, the *amount* of the loan is equal to the amount of the annuity.

Or, $$L(1+i')^n = P.s_{\overline{n}|}, \text{ and } P = \frac{L.(1+i')^n.i}{(1-i)^n - 1}.$$

If the rate of interest paid be equal to the rate at which the Sinking Fund accumulates, *i.e.* $i' = i$,

$$P = \frac{L.i}{1 - v^n} = \frac{L}{a_{\overline{n}|}}$$

which can also be demonstrated directly from theoretical considerations.

*§ The present value of an n-year annuity of 1 to pay i' on the investment during the entire term and replace capital by means of a sinking fund accumulating at $i = i' + \frac{1}{s_{\overline{n}|}} = \frac{1}{a_{\overline{n}|}} + (i' - i)$, $a_{\overline{n}|}$ and $s_{\overline{n}|}$ being calculated at rate i. Of these, i' is said to be the *remunerative* rate and i the *reproductive* rate. The purchaser of such an annuity realizes interest at rate i' on the entire investment, and replaces his invested capital by means of a sinking fund at the usually lower rate i.

*§ **Replacement of Asset.** It is sometimes necessary to ascertain the present value of a Fund which will pay the first cost of an asset and provide the amount necessary to renew the asset indefinitely; for example, when it is desired to find funds sufficient for the construction of a bridge and provide for its replacement at the end of every n years, say.

$$\text{Fund} = \text{Original Cost} + \frac{\text{Present Value of Perpetuity to replace Cost}}{s_{\overline{n}|}}$$

$$= \text{Cost} + \frac{\text{Cost}}{i.s_{\overline{n}|}} = \frac{\text{Cost}}{i.a_{\overline{n}|}}.$$

§ If the annual cost of up-keep has also to be provided by the fund, the total fund necessary

$$= \text{Cost} + \frac{\text{Cost}}{i.s_{\overline{n}|}} + \frac{\text{Up-keep}}{i}$$

$$= \frac{1}{i}\left[\frac{\text{Cost}}{a_{\overline{n}|}} + \text{Up-keep}\right].$$

§ Calculations from these formulæ are made with the help of Tables of Compound Interest and a Table of Logarithms.

The more useful values of $(1+i)^n$, v^n, $a_{\overline{n}|}$ and $s_{\overline{n}|}$ are to be found in the Tables appended.

The work should be carried on in decimals throughout to ensure accuracy. It should be remembered that the final result is ordinarily to be obtained correct to 4 decimal places, and the work contracted accordingly.

It should be noted that a fractional period is accounted for in the last stage. Thus to find the amount after 3¼ years, the amount for 3 years should first be computed, and a quarter year's interest on it added thereto.

*EXAMPLES 25

[*Answer to the nearest rupee unless otherwise required*]

1. **Find the value of an annual payment to a Sinking Fund accumulating at 4% p. a. to liquidate a debt of Rs 50000 in 10 years with interest at 5% p. a. compound. [*Ans.* Rs 6784 p. a.**

2. Find the present value of an annuity of Rs 1000 per annum, the first payment to begin on the maturity of a life insurance policy at the end of 4 years, and to continue for 11 years thereafter. Reckon compound interest at 5% per annum. [*Ans.* Rs 7175

3. A leasehold property is purchased at Rs 3652, when the lease has still 25 years to run. Find the amount that should be set aside each year, beginning from the time of purchase, to recover this value at 3% p. a. compound. [*Ans.* Rs 100

4. An overdraft of Rs 30000 is to be paid back in equal annual instalments over a period of 30 years. Find the value of this payment reckoning compound interest at 4% p. a. [*Ans.* Rs 1735

5. A Municipal Council borrowed Rs 50000 from Government to be paid back in 20 years without interest. Find the amount that should be paid annually into a Sinking Fund, accumulating at 4% p. a. compound. to pay off the loan. [*Ans.* Rs 1679

6. A person borrows Rs 5000 from an Insurance Company against a policy on his life, on the understanding that he will pay it back in 4 equal instalments with interest at 6% p. a., paid along with the half-yearly premium on the policy. If the first instalment is collected at the end of six months after the date on which the loan was taken, find the amount of each instalment. [*Ans.* Rs 1345

7. A motor car is purchased at the list price of Rs 3110 *plus* interest at 5% p. a. compound, if the amount is paid in equal monthly instalments over a period of 3 years, the first payment being made on delivery. Find the amount of an instalment. [*Ans.* Rs 100

8. The production of wheat in a country is 53×10^6 bushels, and its total consumption of wheat per annum is 268×10^6 bushels. During the next 10 years, it is planned to increase the production by 4% every year. If the consumption increase by 2% every year, find the estimated ratio of production to consumption at the end of 10 years. [*Ans* 24%

9. A Life Insurance Company guarantees a reversionary bonus of Rs 50 per thousand compounded at the end of every triennial valuation. A person insures his life for Rs 10000 in 1930, just after a valuation is made. If the policy matures at the end of 30 years, find the amount payable under the policy.

[*Ans.* Rs 16289

10. A person leaves his three sons, aged 6, 10 and 12 years, Rs 10000, Rs 12000 and Rs 15000 respectively, invested at 3% p. a. compound, and payable on each child completing his 18th year. Find how much each gets on attaining majority.

[*Ans.* Rs 14258 ; Rs 15201 ; Rs 17911

11. A business firm sets aside for a reserve the sum of Rs 32000 annually for the replacement after 10 years of certain machinery costing Rs 350000. Assuming that the reserve accumulates at 2½% p. a. compound, find the amount that may be carried over to the P & L account, if at the end of the period exactly similar machinery is purchased at Rs 320000 and the old one is sold at Rs 85000. [*Ans.* Profit : Rs 123508

12. A person desires to donate sufficient funds for the erection of a new building at a cost of Rs 150000, and to provide for its maintenance, repairs, rates and taxes &c. at an estimated annual cost of Rs 5000. Find the total value of the endowment, which may be assumed to be capable of being invested at 4% p. a. [*Ans.* Rs 275000

13. A war pensioner is given an annual pension of Rs 250 in perpetuity, the right of receiving the pension being transferable. Find how much a person should pay him to acquire the pension right when money is at 3½% p. a. [*Ans.* Rs 7143

14. A fixed royalty of Rs 2000 p. a. for 99 years was guaranteed to the holder of certain mining rights by a purchaser. The right of receiving the royalty is put up for auction after 69 years have already passed. Find the price at which it may be sold, assuming that money is worth 3% p. a. [*Ans.* Rs 39201

15. A person desires to create an endowment fund to provide for a prize of Rs 500 every year out of income. If the fund

can be invested at 3% per annum, find the amount of the endowment. [*Ans.* Rs 16667

16. A person desires to pay for the maintenance of a ward in a hospital at an estimated annual cost of Rs 20000. What is the minimum amount of 3½% Government Stock that should be set aside for the purpose ? Find its value at the price of 100½.

[*Ans.* Rs 571500 stock ; Rs 574357 8as

17. A joint stock company decides to raise a loan of Rs 10 lakhs by the issue of debentures at 4% p. a., and to create a Sinking Fund at 3% p. a. compound to pay off the loan in 25 years. What will be the annual cost of this loan to the business ? [*Ans.* Rs 67428

18. A fleet of 10 lorries is purchased at a cost of Rs 3850 each. It is estimated that the average net annual income from the fleet, after providing for all necessary running and incidental expenses, is Rs 9500. If the lorries are to be scrapped at the end of 7 years, find how much should be set aside every year to create a Reserve at 3% p. a. compound for the replacement of the fleet at an estimated cost 10% lower than the present. Is the business profitable ? [*Ans.* Rs 4522 ; Yes

19. A new textile machinery is purchased at a cost of Rs 200000. It is desired to set aside every year a sum of money that would provide for the replacement of the machinery by a new model after 10 years at an estimated price 10% higher than that of the present one. Find the amount that should thus be set aside annually to accumulate at 2½% p. a. compound.

[*Ans.* Rs 19637

20. A philanthropist desires to place in the hands of Government a sum of money sufficient to build a bridge at an estimated cost of Rs 196000, as well as to provide for its replacement at the end of every 30 years. Assume that the investment of the lump sum can be made at 3% p. a. in perpetuity and that the annual dividends may be accumulated also at the same rate ; and the cost does not alter. [*Ans.* Rs 333333

21. A minor aged 10 years is left an estate of Rs 50000. The trustee of the estate invests the money at 4% per annum, and pays the minor for his expenses a sum of Rs 1200 at the end of every year. The expenses of management come to Rs 300 per annum. Find the amount that will be handed over to the minor on his coming of age (18 years). [*Ans.* Rs 54607

22. A person desires to make the following provisions for his children :

1st son, aged 10 years and 2nd son aged 6 years, to receive Rs 400 per annum each towards expenses of their education till each is 21 years old, when they should get a further lump sum of Rss 10000 each.

A daughter aged 4 years to receive Rs 600 per annum towards her educational expenses till she is 18 years of age, when she should get a lump sum of Rs 4000.

Find the amount that has to be set aside for this purpose, assuming that money can be invested at 4% p. a. compound.

[*Ans.* Rs 28648

23. The accumulations in a Provident Fund are invested at the end of every year to earn 5% p. a. A person contributes 12½% of his salary, to which the employer adds 10% every month. Find how much the total accumulations will amount to at the end of 30 years of his service, for every 10 rupees of his monthly salary. [*Ans.* Rs 1793·8478

24. What is the amount and present value of an annuity certain of Rs 150 for 12 years, reckoning interest at 3½% p. a. Given $(1{\cdot}035)^{12} = 1{\cdot}511066$. —I. Com.

[*Ans.* Rs 2190 4as 7p ; Rs 1449 8as

25. What sum will amount to Rs 1000 in 12 years time at 4½% p. a. compound ? —I. Com. [*Ans.* Rs 589 10as 7p

What principal will amount to £720 6s 6d in 4 years at 3% p. a. compound ? —I. Com. [*Ans.* £640

26. Which is better, an annuity of £150 to last for 10 years, or the reversion of a freehold estate of £79 4s per annum to commence 7 years hence, the rate of interest being 5% ?—I. Com.

[*Ans.* First

27. What rate of interest per annum does a person get who is paid at the rate of 5% compound payable half-yearly ? —I. Com. [*Ans.* $5\frac{1}{16}$%

28. A man obtains a loan of Rs 6000 and agrees to repay it in 5 annual instalments at 5%, the first payment to be made at the end of a year. What should each instalment be ? —I. Com. [*Ans.* Rs 1385

29. A man left Rs 18000 with the direction that it should be divided in such a way that his 3 sons, aged 9, 12 and 15 years, should each receive the same amount when they reached the age of 25. If the rate of interest is $3\frac{1}{2}$% p. a., what should each son receive when he is 25 years old ?—I. Com. [*Ans.* Rs 9350 6as

30. The machinery in a factory is valued at £24,537 and it is decided to reduce the estimated values, at the end of each year, by 18% of the value at the beginning of that year. When will the value be (*a*) £20,000 and (*b*) one-tenth of its original value ? —I. Com. [*Ans.* 1·03 yr. ; 11·6 yrs.

31. A man borrows Rs 1000 on the understanding that it is to be repaid with interest in four equal instalments at intervals of six months, the first payment to be made six months after the money was borrowed. Calculate the amount of each instalment, reckoning compound interest at $2\frac{1}{2}$% per half-year. —I. Com. [*Ans.* Rs 265 13as to nearest anna

32. A man wishes to buy a house valued at £1750. He is prepared to pay £800 now and the balance in 9 equal annual instalments. If interest is calculated at $4\frac{1}{2}$% p. a., what should he pay annually ?—I. Com. [*Ans.* £130 13s 11d to nearest penny

33. A person borrows £1500 promising to repay the sum borrowed and the proper interest by ten equal yearly instalments, the first to fall due in one year's time. Reckoning compound interest at 5% p. a., find the value of the annual instalment. Take $(1·05)^{10} = 1·629$. —I. Com. [*Ans.* £194 5s 9d

34. A member of a society borrows £400 and agrees to repay it with compound interest at 5% in five equal annual payments, the first payment to be made at the end of a year. What would each instalment be? —I. Com. [*Ans.* £92 6s 9d

35. A building changed hands 3 times, each agent who sold it charging 25s per cent commission. If each time it was sold for the net amount obtained at the previous sale, calculate its original value if the third sale realised £4622 4s 9¾d? —I. Com.

[*Ans.* £4800.

§ **Yield on Redeemable Stocks.** The valuation of redeemable stocks presents some difficulty. The yield on an irredeemable stock or a perpetuity is directly obtained by dividing the dividend by the price; because the only consideration in respect of an investment in such a stock is the return that it brings to the investor on the amount invested. Its value naturally varies inversely as the money market rate.

Redeemable stocks stand on a different footing, inasmuchas they are repayable at some definite future date, usually another date being mentioned before which payment cannot be tendered by the borrower in repayment. The market price of such stocks is accordingly not determined merely by the dividend it pays. It is influenced, in a market of falling money rates, by the longest possible tenure of the loan, which is determined by the earliest date on which repayment may be tendered by the borrower; and in a market of rising rates, by the latest date on which payment has to be made. For the same reason, the difference between the market value and the repayment value of such stock shrinks to zero at the time when repayment becomes due. While in most cases stocks are issued and are repayable at par, in some instances they are repayable at a premium, and in some others they are issued at a discount. The object is to leave the ruling money rate undisturbed.

§ The yield on an investment in a Redeemable Stock may be computed on the basis of an average return on the investment at an estimated rate of simple interest during the period of maturity. For simplicity, it may be assumed that the dividends received are accumulated without interest.

Thus, a 4% stock repayable at par in 1946, is valued in say 1936, on the basis of the total return during the next 10 years. If its price on the date of valuation be 104, and i be the average yield per unit per annum, at the end of 10 years, the total amount returned on maturity $= 100 + 10 \times 4$ *i.e.*, 140. It is equal to the initial investment of 104 accumulated at simple interest equivalent to the average yield, which $= 104(1 + 10i)$,

$$i.e. \quad 1040i = 36, \; i.e. \; i = \tfrac{36}{1040} = \cdot 0346$$

$$\therefore \quad \text{Average yield} = \underline{3 \cdot 46\%} \text{ p. a.}$$

§ On this basis, if the price of 100 stock be P at a premium $p(=P-100)$ and it carries dividend at D per annum during n years to maturity, when it is repayable at par, the amount of the investment at an average rate of r per cent per annum is

$$P + P \cdot \frac{nr}{100} = 100 + n.D,$$

$$\text{or,} \quad p = P - 100 = n\left[D - P \cdot \frac{r}{100}\right].$$

This gives the premium at which the stock may be valued to yield an average rate of r% per annum.

$$\text{Also,} \qquad r = \frac{100}{P}\left(D - \frac{p}{n}\right).$$

If the stock is at a discount p will be negative.

*§ If, however, dividends can be compared to the interest that could be earned on the investment at the prevailing rate of interest, the premium in price is equated to the present value of an annuity of the difference between the dividend actually paid, and the interest that could thus be obtained.

$$\text{Then,} \qquad p = (D - r).a_{\overline{n}|}$$

It is customary to take the basis of valuation r% p. a. convertible with the same frequency as that with which dividends are paid. In India, Government Stocks pay dividends half-yearly and the premium $p = \frac{1}{2}(D - r).a_{\overline{2n}|}$

* § To obtain the yield from this formula, it is seen that

$$\frac{p}{D - r} = a_{\overline{n}|}$$

It is difficult to evaluate r from this formula by the ordinary methods of Arithmetic. A first approximation is given by the average rate of yield as above. The rate is then determined by methods of successive approximation with the help of a table of the values of $a_{\overline{n}|}$.

A fair approximation, in case where n *is not large*, may be obtained as below :

$$= \frac{D - \frac{p}{n}}{1 + \frac{n+1}{2n} \cdot \frac{p}{100}}$$

Illustrations

1. A stock pays $3\frac{1}{2}\%$ p. a., and is due to mature at the end of 14 years when it is repayable at par. What is the average yield on an investment in this stock at a price of $94\frac{11}{16}$?

Investment ... Rs 94 11as.

Repayment *plus* interest for 14 years = Rs (100 + 49) = Rs 149.

Return on Rs 94 11as in 14 years = Rs 54 5as.

$$\therefore \text{ Average yield } \% = \frac{\text{Rs } 54 \ 5\text{as}}{\text{Rs } 94 \ 11\text{as}} \times \frac{100}{14}$$

$$= \frac{5431{\cdot}25}{14 \times 94{\cdot}6875} = \underline{4\% \text{ p. a.}}$$

*2. To find the yield in the above case by the annuity method, 4% is taken as a first approximation. The formula $p = (D - r).a_{\overline{n}|}$ then, gives $5\frac{5}{16} = (4 - 3\frac{1}{2})a_{\overline{14}|}$ *i.e.* $a_{\overline{14}|} = 10{\cdot}625$.

From the Tables, at $3\frac{1}{2}\%$ p. a. $a_{\overline{14}|} = 10{\cdot}9205$

and at 4% p. a. $a_{\overline{14}|} = 10{\cdot}5631$

Difference for $\frac{1}{2}\%$ $= {\cdot}3574$

Also, trial value of $a_{\overline{14}|} = 10{\cdot}625$

and difference for interpolation $= {\cdot}0619$

$$\therefore \text{ Difference in rate } \% = \frac{{\cdot}0619 \times \frac{1}{2}}{{\cdot}3574} = \frac{{\cdot}0619}{{\cdot}7148} \Rightarrow {\cdot}08$$

$$\therefore \text{ Yield } \Rightarrow (4 - {\cdot}08)\% \Rightarrow \underline{3{\cdot}92\%}.$$

§ An approximation may also be obtained directly as follows : A 5-years' stock carrying 5% p. a. on which interest is payable half-yearly is bought at 97$\frac{5}{16}$. Find the average yield on the investment.

	Rs.
Half-yearly dividend ...	2·500
Total gain on redemption (100 − 97$\frac{5}{16}$)	2·688
Average gain or redemption (2·69 ÷ 10) ...	·269
Average yield per half-year ...	2·769
Capital invested at beginning of 10th period before maturity	97·313
Capital invested at beginning of last period before maturity (100 − ·269) ...	99·731
Total ...	197·044
Average capital at investment...	98·522
∴ Average half-yearly rate of yield on investment *i.e.* average yield ÷ capital (2·769 ÷ 98·522) ...	·0281
∴ Yield% per annum ...	5·62

Such an approximation gives a fairly reliable value when the period to maturity is small. In other cases the first method is recommended.

Note. The same result is obtained by applying the last formula on p. 199. The formula on p. 198 gives 5·69% as an approximation.

EXAMPLES 26

1. Find the average yield on the following to nearest ⅛-per cent :

Stock carrying	Time to Maturity	Purchase price
4 %	10 years	103¾
5 %	10 „	115½
3⅛%	20 „	98
3⅜%	30 „	96

[*Ans.* 3½% ; 3% ; 3¼% ; 3⅝% p. a.

2. A 4% War Loan is issued at par in September 1941, on condition that it will be paid off in September 1961 at a premium of 5%. Find the average return to an investor who buys the stock at 110 in September 1943. [*Ans.* 3·38%

3. The 4% stock (1950-70) is quoting at 102$\frac{1}{2}$ in 1938. Find the average yield on the investment if the Loan is repaid at par (i) in 1950, (ii) in 1960, and (iii) in 1970.

[*Ans.* 3·70% ; 3·79% ; 3·83%

4. A 3% Municipal Loan is issued at 98, being repayable at par in 1960. Find the average yield to an investor who purchases the stock in 1939 at 84. [*Ans.* 4·48%

***5.** Find the value of the following stocks by the annuity method on the basis of valuation noted against each :

Stock carrying	Time to maturity	Basis of valuation
4 %	16 years	3$\frac{1}{2}$% p. a.
4$\frac{1}{2}$%	10 "	4 % "
5 %	10 "	4$\frac{1}{2}$% "
6 %	12 "	5 % "

[*Ans.* 106$\frac{1}{16}$; 104$\frac{1}{16}$, 103$\frac{15}{16}$; 108$\frac{7}{8}$

***6.** The 5% War Loan (1945-55), repayable at par, was offered in 1945 to be converted into a New Loan (1980-90) carrying 3$\frac{1}{2}$%. The rate of conversion would be 105 new stock in exchange for 100 old stock. Find the change in the yield to an investor who bought the stock in 1933 at 103.

[*Ans.* Difference 1·18%

SECTION ELEVEN

PROBABILITIES AND ASSURANCES

§ Past human experience lays the foundations of the science of prediction of the happening of similar events in future. Calculations about likely deaths and survivals amongst insurants have been reduced to a fine art in life insurance ; and similar assessment of the probable happening of a casualty provides the basis of all kinds of insurance.

The problem of insurance is to collect money (premiums) from insurants sufficient to cover the cost of paying out claims as they arise. As, however, there can be no sure knowledge of exactly how many claims will arise and when, the process involves the estimation of probabilities in these respects. Where the number of past observations is sufficiently large the probable experience in the future can be estimated with a considerable degree of accuracy.

Premiums calculated on these bases, from purely mathematical reasoning are called *pure* premiums. The actual premiums charged by insurance companies, however, provide for a sufficient *loading* to cover the costs of running the business and a small return on the capital invested.

§ **Probabilities.** 1. If an event can happen in a ways and fail in b ways, *all these ways being equally likely*, the probability of the happening of the event $p=\frac{a}{a+b}$; and the probability of its failure $q=\frac{b}{a+b}$. It should be noted that there being no third possibility $p+q=1$, representing certainty.

Thus, if a coin is tossed it will either fall with the head up or the tail up. The probability, therefore, of its falling head up $=\frac{1}{1+1}=\frac{1}{2}$; and of the head failing to turn up it is also equal to $\frac{1}{1+1}=\frac{1}{2}$. The truth of the statement may be verified when a sufficiently large number of tosses is made. The chances are even that on the whole the head will come up about as many times as the tail.

2. If there be several independent events, the probabilities of the happening of which are respectively $p_1, p_2 \ldots p_n$, then the probability that all will happen is $p_1.p_2 \ldots p_n$. The probability that none will happen $= q_1.q_2 \ldots q_n$, *i.e.*, $(1-p_1)(1-p_2)\ldots(1-p_n)$; that at least one will not happen $= 1 - p_1.p_2 \ldots p_n$; that at least one will not fail $= 1 - q_1.q_2 \ldots q_n = 1-(1-p_1)(1-p_2)\ldots(1-p_n)$.

3. If there be two or more events which cannot happen together, then the probability that one or other of them will happen is the sum of their probabilities.

Illustrations

A township is divided into four areas as follows, and the experience of past years gives the following averages of fire breaking out in a year in the respective areas causing major and minor damages.

Area	No. of houses	Av. No. of cases of fire annually causing damages		
		Minor	Major	Total
A	10,000	10	1	11
B	20,000	35	3	38
C	30,000	60	6	66
D	15,000	18	2	20

1. In area *A*, the probability of fire breaking out during a year in any house $= \frac{11}{10000}$, *i.e.* ·0011 ; of which the probability of a major accident is only ·0001, that of a minor one being ·001. The probability that no fire breaks out is, of course, very high, being equal to ·9989.

2. The probabitily that there is an outbreak of fire in *all* the areas during the year $= {}^{\cdot}0011 \times {}^{\cdot}0019 \times {}^{\cdot}0022 \times {}^{\cdot}0013 \Rightarrow 6 \times 10^{-12}$ *i.e.* the chances are 6 in a million million, which is a rarity.

3. The probability that there will be an outbreak in B and C and none in A or D during a year $= {}^{\cdot}0019 \times {}^{\cdot}0022 \times {}^{\cdot}9989 \times {}^{\cdot}9987 \Rightarrow 4 \times 10^{-7}$ *i.e.* about 40 in a million.

4. If an average minor damage is estimated to cost Rs 2000 for repairs, and a major damage Rs 20000 in area A, for example, the total probable cost of repairs in a year $=$ Rs $({}^{\cdot}001 \times 2000 + {}^{\cdot}0001 \times 20000)$ *i.e.* Rs 4.

It is evident that if a pool can be formed out of *pro rata* contributions from the owners of the houses in area A, on the basis of the value of each property, and the experience is spread over a sufficient number of years, a small annual fund should be sufficient to make good all losses by fire. It will be easily recognized that this is the basis of fire insurance.

§ **Mortality Tables.** For purposes of Life Insurance, Tables of Mortality have been compiled from which probabilities of survival at different ages can be calculated. Below is an extract from such a Table.

Age x	20	21	22	23	24	25
l_x	96061	95513	94931	94322	93691	93044
d_x	548	582	609	631	647	658
q_x	·00572	·00608	·00643	·00668	·00691	·00707

In the population examined, it appears that out of $l_{20} = 96061$ people at age 20, $l_{21} = 95513$ attained the age 21 and $d_{21} = 548$ died before completing 21.

The rate of mortality q_x at age 20, *i.e.* $q_{20} = \cdot 00572$, being really $= \frac{d_{20}}{20}$.

Based on this tabulated experience we may look to the future and state that in respect of lives similar to those covered by the Tables, the probability of a person aged x years surviving n years $= \frac{l_{x+n}}{l_x}$. This is symbolised by ${}_np_x$ or merely p_x; and the probability of his not surviving that period by ${}_nq_x$ or merely q_x, and $= 1 - {}_np_x = \frac{l_x - l_x}{l_x}$

§ **Expectation of Life.** This expression is generally misunderstood. It does *not* mean the probable length of life of any one person of a given age. It is the average after-lifetime at a given age according to the Mortality Table selected. Below is an extract from a Table of Expectation of Life.

Age Years	Mean Expectation of Life	
	Male	Female
20	40·08	43·04
25	36·15	38·26
30	32·58	34·17
35	28·72	30·86
40	25·12	27·04

This Table has been formed by distributing the excess of ages of those who live long amongst those who die early, so as to place all lives on an equal footing. Very few of the male lives, for example, now at age 35 will fail when they have completed the mean expectation of 28 years odd noted against that age.

Many will fall far short of it, and many will long survive it. With a sufficiently large population, however, the chances are even that the number of earlier deaths at different ages will be balanced by corresponding survivals.

§ Life Annuity. A life annuity is a series of periodical payments dependent on the continued existence of a given life or lives. An *annuity certain* is fully determinate; but as the continuance of a life or lives is a matter of chance, the determination of life annuities entails complicated calculations, providing for *life contingencies.*

The value of a sum, the receipt of which depends upon a contingency, is the present value of the sum taken as a certainty multiplied by the probability of its being received.

Therefore, the value of an endowment of 1 payable at the end of n years, if a person now aged x be then alive $= v^n . {}_np_x = \frac{v^n . l_{x+n}}{l_x} = \frac{v^{x+n} . l_{x+n}}{v^x . l_x}$, and is symbolised by $\frac{D_{x+n}}{D_x}$.

This benefit is known as Pure Endowment; and values of D_n have been tabulated by actuaries for different ages and involving different rates of interest. These are known as *Commutation Tables.*

In the case of a life annuity payable at the end of each year *so long as x remains alive,* we have a series of pure endowments. The present value of such an annuity,

$$a_x = vp_x + v^2 . {}_2p_x + v^3 . {}_3p_x + v^4 . {}_4p_x + \ldots\ldots$$

$$= \frac{D_{x+1} + D_{x+2} + \ldots}{D_x} = \frac{N_x}{D_x},$$

N_x being used for $\sum_{1}^{n} D_{x+r}$.

These values of N_x have also been tabulated corresponding to x, indicating the sum of the relative values of D for ages $x+1$ and upwards.

The present value of a *temporary annuity* of 1 payable annually for n years if a life aged x shall live so long $= \frac{N_x - N_{x+n}}{D_x}$.

§ **Assurances.** The business of life insurance is to spread the risk of loss due to death of insurants at agreed values, which is the Sum Assured.

If A_x represents the present value of 1 payable at the end of the year of death of a life now aged x, the cost per person of paying 1 for each death in the first year amongst l_x persons at age $x = \frac{d_x}{l_x}$; the present value of this $= v \frac{d_x}{l_x}$. The present value of the cost per person of paying for each death in the second and succeeding years are $v^2.\frac{d_{x+1}}{l_x}$, $v^3.\frac{d_{x+2}}{l_x}$, &c. &c.

The total cost of paying 1 for each death is

$$A_x = \frac{v.d_x + v^2.d_{x+1} + v^3.d_{x+2} + \dots}{l_x}$$

$$= \frac{v^{x+1}.d_x + v^{x+2}.d_{x+1} + v^{x+3}.d_{x+2} + \dots}{v^x.l_x} = \frac{M_x}{D_x},$$

where $M_x = v^{x+1}.d_x + v^{x+2}.d_{x+1} + v^{x+3}.d_{x+2} + \dots$

§ It can be easily seen that A_x is the *single premium* for every person aged x for an assurance of 1 payable at the end of the year of his death. It is of, course, the net single premium, and does not take account of *loading* for office expenses &c. that would be added by an insurance company to cover the life.

§ *Annual Premiums.* A single premium in such cases may easily be looked upon as the present value of the annual premiums that the insurant may be called upon to pay. If P_x be the annual premium

$A_x = P_x\,(1 + a_x)$, since premiums are payable *in advance.*

$$\therefore P_x = \frac{A_x}{1 + a_x}.$$

The following extract from Commutation and other actuarial Tables are based on a rate of interest 4% per annum. The values have been approximated for condensation, as necessary.

An Extract from 4-Per Cent Commutation & other Tables

x	D_x	N_x	M_x	a_x	A_x	P_x
10	67557	1355494	12824	20·065	·18982	·00901
15	54528	1058012	11739	19·403	·21528	·01055
20	43841	818165	10687	18·662	·24377	·01240
25	34903	626472	9465	17·949	·27118	·01431
30	27652	474347	8344	17·155	·30175	·01662
35	21828	354066	7371	16·221	·33768	·01961
40	17138	259398	6502	15·136	·37937	·02351
45	13340	185419	5695	13·900	·42692	·02865
50	10243	128258	4916	12·522	·47995	·03550

Illustrations

1. The cost of an annuity of Rs 1000 a year during life to a person aged 30 would be a single payment of $\text{Rs }1000\times\frac{N_{30}}{D_{30}} = \text{Rs }1000\times a_{30} = \underline{\text{Rs }17155.}$

2. The cost of an annuity of Rs 1000 for 10 years to such a person deferred 5 years would be a single payment $= \text{Rs }1000\times\frac{N_{35}-N_{45}}{D_{30}}$ $\text{Rs }1000\times\frac{168647}{27652} = \underline{\text{Rs }6099}$ to the nearest rupee.

3. The annual premium for an assurance of 1 payable at death of a person now aged 30 $= \text{Rs }1000\times P_{30} = \underline{\text{Rs }16·62}$

4. The single premium required to cover a payment of Rs 1000 at the end of the year of death of a person now aged $30 = A_{30}\times\text{Rs }1000 = \underline{\text{Rs }301\ 12\text{as.}}$

SECTION TWELVE

MENSURATION

The problems of mensuration, which means measurement, are twofold. Surfaces and Volumes have to be computed with the help of formulæ obtained by the use of Geometry, Trigonometry and Calculus. Some of the more useful formulæ have been given, mostly without explanation, although a number of them may be obtained geometrically with little difficulty. The second problem relates to methods of calculation with the help of formulæ. This requires more detailed attention.

* § **Duodecimals.** A useful method of computation is by the method of duodecimals.

In the workshop and in freight calculation, the linear, square and cubic foot are subdivided into primes, seconds, etc. Thus,

1 ft., 1 sq. foot, 1 cub. ft. $= 12^{I}$ primes.
$1^{I} = 12^{II}$ seconds.
$1^{II} = 12^{III}$ thirds.
$1^{III} = 12^{IV}$ fourths.
&c. &c. &c.

It may be easily verified that 1 linear prime = 1 inch ; 1 sq. second = 1 sq. inch ; and 1 cub. third = 1 cub. inch.

Illustrations

(i) 2 ft. $6\frac{1}{2}$ ins. $=$ $\underline{2 \text{ ft. } 6^{I}\ 6^{II}}$

[∵ $\frac{1}{2}$ in. $= \frac{1}{2}.1^{I} = 6^{II}$,

(ii) 3 sq. ft. $63\frac{1}{4}$ sq. ins. $=$ $\underline{3 \text{ sq. ft. } 5^{I}\ 3^{II}\ 3^{III}}$

[∵ 60 sq. in. $= 5^{I}$, 3 sq. in. $= 3^{II}$, $\frac{1}{4}$ sq. in. $= 3^{III}$

(iii) 22 c. ft. $533\frac{1}{2}$ c. ins. $=$ $\underline{22 \text{ c. ft. } 3^{I}\ 8^{II}\ 5^{III}\ 6^{IV}}$

[∵ 533 c. in. $= 3^{I}\ 8^{II}\ 5^{III}$ and $\frac{1}{2}$ c. in. $= 6^{III}$

Also, (iv) 5 sq. ft. ft. $7^{I}\ 8^{II}\ 3^{III} =$ 5 sq. ft.

$7^{I}\ 8^{II} =$ 92 sq. in.

[$= 7 \times 12 + 8$.

$3^{III} =$ $\underline{\frac{1}{4} \text{ sq. in.}}$ [$= \frac{3}{12}$

i.e., $=$ $\underline{5 \text{ sq. ft. } 92\frac{1}{4} \text{ sq. in.}}$

(v) 18 cub. ft. $9^{\mathrm{I}}\ 10^{\mathrm{II}}\ 11^{\mathrm{III}}\ 6^{\mathrm{IV}} = 18$ c. ft.

$9^{\mathrm{I}}\ 10^{\mathrm{II}}\ 11^{\mathrm{III}}\ 6^{\mathrm{IV}} =$ 1427 c. in.

$[= [9 \times 12 + 10] \times 12 + 11$

$6^{\mathrm{IV}} =$ $\frac{1}{2}$ c. in. $[= \frac{6}{12}$

i.e. $=$ 18 c. ft. $1427\frac{1}{2}$ c. in.

§ **Calculation of Areas and Volumes.** It will be observed that the indices I, II, III and IV are added in multiplication. Thus $2^{\mathrm{II}} \times 3^{\mathrm{II}} = 6^{\mathrm{IV}}$; $4^{\mathrm{I}} \times 1^{\mathrm{II}} = 4^{\mathrm{III}}$.

Illustrations

(i) Find the area 5 ft. $1^{\mathrm{I}}\ 2^{\mathrm{II}}$ by 3 ft. $2^{\mathrm{I}}\ 3^{\mathrm{II}}$.

5 ft. $1^{\mathrm{I}}\ 2^{\mathrm{II}}$	5 ft. by 3 ft. = 15 sq. ft.
3 ft. $2^{\mathrm{I}}\ 3^{\mathrm{II}}$	3 ft. by $1^{\mathrm{I}} = 3^{\mathrm{I}}$ sq. primes
15 sq. ft. $3^{\mathrm{I}}\ 6^{\mathrm{II}}$	3 ft. by $2^{\mathrm{II}} = 6^{\mathrm{II}}$ sq. seconds
$10^{\mathrm{I}}\ 2^{\mathrm{II}}\ 4^{\mathrm{III}}$	&c. &c.
$1^{\mathrm{I}}\ 3^{\mathrm{II}}\ 3^{\mathrm{III}}\ 6^{\mathrm{IV}}$	
16 sq. ft $2^{\mathrm{I}}\ 11^{\mathrm{II}} 7^{\mathrm{III}}\ 6^{\mathrm{IV}}$	

This is 16 sq. ft. *plus* 35 sq. ins. $= 2 \times 12 + 11$

$\frac{5}{8}$ „ „ $= \frac{7\frac{1}{2}}{12}$

i.e. 16 sq. ft. $35\frac{5}{8}$ sq. ins.

(ii) Find the volume 3 ft. 2^{I} by 1 ft. 5^{I} by 1 ft. 3^{I}.

Now, 3 ft. 2^{I}

1 ft. 5^{I}

3 sq. ft. 2^{I}

1 „ $3^{\mathrm{I}}\ 10^{\mathrm{II}}$

4 sq. ft. $5^{\mathrm{I}}\ 10^{\mathrm{II}}$

1 ft. 3^{I}

4 c.ft. $5^{\mathrm{I}}\ 10^{\mathrm{II}}$

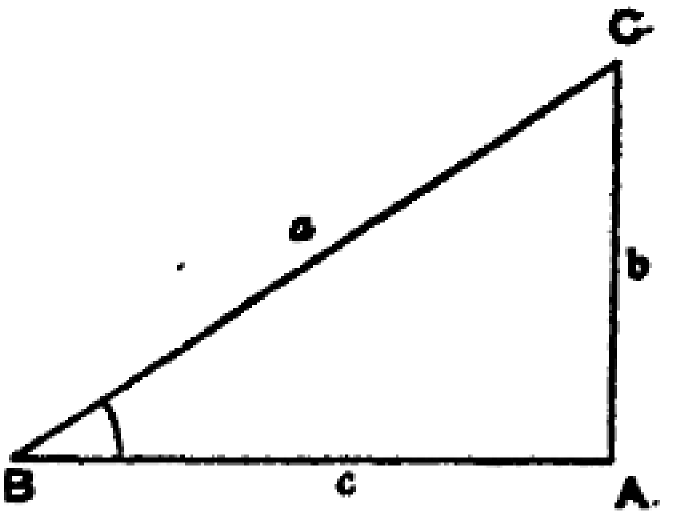
1 „ $1^{\mathrm{I}}\ 3^{\mathrm{II}}$

$2^{\mathrm{II}}\ 6^{\mathrm{III}}$

5 c. ft. $7^{\mathrm{I}}\ 6^{\mathrm{III}}$

Volume = 5 c. ft. 1050 c. in.

§ The inclination of BC to AB is measured by the ratio of $AC : AB$, where CA is perpendicular to AB and is called its *gradient.*



$$\text{The gradient} = \frac{1}{\sqrt{3}} \text{ when the angle at } B = 30^\circ$$
$$= 1 \text{ when the angle at } B = 45^\circ$$
$$= \sqrt{3} \text{ when the angle at } B = 60^\circ.$$

It is customary to indicate slopes in road-making by *gradients* which are taken to correspond to the rate $\frac{AC}{BC}$, indicating the actual rise or fall in level of a road traversing a stated length.

*§ It will be observed that the ratios $\frac{AB}{BC}$ and $\frac{AC}{BC}$ along with $\frac{AC}{AB}$ are constant for a given value of the angle ABC. They are symbolised as below :

$$\sin B = \frac{AC}{BC}, \quad \cos B = \frac{AB}{BC} \text{ and } \tan B = \frac{AC}{AB}, \text{ the } \textit{gradient}.$$

Also,

$\sin 0^\circ = 0$	$\cos 0^\circ = 1$	$\tan 0^\circ = 0$
$\sin 30^\circ = \frac{1}{2}$	$\cos 30^\circ = \frac{\sqrt{3}}{2}$	$\tan 30^\circ = \frac{1}{\sqrt{3}}$
$\sin 45^\circ = \frac{1}{\sqrt{2}}$	$\cos 45^\circ = \frac{1}{\sqrt{2}}$	$\tan 45^\circ = 1$
$\sin 60^\circ = \frac{\sqrt{3}}{2}$	$\cos 60^\circ = \frac{1}{2}$	$\tan 60^\circ = \sqrt{3}$
$\sin 90^\circ = 1$	$\cos 90^\circ = 0$	$\tan 90^\circ = \infty$

§ **Mensuration.** Well-known geometrical theorems give the following relationships :

(i) In a right-angled triangle, $(\text{hypotenuse})^2 =$ sum of squares of sides. Height of an equilateral triangle of side $a = \frac{\sqrt{3}}{2} \cdot a$.

(ii) Area of a rectangle with sides a and $b = ab$.

(iii) Area of a parallelogram = area of rectangle on the same base a and of the same height $b = ab$.

(iv) Area of a triangle of base b and height $h = \frac{1}{2}bh$.

(v) Area of a triangle of sides a, b and c

$$= \sqrt{s(s-a)(s-b)(s-c)}, \text{ where } 2s = a+b+c.$$

(vi) Area of an equilateral triangle of side $a = \frac{\sqrt{3}}{4}.a^2$

(vii) Area of a trapezium of height h, and parallel sides a and $b = \frac{1}{2}(a+b).\ h$.

(viii) Area of 4 walls of a room l by b by $h = 2h\ (l+b)$.

(ix) Area of a regular polygon of n sides each of length a

$\overset{-}{\rightarrow} a^2 \times \cdot 433,$ if $n = 3$

$\overset{-}{\rightarrow} a^2 \times 1\cdot 7205,$ if $n = 5$

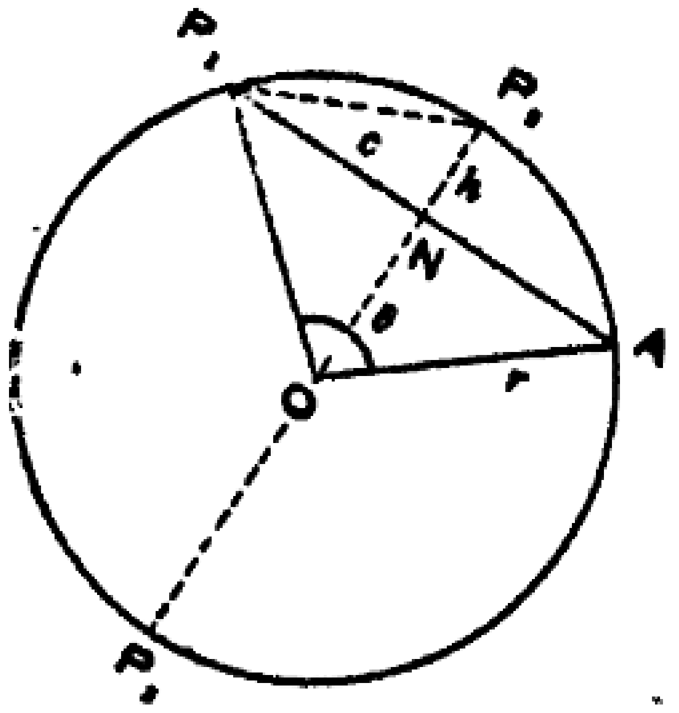
$\overset{-}{\rightarrow} a^2 \times 2\cdot 5981,$ if $n = 6$

$\overset{-}{\rightarrow} a^2 \times 3\cdot 634,$ if $n = 7$

$\overset{-}{\rightarrow} a^2 \times 4\cdot 8284,$ if $n = 8$

$\overset{-}{\rightarrow} a^2 \times 6\cdot 182,$ if $n = 9$

§ **The Circle.** The circumference of a circle bears a constant ratio to its diameter, which is universally symbolised by π. The value of π, which is incommensurable, may be taken to be roughly equal to $3\frac{1}{7}$, a better value being $3\cdot 1416$, or, $3\cdot 1\frac{1}{24}$. Approximately, $\pi \overset{-}{\rightarrow} 3 + \underline{4}\% + \underline{6} + \underline{8}\%$, also $\overset{-}{\rightarrow} \frac{22}{7} - \cdot\underline{4}$‰.



For a circle of radius r,

(i) the length of the circumference $= 2\pi r$.

(ii) the length of an arc AP_1, subtending an angle θ° at centre $= \pi r . \frac{\theta^\circ}{180^\circ}$

(iii) the length of an arc $AP_1 = \frac{8 \times \text{chord of half arc} P_1P_2 - AP_1}{3}$.

(iv) the length of chord $AP_1 = 2r.\sin\frac{\theta^\circ}{2}$, also $= 2\sqrt{P_2N.P_3N}$

(v) the area of the circle $= \pi . r^2$ or, $(\text{diameter})^2 \times {}^{\cdot}7 + \underline{10} + \underline{5} + \underline{10}$.

(vi) the area of a sector subtending an angle θ° at centre

$$= \pi r^2 . \frac{\theta^\circ}{360^\circ}.$$

(vii) the area of a sector containing an arc $b = \frac{1}{2}. b. r.$

(viii) the area of a segment containing an arc forming a chord c

$$= \frac{h^2}{2c} + \frac{2}{3} c.h,$$

where the height of the arc $h = r -$ ON.

(ix) chord of half-arc $= \sqrt{2rh}$

(x) the area of an annulus or a flat ring of which the internal and external radii are r_1 and r_2 respectively $= \pi(r_2 + r_1)(r_2 - r_1)$.

(xi) the area of a regular polygon $= \frac{1}{2}$ perimeter × radius of inscribed circle.

(xii) Side of square inscribed in a circle $= {}^{\cdot}707 \times$ diam.

(xiii) Diam. of circle circumscribing square $= 1{}^{\cdot}414 \times$ side of square

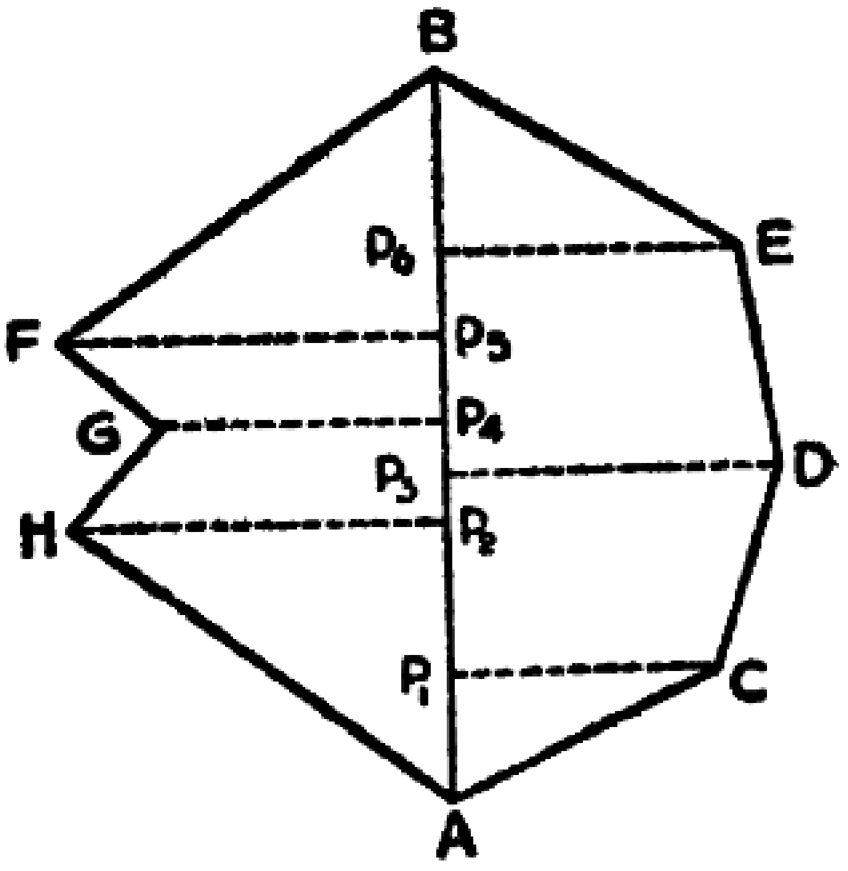
*§ **The Ellipse.** The length of the circumference is roughly $= \pi\left(\frac{3a}{2} + \frac{b^2}{2a}\right)$, where a and b are the semi-major and semi-minor axes, and the ellipse is nearly circular. The area of the ellipse $= \pi ab$.

§ These formulæ are applied usually with the help of Logarithms. In measuring irregular areas, it is convenient to split them up into triangles and trapeziums of measured dimensions, and then to add up the constituent areas. The method of breaking up an area into triangles is known as the Method of Triangulation.

*§ **The Field Book.** The land surveyor makes his notes for the calculation of areas in his Field Book in the following manner.

The Field Book is read upwards. Thus, the directions will mean, from A go North 30 links to P_1, whence go 70 links to the East to get C. From P_1 go North 35 links to P_2, whence go 90 links to the West to get H. And so on, till B is reached.

AB is called the *base line* or the *chain line* ; P_1C, P_2H, etc. which are normal to AB are known as *offsets*.



	LINKS to B	
	15	
	40	50 to E
100 to F	10	
80 to G	9	
	10	85 to D
90 to H	35	
	30	70 to C
From	A	Go North

The entire area has now been broken up into right-angled triangles and trapeziums, the total value of which can be directly obtained.

sq. links.

ΔAP_1C has an area $= \frac{1}{2}AP_1.P_1C = \frac{1}{2}.\ 30.70 = 1050$

ΔAP_2H " $= \frac{1}{2}AP_2.P_2H = \frac{1}{2}.\ 65.90 = 2425$

Trap. P_1CDP_3 " $= \frac{1}{2}(P_1C + P_3D).P_1P_3 = \frac{1}{2}.\ 155.45 = 3487{\cdot}5$

" P_2HGP_4 " $= \frac{1}{2}(P_2H + P_4G).P_2P_4 = \frac{1}{2}.\ 170.19 = 1615$

" P_4GFP_5 " $= \frac{1}{2}(P_4G + P_5F).P_4P_5 = \frac{1}{2}.\ 180.40 = 3600$

" P_3DEP_6 " $= \frac{1}{2}(P_3D + P_6E).\ PP_6 = \frac{1}{2}.\ 135.59 = 3982{\cdot}5$

ΔAP_6E " $= \frac{1}{2}BP_6.P_6E = \frac{1}{2}.\ 15.50 = 375$

ΔBFP_5 " $= \frac{1}{2}BP_5.P_5F = \frac{1}{2}.\ 55.100 = \underline{2750}$

19285

$\therefore$ Total area = ·193 acres

§ Cost of Carpeting Floor, Papering Walls, &c. When carpet of a stated width is to be bought to cover a definite area, say the floor of a room, an analysis as in the following examples is to be made.

Illustrations

1. Find cost of carpeting a room 28′ 3″ by 15′ 6″ with carpet 1 yd. wide.

(i) If the carpet be laid along the length, $\frac{15\frac{1}{2}}{3}$, that is not less than 6 pieces, each of length $28\frac{1}{4}$ ft of carpet will be required; for lengths are not sold cut along the width of the carpet. $\therefore$ Length of carpet required $= 6 \times 28\frac{1}{4}$ ft $= \underline{56\frac{1}{2}\text{ yds.}}$

(ii) If, again, it be laid along the breadth, $\frac{28\frac{1}{4}}{3}$, that is not less than 10 pieces of carpet, each of length 15′ 6″, will be required. Hence length of carpet required $= \underline{51\frac{2}{3}\text{ yds.}}$

Evidently, the carpet will then be laid along the breadth; and $\underline{\text{51 yds 2 ft}}$ of carpet will be required.

N.B. It will be observed that *theoretically* $\frac{28\frac{1}{4} \times 15\frac{1}{2}}{3}$ ft, *i.e.*, $48\frac{2}{3}$ yds nearly will be required. As will be seen from above, this length will be inadequate for practical purposes, and such an answer will be *wrong* for such purposes. The practical answer 51 yds. 2 ft. will involve some wastage; but that is unavoidable.

2. Find the cost of papering the walls of a room 32 ft. by 16 ft. by $15\frac{1}{2}$ ft. with paper of width 2 ft 8 in. costing 3as per yd.

(i) If the paper were laid parallel to the edge of the floor, $\frac{15\frac{1}{2}}{2\frac{2}{3}}$, *i.e.*, 6 pieces will be required, each of length equal to the perimeter of the floor, *i.e.*, 96 ft.

$\therefore$ Total length of paper required $= 96 \times 6$ ft $= 192$ yds.

(ii) If again, the paper were laid from floor to ceiling the number of pieces required will be $= \frac{96}{2\frac{2}{3}} = 36$ pieces, each of length equal to the height of the room.

∴ Total length of paper required $= 36 \times 15\frac{1}{2}$ ft $= 186$ yds.

∴ Length that will actually suffice $= 186$ yds.

∴ Cost $=$ Rs 34 14as

§ **Customary Measures in Trade.**

1. The volume of a cask or barrel is obtained for many purposes by taking the volume of a cylinder of section equal to that of the cask at its widest, and deducting 5% therefrom.

2. The cubical content of a round log of wood is taken to be $(\frac{1}{4}\text{ mean girth})^2 \times \text{length}$. More accurately, it is equal to length $\times \left(\frac{\text{mean girth}}{3\cdot545}\right)^2$; $= \text{length} \times (\text{mean girth})^2 + [\frac{1}{2} \times \underline{2} + \underline{6} + \underline{10}]$. Roughly, it is sometimes taken to be equal to length × mean cross-section, allowing about $21\frac{1}{2}\%$ for waste in squaring the timber.

3. The cubical content of a tapering log of wood is obtained from the above formula, taking the average of measurements of the girth at 3 or 4 points.

4. The St. Petersburg Standard Hundred is taken to be 120 pieces of timber 12 ft × 11 inches × $1\frac{1}{2}$ inches, the solid content of which is 165 cu. ft.

5. 1 cubic foot of timber = 144 Board feet. 1 Board foot = 1 sq. ft board 1″ thick.

6. Number of Board feet (1″ thick) that can be sawed from a round log $= \frac{\{\text{Diam. in inches} - 4\text{ inches}\}^2}{16} \times$ length in feet.

7. 1 Gross Registered Ton in Shipping represents a displacement of 100 c. ft. of water.

8. For shroud or hawser-laid ropes, the weight W in *cwt.* of cordage = square of circumference C in *inches* × length of cordage in *fathom* ÷ 420. For Hemp ropes W in lb per yd $= \frac{C^2}{8}$; for steel ropes it $= \frac{C^2}{2}$.

9. Breaking strain of round ropes $= B$ tons; $C =$ circumference in inches. Hemp...$B = C^2 \times \cdot3$; Iron...$B = C^2 \times 1\cdot5$;

Mild Steel...$B = C^2 \times 2$; Plough Steel...$B = C^2 \times 4$.

10. Multiply the volume of a grains stack in cubic feet by ·779. Result will give the value in Imperial Bushels.

Illustrations

1. Find the area of a figure combining an equilateral triangle of side 6′ with a square on the base thereof drawn outwards.

Area of triangular portion = $\frac{\sqrt{3}}{4}$ · 66 sq. ft.
" square = 36 sq. ft.
Total = ·43 × 36 sq. ft.
= 51½ sq. ft. nearly.

2. Find the area of an equilateral triangle of side 6′ with a semicircle drawn on the base outwards.

Area of triangular portion = $\frac{\sqrt{3}}{4}$ · 36 sq. ft.
" " semicircular " = ½ . π. 36 = $\frac{11}{7}$. 36 sq. ft.
∴ Area of whole = (·43 + 1·57). 36 sq. ft.
= 72 sq. ft.

3. Find the area left of a circle of radius 4′ from which another circle of radius 2′ has been cut out.

Area of original circle = π. 16 sq. ft.
" of portion cut out = π. 4 " "
∴ Area of remainder = π. 12 sq. ft.
= $\frac{264}{7}$ sq ft = 38 sq. ft. nearly.

4. The parallel sides of a trapezium are 12′ and 18′, and the perpendicular distance between them = 9′. Find its area.

Area = ½(12 + 18). 9 sq ft = 135 sq. ft.

5. Find the cost of flooring a room 16′ by 12′ at the cost of Rs 18 per 100 sq. ft.

Area of floor = 192 sq. ft.
Cost of 200 sq. ft. = Rs 36
Cost of 8 " " = 1 7as
∴ Required cost = Rs 34 9as

6. **Find the cost of papering the four walls of a room 24 ft by 18 ft by 16 ft at the rate of 1a 6p per sq. ft.**

Area of four walls $= 2 \times 16\ [24 + 18]$ sq. ft.

$= 1344$ sq. ft.

Cost of covering 1344 sq. ft. @ 1a per sq. ft. = Rs 84

Cost of " " @ 6p " " " = 42

∴ Total cost = Rs 126

7. Find the area of a triangular plot of which the sides are 64′, 48′ and 38′.

Here $2s = (64 + 48 + 38)' = 150'$

$\therefore \quad s = 75',\ s - a = 11',\ s - b = 27',\ s - c = 37'.$

$\therefore$ Area $= \sqrt{75 \times 11 \times 27 \times 37}$ sq. ft. $= \sqrt{825 \times 999}$ sq. ft.

$= \sqrt{824175}$ sq. ft.

$=$ 908 sq. ft. nearly.

N. B. The result should be obtained with the help of a Log. Table.

8. Find the area of a circular sheet of radius 8 ins. from which a smaller circle of radius 3 ins. has been punched out.

Area of whole sheet $= \pi.\ 8^2$ sq. in.

" " smaller circle $= \pi.\ 3^2$ " "

$\therefore$ Area of remaining portion $= \pi\ (8^2 - 3^2)$ sq. in.

$= \pi.\ 55$ sq. in.

$= \frac{1210}{7}$ sq. ins. nearly.

$= 172\frac{6}{7}$ sq. ins.

9. **Find the area covered by a door-frame 10′ by 4′ with a semi-circular top.**

Area of rectangular portion $= 10' \times 4' = 40$ sq. ft.

Area of semicircular " $= \frac{1}{2}.\pi.2^2 = \frac{44}{7}$ sq. ft. $= 6\frac{2}{7}$ sq. ft.

$\therefore$ **Total area** $= 46\frac{2}{7}$ sq. ft. nearly.

Solids

§ 1. Rectangular parallelopiped of dimensions l, b and h.

Volume $= l.\ b.\ h.$ **Area of six faces** $= 2[h(l + b) + lb]$.

Area of four walls $= 2h(l + b)$.

2. **Right prism,** formed with two polygonal faces. (Normal sections are all equal).

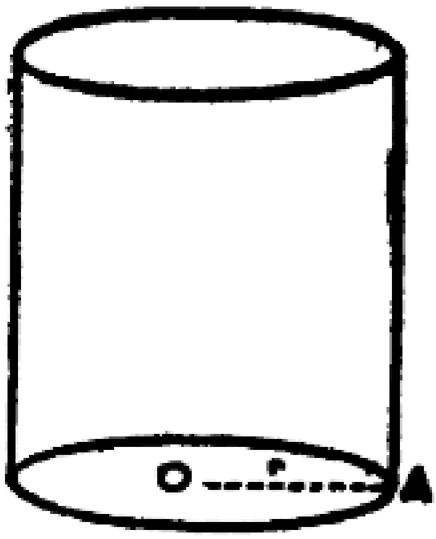
Volume = area of polygon × length.

Area of lateral surface = perimeter of polygon × length.

3. **Right cylinder.**

Volume = base × height $= \pi r^2 . h$, when base is a circle of radius r.

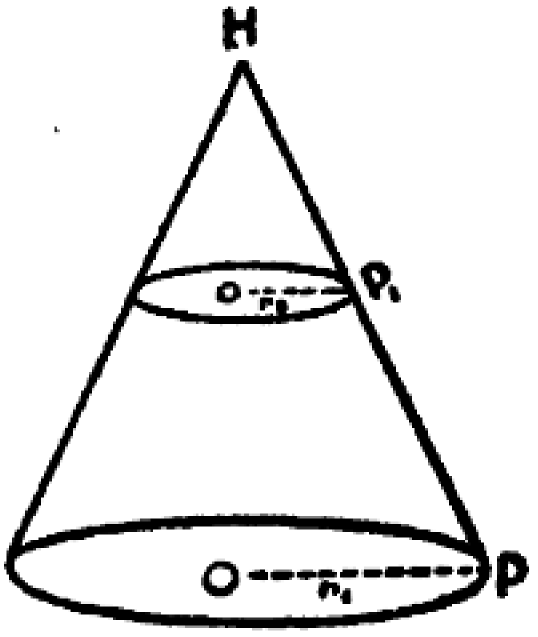
Area of curved surface = perimeter × height $= 2\pi r . h$, when the base is a circle.



4. **Right circular cone.** Volume $= \frac{1}{3} \pi r^2 . h$.

Area of slant surface $= \pi r . h'$, where h' is the slant height $= \pi r \sqrt{r^2 + h^2}$.

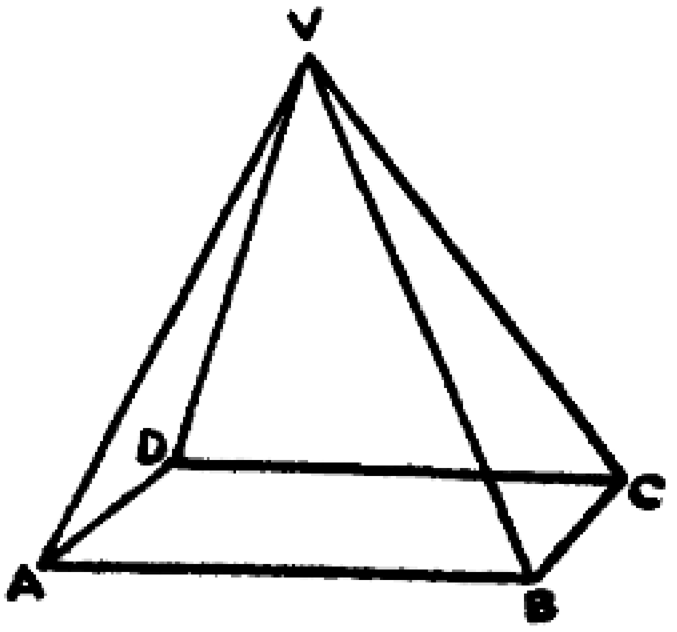
Total superficial area $= \pi r(h' + r)$.



5. **Pyramid.**

Volume = $\frac{1}{3}$ base × altitude.

Lateral surface = $\frac{1}{2}$ perimeter of base × slant height.



6. **A frustum** of a cone or a pyramid.

Volume = (sum of area of two bases *plus* the geometrical mean thereof) × $\frac{1}{3}$rd altitude $= \frac{\pi h}{3}(r_1^2 + r_2^2 + r_1 . r_2)$ for a cone.

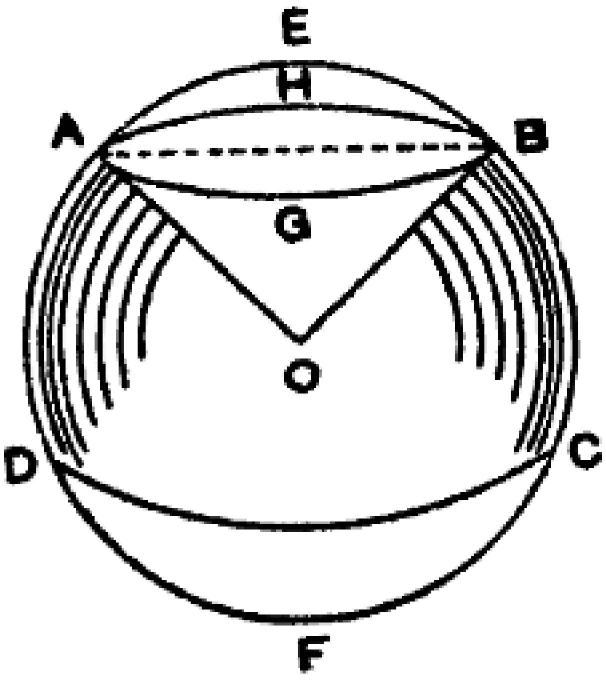
Area of convex surface = $\frac{1}{2}$ sum of perimeters of bases × slant height $= \pi h'(r_1 + r_2)$ in the case of a cone.

Note. These may also be obtained by deducting the volume or area of the portion cut off from the whole, if the required data are given.

If h be the height of the frustum and r_1, r_2 are the respective radii of the top and bottom, the height of the portion cut off $h' = \dfrac{h}{\dfrac{r_2}{r_1} - 1}$ and the height of the complate cone $H = \dfrac{r_2}{r_2 - r_1} \cdot h$.

7. **Sphere** of radius r. Volume $= \frac{4}{3} \pi r^3$ and surface $= 4\pi r^2$.

The solid portion $ABCD$ of the sphere lying between two parallel planes is known to be a *zone*.



If the thickness of the zone be d, the curved surface has an area $2\pi rd$; and the volume of the zone $= \frac{xd}{6}[3(r^2 + r'^2) + d^2]$, r and r' being the radii of the two ends.

A *segment* of a sphere is a portion cut off by a plane section, that is practically a zone with one end a point.

If d be the depth of the segment, the radius R of its base is given by $R^2 = d(2r - d)$.

The curved surface of the segment $= 2\pi r.d$

and volume $= \frac{\pi d}{6} (3r^2 + d^2)$. r' being $= 0$.

A *sector* of a sphere, *e.g.* $AOBE$, is made up of a cone formed by radii with a plane section of the sphere as base, together with an adjoining segment.

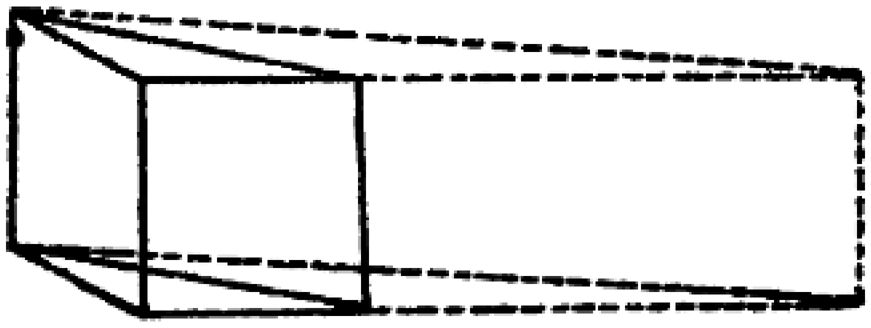
8. **Solid (circular) anchor ring** of radii r_1 (internal) and r_2 (external).

$$\text{Volume} = \frac{\pi^2}{4}(r_2 - r_1)^2 \times (r_2 + r_1).$$

$$\text{Area of outer surface} = \pi^2(r_2{}^2 - r_1{}^2).$$

9. **Solid flat ring.** Volume $= \pi h.(r_2 - r_1)(r_2 + r_1)$.

10. **Wedge.** Volume $= \frac{1}{2}$.Area of base × perpendicular height, if the length of the base equals the thin edge. Generally, it is equal to $\frac{\text{breadth of base} \times \text{height}}{6} \times (2 \text{ length of base} + \text{edge})$.



§ **Similar figures.** The areas of figures, whose sides are proportional, are proportional to squares of their dimensions; and their volumes to the cubes of their dimensions.

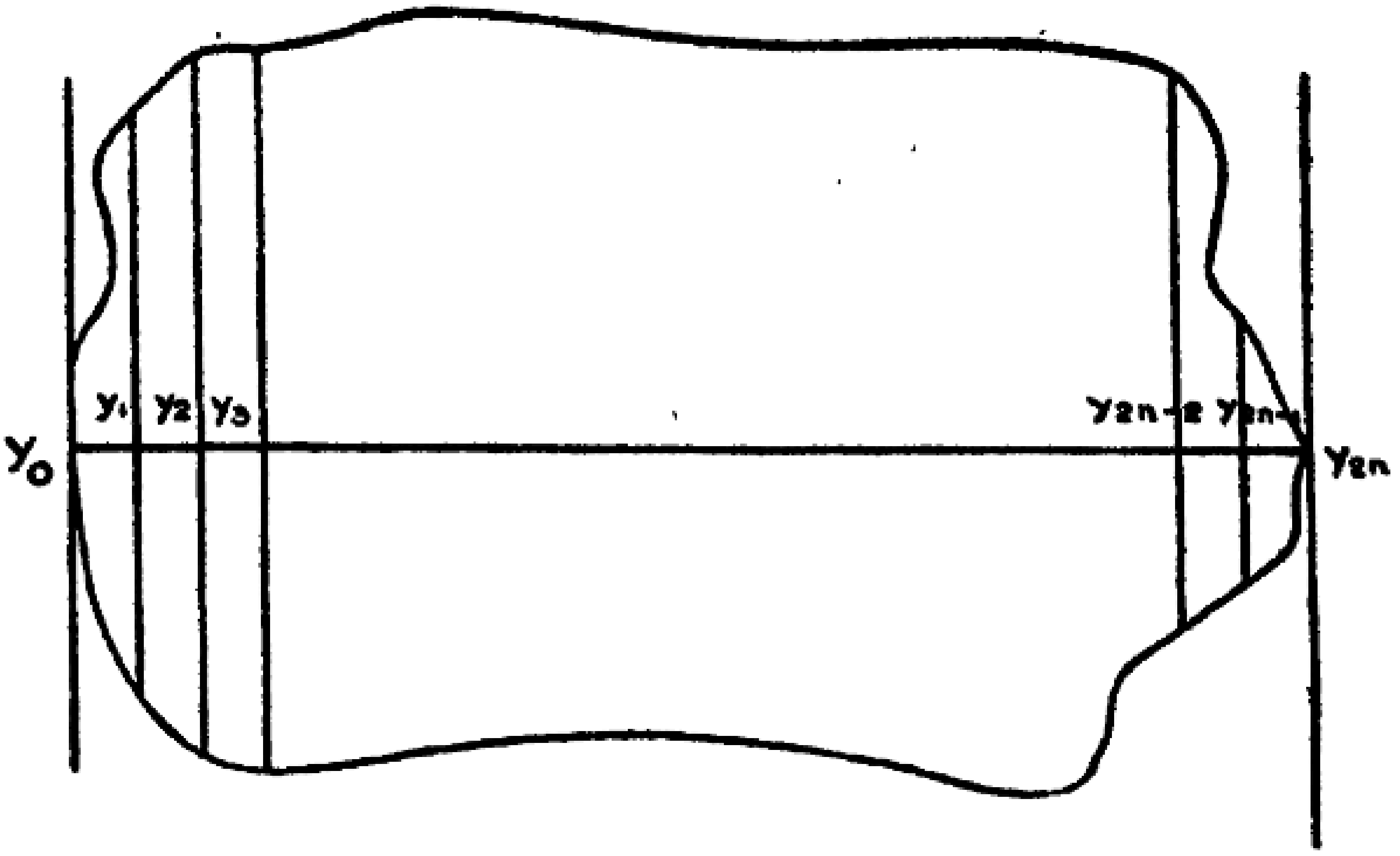
§ The following generalised theorems may be noted. Calculations by the first two methods are made by plotting points from data obtained by actual measurement at suitable points on a graph paper, and reading off dividing ordinates from the graph.

§ **Simpson's Rule.** The area of an irregular figure is usually obtained by this Rule, which requires a division of the area into an *even* number of parts by a number (necessarily odd) of equidistant ordinates.

Area = [sum of extreme ordinates + 4 × sum of even ordinates + 2 × sum of remaining odd ordinates] × $\frac{1}{3}$ distance between ordinates.

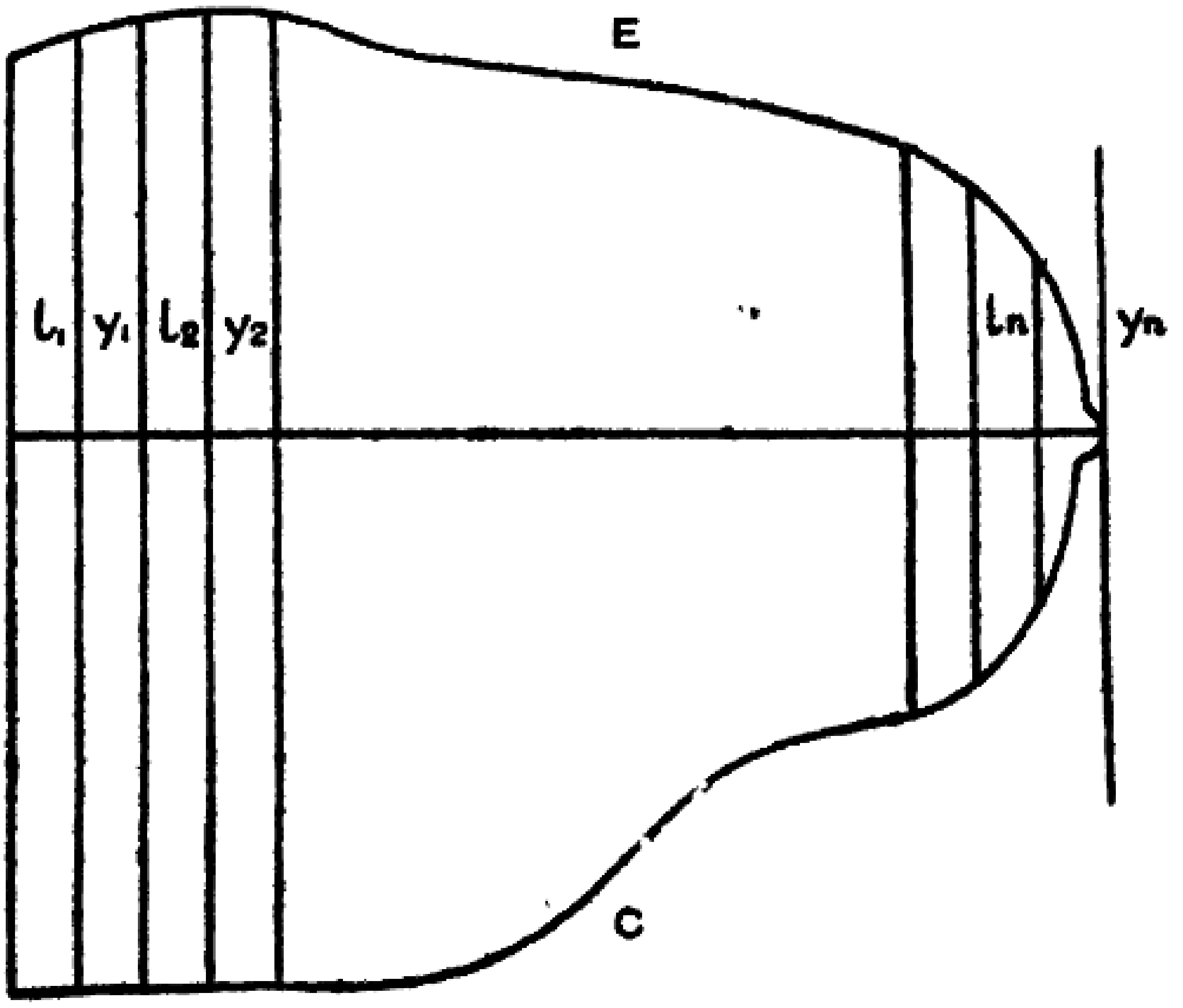
h = distance between ordinates.

$$\text{Area of the figure below} = \frac{h}{3}\left[4(y_1 + y_3 + \cdots + y_{2n-1}) + 2(y_2 + y_4 + \ldots + y_{2n-2})\right],$$ y_0 and y_{2n} being zero.



*§ If this rule be interpreted in relation to a solid, the average cross-section of a solid, of which the two faces and a middle section are known, is found to be equal to $\frac{1}{6}$ [sum of two faces + 4 times middle section].

* **Mid-ordinate Rule.** Divide the area into any number of parts by equidistant ordinates, the first and last touching the boundary. Ascertain the breadth of the area between every



consecutive pair of ordinates. Multiply the sum of these breadths by the distance between each such pair of ordinates, to obtain the area.

Area—*ABCDE*

h = distance between ordinates. $y_n = 0$.

Breadths of area between ordinates $y_0, y_1, y_2 \ldots\ldots$ are $l_1, l_2 \ldots\ldots$ $\therefore$ Area $= h(l_1 + l_2 + \cdots)$

This rule is applied in finding areas of closed curves. For convenience the extreme ordinates are tangents at two extremes of the curve.

* **Guldin's or Pappus' Theorems.**

(i) The *area of the surface* traced out by the revolution of a curve about an external axis in its own plane is equal to the product of the perimeter of the curve and the distance moved through by the centre of gravity of the curve.

(ii) The *volume* generated by the revolution of any plane figure about any external axis in its plane, is equal to the product of the generating area and the distance moved through by the centre of gravity of the area.

Illustrations

1. An area is bounded by two ordinates, the horizontal axis and a curve. Apply Simpsoson's Rule to find the area, given the following ordinates; the distance between a pair of them being 2 ft.

Ordinates—0, 18, 23, 16, 17, 25, 22, 0 ft.

From the formula, required area

$$= \tfrac{2}{3}[4(18 + 16 + 25) + 2(23 + 17 + 22)] \text{ sq. ft.} = \underline{240 \text{ sq. ft.}}$$

2. An irrigation canal is measured for the purpose of contructing a sluice across it. The height of the sluice is to be 10 ft above the edge of the canal on either side, and it would be just submerged in water. The soundings taken at distances of every 2 ft from the edge are—3, 8, 9, 10, 13, 12, 8 ft. Find the area of the sluice gate.

Applying the Mid. Ordinate Rule,

area $= 2(10 + 3 + 8 + 9 + 10 + 13 + 12 + 8 + 10)$ sq. ft. $= \underline{166 \text{ sq. ft.}}$

EXAMPLES 27

1. Find the area of a triangular area with a base 42 chains, and altitude 63 ch. 11 yds. [*Ans.* 133·35 acres

2. A square field, of which one side is 120 yds is to be turfed at the rate of 6p per sq yd and enclosed by railing at a cost of Rs 2 12as per yd. What is the total cost ? [*Ans.* Rs 1770

3. A path 3 ft wide is to be laid round a rectangular grass plot 80 yds by 60 yds. Find the cost of macadamising it at the rate of 3as 9p per sq yd. [*Ans.* Rs 66 9as

4. A plot of land 150 yds by 120 yds is to be sown with cabbages 1½ ft apart in lines 2½ ft apart. Find the number of plants that can be sown. [*Ans.* 43645

5. The cost of removing the flooring of a room 20 ft by 16 ft is Rs 12 8as per 100 sq ft ; and that of covering it with art tiles 1½ ft by 1½ ft is Rs 2 2as per tile. Find the cost of renovation. Assume that these tiles can be fitted in pieces. [*Ans.* Rs 342 4as

6. A triangular plot of land has three sides $181' \times 125' \times 116'$ Calculate its area. [*Ans.* 800 sq yds app.

7. An irregular figure $ABCD$ was measured to obtain $AB = 160$ ft, $BC = 86$ ft, $AC = 90$ ft, $CD = 63$ ft and $AD = 125$ ft. Find its area. [*Ans.* 625 sq yds app.

8. A surveyor takes the following measures of a quadrilateral $ABCD$. Diagonal $AC = 216$ ft 3″. Perpendiculars from B and D on AC are equal to 96′ and 102′ respectively.. Calculate the area $ABCD$. [*Ans.* 2378¾ sq yds

9. From a rectangular plot of land $ABCD$, of which AB is 100 yds and BC is 60 yds, a portion BCE is to be cut off equal in area to ⅛th of the total. Find the position of E on CD.

[*Ans.* 25 yds from C

10. To an equilateral triangular field of side 90·34 m is added an area in the shape of a square on its base. Find the total area of the field. [*Ans.* 116·95 Ares

11. The perimeter of a rectangular field is 186·60 m and the diagonal measures 66·30 m. Find the sides, and the area.
[*Ans.* 51·3 m ; 42 m ; 21·55 Ares

12. A map is drawn to a scate of 50 inches to a mile ; how many square inches on the map will represent 10 acres on the ground ? [*Ans.* 39·06 sq in.

13. The length of a hall is 3 times the breadth ; the cost of whitewasing the ceiling at 5½d per sq yd is £4 12s 7·1d ; and the cost of papering the four walls at 1s 9d per sq yd is £35 ; find the height of the hall. —I. Com. [*Ans.* 18 ft

14. A tank is capable of holding 10 Kilolitres. If it has a rectangular base 3·60 m by 4·53 m, find its height. [*Ans.* 61·32 cm

15. Find the cost of painting the walls of a room 10·36 m by 8·50 m by 4·65 m @ 14·56 fr. per sq metre, allowing for 8 openings each 2·8 m by 1·3 m. [*Ans.* 2129·84 francs

16. Find the cost of carpeting a room 22 ft by 16 ft with carpet 26 ins. wide costing Rs 5 13as a yard, leaving a space all round 18 ins. in width. [*Ans.* Rs 220 14as

17. Find the cost of papering the walls of a room 18′6″ by 12′ by 11′ 6′ with paper 1 yard wide at 10as per yd. [*Ans.* Rs 50 5as

18. Find whether it is cheaper to lay carpet on the floor of a room 21′ by 16′ along the length or along the width, if the carpet has a width of 33 ins. [*Ans.* Lengthwise

19. A person is shown papering for the walls in rolls 24″ and 30″ wide. The price of the first is 3as per yard, and that of the latter 3as 6p per yard. Which size should he select to cover the walls of a room 24 ft by 16 ft by 12 ft ? [*Ans.* Latter

20. The cost of papering the four walls of a room 12 ft high is Rs 8 7as. If a margin is left over along the top, the cost comes to Rs 7 0a 6p. Ascertain the width of the margin if papering costs 3as per square foot. [*Ans.* 2 ft

21. A path 5 ft wide is to be laid round a rectangular garden on its inner side. It is estimated that the area of the path will be 1400 square feet. Find the cost of fencing the garden at the rate of 12 annas per running foot. [*Ans.* Rs 225

22. A circular grass plot has a diameter of 28 yards. Find the cost of gravelling a path 6 ft wide round the plot on the outside at the rate of 7as per square yard. [*Ans.* Rs 82 8as

23. A rectangular case weighs 3 cwt 3 qr when lined with lead. A similar case weighs 2 cwt 1 qr being lined with copper. If the lead sheet weighs 6 lb per sq ft and the copper sheet half as much, find the weight of an unlined case of equal dimensions. [*Ans.* $\frac{3}{4}$ cwt

24. A metal scale of 40 inches increases $\frac{1}{10}$th inch by expansion due to a rise of 1°*C* in the temperature. Find by how much a sheet of the same metal and a cube of the same metal will expand in the same circumstance. [*Ans.* $5\frac{1}{160}$‰ and $7\frac{83}{160}$‰

25. Compare the area covered by two sectors each subtending an angle 37° at the centre, the radii of the two being proportional to 3 : 7. [*Ans.* 9 : 49

26. A glass vessel in the shape of a tumbler 9 inches high has a base of diameter $1\frac{1}{2}$ in. and a top of diameter $4\frac{1}{2}$ in. Find the cubic content of the glass, assuming that 1 pint contains ·0201 cu ft. [*Ans.* $7\frac{7}{8}$ gills app.

27. A cone-shaped vessel is half-filled with water. The base has a diameter of 10 inches and the height of the vessel is 16 inches. The contents are emptied into a cylindrical vessel in which it rises $7\frac{1}{2}$ inches. Find the diameter of the base of the cylinder. [*Ans.* 3·2 in. app.

28. Lead is known to weigh 710 lbs per cubic foot. A square foot of a lead sheet ·1014 in. thick is made into a box. Find the weight of the box. [*Ans.* 6 lbs.

29. A circular disc of metal 14 ft in diameter weighs 1 qr. Seven equal circular discs punched out of the sheet have a total weight of 9 lbs. Find the diameter of these discs. [*Ans.* 3 ft

30. A circular copper plate 2·7 mm thick weighs 1 Kg 485 gm. 1 cu. cm. of copper weighs 8·8 gm. Find the radius of the plate to the nearest millimeter. ($\pi = 3·1416$) —P. S. C. [*Ans.* 14·1 cm

31. Two open cylindrical metal pipes have equal internal volumes. The internal and external diameters of one pipe are 12″ and 13″ respectively, and the corresponding diameters of the other are 6″ and 6¾″. Express as a fraction in its lowest terms the ratio of the quantity of metal in the first pipe to the quantity in the second. —P. S. C. [*Ans.* 1600 : 153

32. A cylindrical copper wire 0·028 inch in diameter, weighs 3 lb and is to be covered with a layer of silver 0·012 inch thick. What volume of silver will be required, if one cubic foot of copper weighs 648 lb. —P. S. C. [*Ans.* 13$\frac{4}{7}$ cu. in.

33. Flat sheets of metal 5 ft 6 in. wide are made into corrugated sheets of which the curved portions are semicircular. Allowing 10% for overlap, how many sheets will be required to make a fence 35ft long ? —P. S. C. [*Ans.* 11

34. A hollow rectangular vessel formed of material 1 inch thick has external dimensions 13 ft 6 in, 6 ft 8 in, and 6 ft 11 in respectively. It weighs 1 cwt 1 qr 10 lb. Find the weight of a solid mass of the same material and of the same dimensions.
—I. Com. [*Ans.* 1 ton 2 cwt 1 qr app.

35. The Great Wall of China is said to be 2400 Km long, 61 dm high and 457·5 cm thick at the top and 7625 mm thick at the bottom. How many cubic metres of building materials does it contain ? —I. Com. [*Ans.* 8930400 cu. m.

36. 20 equal steel bearing balls are found to weigh ·934 lb. Find as accurately as possible the diameter of one, being given that 1 cu. in. of steel weighs ·288 lb. [*Ans.* ·676 in.

37. Show that the weight of a sphere of diameter D inches equals $\frac{D^3}{6} \times \frac{\text{sp. gr. of the material}}{\text{sp. gr. of copper}}$. Given that 1 cu. in. of copper weighs ·318 lb (app.).

38. A hemispherical dome with an inside diameter of 8 ft is to be whitewashed both on the outside and inside. If the thickness of the wall be 9 inches, find the cost at Rs 14 per 100 sq. ft. [*Ans.* Rs 33 15as

39. Find the number of gallons of water a cistern, measuring inside 3ft by 2 ft 6 in by 7 ft in height will hold when full ; find also the weight of the contents, having given 1 pint equals 34·7 cubic inches and that the weight of 1 gallon of water is 10 lb. —I. Com. [*Ans.* 326·8 gall. app.

***40.** A metal rod with a circular cross-section is made into a ring of which the outer diameter is 3 ft and the inner 2 ft 10 ins. If a cubic foot of the metal weighs $1\frac{23}{121}$ lb and costs Rs 3 8as per lb, find the cost of the ring. [*Ans.* 3as 4p app.

41. A reservoir is 22 ft by 16 ft. If 1800 gallons of water are poured into it, find how high it will rise in the cistern. Assume that 1 gallon of water weighs 10 lbs and a cubic foot of water weighs 1000 oz. [*Ans.* $9\frac{9}{11}$ ins.

***42.** 200 ft of a metal wire weigh 3·795 lbs. If the thickness of the wire be ·08 in., find the weight of a cubic foot of the metal. [*Ans.* $543\frac{3}{8}$ lbs

43. A lb tin of tobacco $6\frac{1}{4}$ in. high has an internal diameter of $4\frac{2}{5}$ in. Find the height of another tin with an internal diameter of $2\frac{3}{4}$ in. to hold 2 oz. of tobacco. [*Ans.* 2 ins.

44. A cylindrical tin of milk powder is covered with print $1\frac{1}{3}$ ft long and 5″ wide. Assuming that $\frac{1}{2}$″ of a sheet of print is used up in gumming, find the contents of the tin. [*Ans.* $95\frac{1}{2}$ c. ins.

***45.** A metal tube is sold at $5\frac{3}{4}$as per running foot (r. ft.) or Re 1 12as per lb. If 1 c. ft of the metal weighs 528 lbs, find the thickness of metal in the tube, which has an external diameter of $\frac{1}{2}$″. [*Ans.* 0″·04

***46.** Calculate areas from the following entries in a surveyor's Field Book. (10^5 sq. links = 1 acre = 4840 sq. yds.)

(i)

	Links to *B*	
	840	
510 to *D*	660	
	360	250 to *C*
From	*A*	go North

[*Ans.* 7·07 acres

(ii)

	Links to *B*	
	53	
42 to *E*	46	
	35½	17½ to *D*
	28	20½ to *C*
From	*A*	go North

[*Ans.* 253·6 sq. yds.

***47.** A semicircular riding track has a width of 20 ft. If the straight edge has an outside length of 560 yds, find the average length of the track. Find also the area of the field surrounded by the track. [*Ans.* 3·9 fur. ; 24½ acres app.

***48.** A rectangular garden 820 ft by 640 ft has a circular tank at the centre. The diameter of the enclosure round the tank is 120 ft. The path running round the enclosure is 6 ft wide, and that running along the boundary of the garden on the outside has a width of 12 ft. Find the cost of covering the paths with patent stone @ Rs 16 per 100 sq ft. What is the area covered by the remaining garden plot ?

[*Ans.* Rs 6078 11as 6p ; 11·73 acres app.

***49.** A field in the shape of a regular hexagon is planted with a palm tree at each vertex of the outer hexagon and of all similar hexagons inscribed within it, each at a distance of 20 ft from the last. Find the total number of trees that may possibly be planted if a side of the field measures 480 ft. What is the distance between two consecutive trees nearest the centre ?

[*Ans.* 144 ; 20 ft

***50.** Find the cost of 27 sets of windows, from the following details :

Window frames 8 ft by 4 ft made of wood 5″ × 4″ @ Rs 7 per c. ft.

Venetian shutters (inside measurements of frame) @ Rs 2 per sq ft. [*Ans.* Rs 1915

***51.** A round log of wood 24 ft in length has the following measurements of its girth : 42 in., 48 in., 46 in., 40 in. Find its cubical content. [*Ans.* 26 c. ft. app.

***52.** A barrel 4 ft 6 in high has a diameter of 3 ft at its widest. Can it contain 1800 lbs of an oil, a cubic foot of which weighs 56 lbs ? [*Ans.* Capacity 1693 lb app.

***53.** A plain galvanized iron sheet 52 ft. 3″ long is corrugated so that each curved portion is a semicircle with a diameter of 3 inches, and the distance between any two curved portions is also 3 inches. What size should the sheet be now cut into, so that there may be the minimum loss ?

[*Ans.* 9 pieces @ 4 ft 6 in each

***54.** A tin sheet in the form of a circle of radius 30 inches is cut to form a sector containing an angle of 35° at the centre. The piece is cut off along the edge of a concentric circle of radius 18″. The outer portion is bent into a bucket shape. If a bottom is now fixed, find the total surface of the tin required to make the bucket. [*Ans.* $185\frac{5}{8}$ sq in.

SECTION THIRTEEN

SOME PROBLEMS OF BANKING AND ACCOUNTANCY

§1 Goodwill. Goodwill may be valued on the basis of

(i) an expert appraisal, or valuation by mutual agreement ;

(ii) a number of years' purchase of the net profits ;

(iii) a number of years' purchase of profits in excess over interest on net assets.

(i) The appraisal of goodwill by an umpire or by mutual agreement generally proceeds on the basis of a valuation of the entire business as a running concern, and finding the excess thereof over the net assets.

If a running business, including the clientele, the established advantages of sale and purchase dependent on it, is valued at Rs 50 000, and the assets to be transferred are valued at Rs 30 000, the difference of Rs 20 000 is the value of the goodwill.

(ii) The net profits may be that of a particular year, or of the average of a number of years ; and the number of years' purchase is usually fixed by custom.

Illustration

Profits (net) during the past three years were as follows :

1st year	...	Rs 10 000
2nd "	...	Rs 8 000
3rd "	...	Rs 12 000

If the net assets of the business are valued at Rs 48 000, and goodwill is valued at 5 years' purchase of average net profits, the price of the business is Rs 48 000 *plus* 5 × Rs 10 000 = Rs 98 000.

(iii) A minimum return on capital equivalent to the ruling rate of interest is sometimes deducted from the average net profits to obtain an estimate of the real earning capacity of the business; and this is capitalised at a number of years' purchase.

Illustration

A purchaser agrees to pay full value of the assets, and value at 6 years' purchase of the excess of net profits over 6% on the assets.

Obtain the purchase price from the data given below :

Assets	Rs 45000
Average Net Profits ...	Rs 10000

The interest on Rs 45000 @ 6% p. a.	= Rs 2700
Excess of Net Profits over interest	= Rs 7300
Goodwill (captialised at 6 years' purchase)	= Rs 43800
Value of assets	= Rs 45000
∴ Purchase price	= Rs 88800

*(iv) The valuation of the goodwill of a partnership presents difficulties of detail. The following illustration shows a simple way of assessment of goodwill.

Illustration

Estimate the value of a business at the value of the capital *plus* goodwill at 4 years' purchase from the following abstract :

Year	Total Profits Rs	Drawings Rs
1928	10500	7634
1929	12618	8246
1930	15505	10575
1931	13224	11230

The capital at the beginning of 1928 was Rs 20000; the profits less drawings were used in the business as capital. Allow interest at 5% p. a. on capital.

	Rs	Capital Rs
Capital at beginning of 1928 ...	20000	
Add profits for 1928 ...	10500	
	30500	
less drawings for 1928 ...	7634	
Capital at beginning of 1929 ...	22866	22866
Add profit for 1929 ...	12618	
	35484	
less drawings for 1929 ...	8246	
Capital at beginning of 1930 ...	27238	27238
Add profits for 1930 ...	15505	
	42743	
less drawings for 1930 ...	10575	
Capital at beginning of 1931 ...	32168	32168
Add profits for 1931 ...	13224	
	45392	
less drawings for 1931 ...	11230	
Capital at end of 1931 ...	34162	34162

Thus,

Capital in 1928 ...	20000
„ 1929 ...	22866
„ 1930 ...	27238
„ 1931 ...	32168
Average Capital (1922-31)...	25568

Interest @ 5% p. a. on average capital = Rs 1278 6as 6p.

Profit in 1928 ...	10500
„ 1929 ...	12618
„ 1930 ...	15505
„ 1931 ...	13224
Total Profits ...	51847

Average Profits = Rs 12961 12as

Excess of profits over interest = Rs 11683 5as 6p.

Goodwill @ 4 years' purchase = Rs 46733 6as

Capital = Rs 34162

∴ Total value = Rs 80895 6as

EXAMPLES 28

1. A person agrees to take up a Motor Bus Building Concern, paying the full value of the net assest in cash, and a cash value of the goodwill at six years' purchase of the average net profits reduced by 6% p. a. interest on capital. The assets are appraised at Rs 1 lakh; average annual profits are estimated at Rs 18000. Find the purchase price. [*Ans.* Rs 172000

2. A watch-dealer's shop is sold at the net value of the assets plus the cost of goodwill at 4 years' purchase of the last 3 years' average of net profits. The inventory at list price is Rs 150000, on which an average discount of 33⅓% is allowed to obtain the net value. The past three years' profits were as follows: Rs 20000, Rs 15000 and Rs 28000. Find the value of the business. [*Ans.* Rs 184000

3. A mortgagee of a bookseller's business desires to dispose of it as a going concern on the failure of the mortgagor to pay up dues. He accepts a tender for the inventory value of the stock reduced by 40%; publisher's rights of certain books at 8 years' purchase of the average net profits on the publications, goodwill at 3 years' purchase of average net profits of the selling business. Ascertain the value of the tender from the following data: Inventory Rs 40800; average annual profits on publisher's business Rs 4600; average annual profits on selling business Rs 2500. [*Ans.* Rs 68780

4. A circulating library changes hands on the following conditions. Books are to be valued at their cost price less depreciation at 50%. Goodwill is to be valued at 5 times the average of last 3 years' subscription lists. The cash value will be the sum of these and other assets reduced by outsandindg liabilities amounting to Rs 5600. Find the value of the transfer from the following data: Inventory value of Books Rs 54000; Furniture etc. less depreciation Rs 8500. Subscription received during past 3 years were Rs 5600, Rs 6300, Rs 5800. [*Ans.* Rs 59400

5. A person buys the entire business of a manufacturer of fireworks for Rs 30000. The stock was valued at Rs 10735; plant and machinery at Rs 3485: land, buildings, etc. at Rs 2456.

The books of the business showed an average profit of Rs 3331 during the past 4 years. What was the consideration paid for the goodwill? How many years' purchase did it represent?

[*Ans.* Rs 13324 : 4 years

§ **Equation of Payments.** Other things being equal, the earning capacity of Rs 500 in use for 4 months may be looked upon as equivalent to that of Rs 2000 in use for 1 month. In general, the total earning capacity of a series of payments $s_1, s_2, \ldots$ made at the end of periods $n_1, n_2 \ldots$ respectively, may be regarded as equivalent to that of a different series of payments $\delta_1, \delta_2, \delta_3 \ldots$ at the end of periods $v_1, v_2, v_3 \ldots\ldots$ respectively, if

$$\Sigma s.n = s_1.n_1 + s_2.n_2 + \cdots = \delta_1.v_1 + \delta_2.v_2 + \cdots \text{(A)}$$
$$= \Sigma \delta.v.$$

provided, of course, $\Sigma s = s_1 + s_2 + \cdots = \delta_1 + \delta_2 + \cdots = \Sigma\delta$.

The equation (A) is known as an equation of payments, from which the value of v_r, corresponding to a known value of δ_r may be directly obtained, when all the other remaining items are known.

The value of v thus obtained is known as the *equated time of payment*, the date thereof being known as *average due date* of payment of the corresponding δ.

It is evident that by equating the two series of payments, loss or gain to either party is eliminated, if the outstanding amount δ be paid on the average due date.

The periods may be calculated from any date conveniently fixed, which is known as the *zero date*.

Illustrations

1. On the 13th March 1940 a trader accepted bills in favour of another as follows: Rs 500 payable at end of 2 months; Rs 400 at end of $2\frac{1}{2}$ months; Rs 800 at end of 3 months and Rs 600 at end of 4 months. If he wishes to arrange for the payment of the entire amount in a lump sum, what is the latest date when such payment may be accepted? (Neglect the grace period)

The lump sum payment of Rs 2300 may be made on the average due date without involving either party in gain or loss. The equated time of payment is obtained as below :

Amount payable	Period (months)	Product
Rs 500	2	1000
" 400	$2\frac{1}{2}$	1000
" 800	3	2400
" 600	4	2400
Total 2300		Total 6800

$$\text{Equated time} = \tfrac{6800}{2300} \text{ months}$$
$$= 2 \text{ months } 28\tfrac{16}{23} \text{ days}$$
$$= 2 \text{ months } 28 \text{ days,}$$

for any fraction in such cases is ignored in favour of the creditor.

It will be noted that the equation of payments should give $2300 \times \text{equated time} = 6800$ which explains the above method.

The required date is 10th June 1940.

2. On the 15th March 1940, a person buys a Radio set on instalment terms : Rs 20 down and there after 12 monthly payments of Rs 15 each. After the third monthly instalment has been paid, the purchaser desires to pay the balance in a lump sum. What is the latest date on which such payment may be accepted ?

The method indicated above takes note only of those dues that are outstanding ; those that have already been paid up have to be ignored.

The date of purchase is taken to be the *zero date*.

Amount due Rs	Month	Product
15	4	15×4
15	5	15×5
15	6	15×6
15	12	15×12
Total 15×9		Total $15\{4+5+6+\cdots+12\}$,
		$= 15 \times 8 \times 9$

$$\therefore \text{ Equated time} = \frac{15 \times 8 \times 9}{15 \times 9} = 8 \text{ months.}$$

Due date is 15th November 1940.

3. On January 13, 1941 a trader hands over the following bills to his banker for settlement on due dates and adjustment.

Bills Payable	Bills Receivable
£200 after 17 days	£600 after 38 days
£350 " 24 "	£400 " 42 days
£300 " 39 "	£200 " 48 days
£400 " 67 "	

If the banker agrees to postpone debiting the trader's account with the balance of £50 to the average due date, how much does he save in interest, money being worth 4% per annum ?

The trader saves interest on £50 of which he has the use till the average due date.

Debits			*Credits*		
Amount	Time	Product	Amount	Time	Product
200	17	3400	600	38	22800
350	24	8400	400	42	16800
300	39	11700	200	48	9600
400	67	26800	To Balance		1100
	Total	50300	Total		50300

The balance product 1100 = 50 × equated time

∴ Equated time = $\frac{1100}{50}$ = 22 days.

∴ The trader gains interest on £50 for 22 days at 4% p.a., which = 1s 2½d.

4. A merchant has accepted the following bills :

£320 drawn on 4th March 1940 at 3 m/s after date :
£400 " " 6th April 1940 at 2 m/s after date ;
£250 " " 15th April 1940 at 4 m/s after date.

If he desires to retire the bills under rebate @ 4% p. a. on the 2nd June 1940, what funds should he place with his banker for the purpose?

For convenience, take 2.6.1940 as the *zero date.*

Bill	Due date	Period d/s	Product
£320	June 7	5	1600
£400	" 9	7	2800
£250	Aug. 18	77	19250
£970			23650

Equated time = $\frac{2365}{97}$ d/s = 24·4, *i.e.*, 24 days.

∴ Amount of rebate = interest @ 4% p. a. on £970 for 24 days = £2 11s.

∴ He pays banker £967 9.

5. A dealer offers to sell a motor car on the following terms : Rs 2000 down and Rs 200 per mensem for 12 months following. The buyer proposes to pay as follows : Rs 1400 down and Rs 300 per mensem for 10 months following. Would the alternative terms be acceptable to the dealer ?

$$\text{Here, } \Sigma s_r . n_r = 200 \times \frac{12 \times 13}{2} = 15600$$

$$\text{and } \Sigma \delta_r . v_r = 300 \times \frac{10 \times 11}{2} = 16500$$

Although $\Sigma s_r = \Sigma \delta_r$, that is the total amount receivable by the dealer is the same, he is deprived of the use of a larger equivalent amount under the alternative terms.

∴ He would not accept the offered terms. If money be worth, say 4% p.a., the loss in interest will amount to that on Rs 900 for 1 month at this rate, *i.e.*, to Rs 3.

***§ Repayment of Loan by Instalments.** The practice in regard to the calculation of interest by money-lenders is hardly uniform. A reasonable practice would be to calculate interest on the basis of calendar months. Frequently days are eliminted from the calculation in favour of the lender. When a part payment covers more than the interest due on the date, the excess is usually taken towards the reduction of the principal outstanding. It is also customary on the part of the lender to stipulate, that such reduction of principal, for the purpose of calculating interest only, would not be considered except in certain named lump sums.

Under legislation recently passed in many parts of India, a money-lender is not now allowed to charge compound interest. In Bengal, he is not permitted to seek recourse to law for payment of simple interest at a rate exceeding 10% p.a. on unsecured and 8% p.a. on secured loans. There is the further limitation that on no date should the outstanding interest exceed the principal outstanding, and the total amount recoverable on a loan would in no case exceed double the original principal. The borrower may be granted by a competent court an extended time up to 12 years in case of secured loans, and to 20 years in case of unsecured loans, repayment being made in suitable instalments. In case of default of an instalment, interest may be added thereto at the rate of 3% p.a.

§ Much of the legislation in India on the subject follows the English Law ; and it may be presumed that in cases where the rate of interest is not stipulated, the English precedent would be followed. In such cases, in England, any partial payment towards the liquidation of a loan is apportioned between interest payment and principal repayment in the proportion of the total amount of interest payable to the original principal. The problem of ascertaining the actual rate earned by the money-lender in such cases is interesting.

If a series of payments $S_1, S_2 \ldots S_n$ are made in repayment of a loan C together with interest D, each payment S_r is split up into interest payment of $\frac{D}{C+D} \cdot S_r$, and principal repayment $\frac{C}{C+D} \cdot S_r$.

These principal repayments may be looked upon as being equivalent to a single payment of C at the end of the equated time $\frac{\Sigma S_r . n_r}{\Sigma S_r}$.

Assuming that the interest is not compounded, the total interest paid $D =$ interest on C for the period $\frac{\Sigma S_r . n_r}{\Sigma S_r}$; *i.e.*,

$D = C . i . \frac{\Sigma S_r . n_r}{\Sigma S_r}$, where i is the rate of interest charged per unit per period.

$$\text{Hence } i = \frac{D}{C} . \frac{\Sigma S_r}{\Sigma S_r . n_r} = \frac{D}{C} . \frac{C+D}{\Sigma S_r . n_r} \quad (\because \Sigma S_r = C + D)$$

If the instalments are payable monthly and are equal,

$\Sigma S_r . n_r = S[n + (n-1) + (n-2) + \cdots + 1] = S \cdot \frac{n(n+1)}{2}$; and

$$i = \frac{2}{n+1} . \frac{D}{C} ; \quad \because \Sigma S_r = n . S = C + D.$$

Note. It will be observed that the English rule amounts to the simple practice of dividing up an instalment into $\frac{C}{n}$ and $\frac{D}{n}$, when the repayment is made in n equal periodical instalments.

**Illustrations*

1. A note for Rs 3000 dated 1st Jan. 1940 was endorsed as follows :

Rs 200 on 5. 3. 40 ; Rs 600 on 18. 5. 40 ; Rs 1400 on 6.6. 40 ; Rs 1000 on 9. 7. 40.

The last payment was accepted in full discharge. Find the rate of interest charged.

Here, $C = 3000$, $D = 200$, and $\Sigma S_r n_r = 200 \times 63 + 600 \times 137 + 1400 \times 156 + 1000 \times 189 = 502200$; and

$i = \frac{200}{3000} \times \frac{3200}{502200}$ per unit per diem.

∴ Rate charged = 15½% p. a. nearly.

2. The amount on a mortgage bond for Rs 5000 dated the 1st Jan. 1940 was increased by another Rs 1000 on the 3rd May following. The following payments were endorsed, the last being received in full discharge. Find the rate of interest charged.

Rs 2000 on 8. 1. 40 ; Rs 200 on 10. 3. 40 ; Rs 500 on 5. 5. 40 ; Rs 800 on 3. 6. 40 ; Rs 1000 on 4. 7. 40.

Reducing data to 4. 7. 1940 as *zero date*,

Dr.							*Cr.*
Date	Amount	Days to zero-date	Product	Date	Amount	Days to zero-date	Product
1.1	5000	184	920000	8.1	2000	178	356000
3.5	1000	62	62000	10.3	2000	116	232000
Intt.	300		———	5.5	500	60	30000
		Total	982000	3.6	800	31	24800
		Cr Total	642800	4.7	1000	0	—
		Difference	339200			Total	642800

∴ Interest of Rs 300 = interest on Rs 339200 for 1 day.

∴ Rate % p.a. = $\frac{300 \times 365}{3392}$ = 33% p.a. app.

§ **Instalment Purchase.** Under an agreement in an instalment purchase the buyer becomes the owner of the property directly on its transfer to his possession. In case of failure of payments under the contract, outstanding instalments only may be demanded.

When the balance outstanding under a system of instalment purchase is offered in the lump, before the due date, the buyer is usually offered a kind of discount, or a reduced amount (within reasonable limits) may be accepted from him in accordance with the theory of equation of payments.

Illustration

Furniture valued at Rs 350 is purchased on 1st Jan. 1937 on the instalment plan. Rs 50 is paid down and the balance is agreed to be paid in 12 equal monthly payments. The first monthly instalment is paid in time; when should the balance be offered in lump sum? If money be worth 4%, what discount may be offered to the buyer if he proposes to pay off the balance on 1st April 1937?

The sum of the equivalents of the 11 *remaining* instalments

$$= 25[2+3+\cdots+12] = 25 \times \left(\tfrac{12 \times 13}{2} - 1\right) = 25 \times 77 = 1925.$$

The balance due = Rs 275.

$$\therefore \quad \text{Equated time} = \tfrac{1925}{275} \text{ months} = \underline{7 \text{ months.}}$$

If payment be offered before time, the dealer may offer by way of discount the interest on Rs 275 for 4 months @ 4%, *i.e.*, Rs 3 11as.

§ **Hire-Purchase.** The agreement in a Hire Purchase sale is similar to that in an Instalment Sale, with the fundamental difference that the buyer has no title to the goods, but is merely a hirer thereof, till he chooses to exercise his option of buying them up on payment of a stipulated sum, usually ending with the payment of the last instalment.

The term Hire Purchase is also loosely used for Instalment Sale.

§ **Interest on Instalment Sale.** The rapid growth of sale of goods on instalment terms has brought to the fore the question of the cost of this accommodation to the buyer. The problem, however, is not as easy as it may appear at first sight ; for in addition to a legitimate expectation of return on his capital the seller naturally seeks to provide for the considerable increase in incidental office expenses in maintaining records, cost of collection of instalments, insurance against bad debts, as also cost of legal expenses in realising dues in some cases.

The arrangement usually is that a substantial *deposit* is called for at the time of delivery, which is really part payment of the value. The balance of the Instalment price is then paid in easy instalments, usually equal in value, spread over a length of time.

In law, the sale is completed at delivery and neither has the seller a right to recall the goods, nor has the buyer the right to return them and ask for refund of the deposit.

The cost of this accommodation to the buyer is represented by a difference in the Cash and Instalment prices.

§ In accounting practice, an asset purchased on hire-purchase or instalment terms has to be shown at its cash value, the excess paid over that value being charged to an Interest Account. Periodical payments are split up into interest and principal repayment and charged accordingly in accounts.

Under English Law a seller of goods on instalment terms is in the position of a money-lender, and instalment payments are apportioned as shown above (p. 240), the principal outstanding being the cash value reduced by the initial deposit (h, say) on completion of a sale.

The average rate of interest charged for the accommodation will be $= \frac{2}{n+1} \cdot \frac{D}{C-h}$ per unit per period for n equal periodical payments.

Illustrations

1. The cash price of a gramophone is Rs 150. A buyer desires to pay on instalment terms—Rs 35 cash down and

20 monthly instalments of Rs 7 each. What is the average rate of interest paid by him ?

Here, difference between instalment and cash price, $D=$ Rs 25, the outstanding balance when the first deposit is made, $C-h$ $=$ Rs 115 ; and $n=20$.

$$\therefore \quad i=\tfrac{2}{21}\times\tfrac{25}{115} \text{ per unit per mensem}$$

$$=\underline{25\% \text{ p. a.}} \text{ app.}$$

2. A Diesel Engine is listed at Rs 1005 net cash or on instalments as follows : Rs 200 on delivery, Rs 300 after 3 months, Rs 400 after 6 months and Rs 200 at the end of a year. What is the average rate of interest charged ?

The instalments, excluding the deposit, are equivalent to a single payment of Rs 900 at the end of the equated time

i.e., after $\dfrac{300\times3+400\times6+200\times12}{900}$ or $\dfrac{19}{3}$ months.

$\therefore$ Rs 95 may be regarded as the interest on Rs 805 for $\tfrac{19}{3}$ months.

$\therefore$ Average rate of interest per unit per month

$=\dfrac{95}{805}\times\dfrac{3}{19}=\dfrac{3}{161}=22\%$ p.a. The instalments may now be split up us under :

Pl. outstanding	Mos.	Instalments	Pl. repayment	Interest
Rs 1005	—	Rs 200	Rs 200	Rs—
805	3	300	255	45
550	3	400	370	30
180	6	200	180	20
		Total 1100	1105	95

3. The cash price of a machinery is Rs 1,60,000. It is sold on the following terms : Rs 20,000 on delivery ; Rs 20,000 after a month ; Rs 15,000 a month for 4 months following ; and Rs 10,000 a month therefter for the following 7 months. Apportion the payment between capital and interest.

The instalmen price	=	Rs 1,70,000
First deposit	=	20,000
Principal outstanding	=	1,50,000
Total interest	=	10,000

$$\therefore \quad \text{Rate of apportionment}=\frac{10{,}000}{1{,}50{,}000}=\frac{1}{15} \text{ p. m.}$$

			Interest	Principal
First payment		Rs 20,000	—	20,000
2nd	„	20,000	1,333	18,667
3rd-6th	each	15,000	1,000	14,000
7th-12th	„	10,000	667	9,333
13th	„	10,000	665	9,335

The last instalment is treated separately to allow of the necessary adjustment being made for reconciling the totals.

§ The true gross return to the seller on a Hire-Purchase sale, however, is a matter of some difficulty in theory.

Let i be the return to the seller per unit per period of instalment, and

H = Hire or Instalment price,
C = Cash price,
h = Initial deposit
n = No. of instalments,
d = An instalment,
D = Hire Price − Cash Price

With the payment of the initial deposit h, the balance of cash value outstanding becomes $C - h$, and it may be looked upon as the present value of an immediate annuity of d for n periods.

$$\text{Thus,} \quad C - h = d.a_{\overline{n}|}.$$

The value of i may be obtained from the above with the help of a Table of Annuities and the method of Interpolation. Linear interpolation would ordinarily give fairly dependable results.

N. B. Representing $\frac{C-h}{d}$ by a, a first approximation from Tables by i_0, the value of $a_{\overline{n}|}$ at i_0 by a_0, a close approximation will be obtained from the following formula, v^{n+1} being obtained from the Tables at rate i_0,

$$\frac{i}{i_0} - 1 = \frac{a_0}{a} \cdot \frac{a_0 - a}{a_0 - n.v^{n+1}}.$$

Illustration

A suite of furniture marked at Rs 400 net cash is offered also on the following terms : Rs 50 down and twenty monthly instalments of Rs 21 each. Find the gross return to the seller.

Here $C-h=$ Rs 350 and $d=$ Rs 21.

$$\therefore\ 350=21.a_{\overline{30}|}$$

$$\text{or,}\quad a_{\overline{30}|}=16{\cdot}666667$$

From the Tables,

@ 1½% ... $a_{\overline{30}|}=17{\cdot}168639$

@ 2% ... $a_{\overline{30}|}=16{\cdot}351433$

Diffce. for ½% = ·817206

Proportional diffce. for 1% 1·65441

Also, given ... $a_{\overline{30}|}=16{\cdot}666667$

and @2% ... $a_{\overline{30}|}=16{\cdot}351433$

Diffce. = ·315234

This difference is due to a difference in the rate of interest %.

$$\text{which }=\frac{{\cdot}315234}{1{\cdot}65441}\%={\cdot}19\%$$

$$\therefore\ i\equiv(2-{\cdot}19)\%=1{\cdot}81\%\text{ per mensem.}$$

∴ Effective rate of gross return to seller per annum = 24% app.

*§ For practical purposes, however, it would generally be sufficient to obtain an approximate value of i in the following manner.

From, the formula $C-h=d.\ a_{\overline{n}|}$, we have,

$$\frac{C-h}{d}\ (=a,\text{ say})=\frac{1-v^n}{i};\ \text{or, } 1-ia=v^n$$

$$=(1+i)^{-n}=1-ni+\frac{n(n+1)}{2}.\,i^2-\frac{n(n+1)(n+2)}{6}.\,i^3+\cdots\cdots$$

if $n.\left(\frac{ni}{3}\right)^3$ and smaller values can be neglected.

Collecting co-efficients,

$$i^2-\frac{3}{n+2}.\ i+\frac{6(n-a)}{n(n+1)(n+2)}=0,$$

$$i.e.,\quad i^2-\beta.i+\frac{2\beta(n-a)}{n(n+1)}=0,\ \text{putting }\beta=\frac{3}{n+2}.$$

For a first approximation,

$$i_0 = \frac{2(n-a)}{n(n+1)} = \frac{2}{n(n+1)}\left[n - \frac{C-h}{d}\right]$$

$$= \frac{2}{n(n+1)}\cdot\frac{D}{d}. \quad \because \quad nd = H - h, \text{ and } D = H - C.$$

Cor. 1. It follows, that $D = (C-h)\cdot\frac{(n+1).i_0}{2-(n+1)i_0}$.

Cor. 2. If the initial deposit $h = d$, $H = (n+1).d$,

$$\text{and, } i_0 = \frac{2}{n}\cdot\frac{D}{H}.$$

*§ This value of i_0 may be improved by adding to itsel $\frac{100}{\frac{3}{2}\cdot\frac{H}{D} - 2}$ % of it, *if n is fairly large*. With a little more accurac

$$i = i_0\left[1 + \frac{i_0}{\beta - 2i_0}\right] = i_0\left[1 + \frac{1}{\frac{\beta}{i_0} - 2}\right]$$

Cor. If the first deposit be equal in value to an instalment,

$$H = (n+1).d \quad \text{and} \quad \frac{\beta}{i_0} = \frac{3}{2}\cdot\frac{n}{n+2}\cdot\frac{H}{D}.$$

Illustrations

1. A gramophone is listed at Rs 216 to be paid at the rat of Rs 6 a month. If its net cash value is Rs 210, what is th return to the dealer on the instalment sale ?

The number of instalments (besides the first payment of Rs 6 is 35, and $i_0 = \frac{2}{35}\cdot\frac{6}{216} = {}^{\cdot}16\%$ per mensem $= 1{}^{\cdot}92\%$ p. a.

$$\text{The correction} \overset{=}{\rightarrow} \frac{100}{\frac{3}{2}\cdot\frac{216}{6} - 2}\,\%\text{ of } i_0 \overset{=}{\rightarrow} 2\%\text{ of } i_0$$

$$\therefore \quad i = 1{}^{\cdot}92\% + \underline{2}\% = \underline{\underline{2\%}} \text{ p.a.}$$

2. The cash price of a sewing machine is Rs 150. A buye chooses to pay on the instalment system as follows : Rs 35 casl down and 20 monthly instalments of Rs 7 each. What is th cost of this accommodation to the buyer ?

Here $n=20$, $D=25$, $H=150$, $d=7$

and $\beta=\frac{3}{22}=\cdot136$, $i_0=\frac{2}{20.21}\cdot\frac{25}{7}=1\cdot7\%$ p. a. $=23\%$ p. a.

$$\text{Correction} \rightrightarrows \frac{100}{\frac{3}{2}\cdot\frac{150}{25}-2}\ \%,\ i.e.,\ 14\tfrac{2}{7}\%.$$

Then, $i_0=23$

Correction $=\ 3$

$\quad\quad 26$ $\quad$ *i.e.*, $i=\underline{26\%}$ p.a.

3. A dealer desires to make 6% p. a. on instalment terms. An article is listed at Rs 200, and he desires a deposit of at least Rs 50. What will be the terms of instalment, spread over 2 years ?

Taking $i_0 \rightrightarrows \cdot005$ per unit per mensem

$$D=\text{Rs }150\times\frac{(24+1)\times\cdot005}{2-(24+1)\times\cdot005}=\text{Rs }10$$

$$\therefore\ \text{Each instalment}=\text{Rs. }\frac{200+10-50}{24}=\text{Rs }6\tfrac{2}{3}$$

$$=\underline{\text{Rs }6\ 11\text{as}}\ \text{per mensem.}$$

4. A cycle listed at Rs 60 net cash is to be sold to a customer on instalment terms. The dealer desires to clear at least 10% p. a. and to obtain full payment in 11 equal monthly payments. Find the instalment price.

Here, the instalment is also equal to the first deposit, and $n=10$, excluing the the deposit.

Takhing $i_0=10\%$ p. a., *i.e.*, $=\frac{1}{120}$ p.m., which is also $=\frac{2}{10}\cdot\frac{D}{H}$.

$$\text{Whence,}\quad H=24D=24(H-60)$$

$$\therefore\quad H=\text{Rs }60\times\frac{24}{23}=\text{Rs }62\ 10\text{as}$$

This will yield the dealer slightly more than 10% since $i>i_0$. Each monthly instalment should be $\frac{1}{11}$th of Rs 62 10as *i.e.* Rs 5 11as.

EXAMPLES 29

1. Bills for Rs 1250, Rs 1800 and Rs 1600 are to be paid by a merchant to the same dealer on 14th Jan., 3rd Feb. and 24th Feb., 1937 respectively. When would a payment of the total value of the bills be made without involving either party in loss ?
[*Ans.* 5th Feb. 1937

2. Find the average due date for the following payments : Rs 500 payable on 1st Jan. ; Rs 1500 on 18th Jan. ; Rs 700 on 11th March ; Rs 1100 on 3rd May and Rs 1800 on 30th June.
—I. Com. [*Ans.* 6th April

3. An up-country merchant requests his suppliers to make out a single bill in lieu of the following :

Bill for Rs 2585 payable 6th Jan. 1936
" " 3860 " 13th Feb. 1936
" " 5000 " 26th Feb. 1936
" " 1650 " 6th Mar. 1936

What would be the due date of the new bill ?

[*Ans.* 14th Feb. 1936

4. Calculate the equated date of payment in the following case : Rs 58 payable in cash on the 1st Jan. 1936 ; Rs 24 4as on the 2nd March 1936 ; Rs 28 12as on the 11th April 1936 ; Rs 20 on the 15th May 1936. [*Ans.* 24th Feb. 1936

5. A merchant has to pay the following bills to an up-country dealer : Rs 3520 on April 1, Rs 6200 on April 18, Rs 5400 on May 3, 1940. He remits Rs 8000 on account to reach the dealer on April 10, 1940. What is the latest date when the balance due would be acceptable by the latter in a lump sum ? If the merchant pays up on April 17, 1940, what discount may be allowed to him, money being worth 5% p. a. ?
[*Ans.* 30th April 1940 ; Rs 12 11as

6. A piano is bought on the 1st January 1936 for Rs 1400 on the instalment system with Rs 200 down, the balance being payable in six equal monthly instalments. After the third monthly instalment has been paid, the buyer decides to pay up the balance in a lump sum. What is the equated date of payment ? If, however, he offers to pay lump sum on the 1st April, and if the market rate of interest on that date is 4% p. a.,

what would be the limit of rebate that may be allowed to him by the seller ? [*Ans.* 1st June 1936 ; Rs 4

7. A debt is to be discharged thus :

20% at the end of 1 month
15% ... ... 2 months
30% ... ... 3 "
10% ... ... 4 "
20% ... ... 5 "
5% ... ... 6 "

Instead, 40% was paid at the end of 1½ months ; 30% was paid at the end of 3 months. What is the latest date when the balance will be acceptable in the lump ? [*Ans.* 5 months 10 days

***8.** A motor cycle is offered for Rs 660 cash, or Rs 700 on instalments on the following terms ; Rs 100 cash down and the balance in 12 equal monthly instalments. What is the average interest charged ? [*Ans.* 13% p. a.

***9.** A set of books is priced at Rs 230 cash. If sold on instalment terms an extra Rs 50 is charged, payment being made as follows : Rs 40 down and 48 monthly instalments of Rs 5 each. What effective rate of interest has the buyer on instalment terms to pay ? [*Ans.* 12% p. a.

***10.** A sewing machine was sold for Rs 170 on hire purchase terms : Rs 20 down and the balance in monthly payments of Rs 5 each. After the fourth instalment was paid, payment was stopped. Under the agreement the entire balance then due became payable, and was actually realised at the end of the sixth month from the date of purchase. What effective rate of interest was charged under instalment terms, and what rate was earned by the seller, if the cash price was Rs 130 ? [*Ans.* 27% p. a. ; 75% p. a.

***11.** The cash price of a suite of furniture is Rs 350. It is also to be had on instalment terms for Rs 407, of which Rs 107 is to be paid cash down and the balance is to be paid in 10 equal monthly instalments. A prospective buyer, however, offers the following terms : Rs 157 down and the balance to be paid at the rate of Rs 25 per month for 10 months. Should the dealer accept his terms ? [*Ans.* Yes.

[Effective interest on original terms 60% p. a. ; revised offer 80% p. a.

***12.** The catalogue price of a Grandfather Clock is Rs 380. A dicount of 10% is allowed for cash ; but an extra 5% is charged on instalment terms as follows : Rs 99 down and the balance in 10 equal monthly instalments. What is the effective rate of interest charged ? [*Ans.* 60% p. a.

***13.** A gramophone is listed at Rs 160 net. A buyer proposes to take it on instalment, and the dealer desires to clear 13%. If the instalments are all equal, and spread over a year, find the value of an instalment. [*Ans.* Rs 14 3as

***14.** A secondhand motor car is priced at Rs 2000 less 10% for cash. A purchaser offers to take it on the following instalment terms : Rs 500 down, and Rs 125 p. m. for the following 12 months. The dealer's original net price was marked to clear 20%. Find what return on his money is represented by the proposed terms. [*Ans.* 4% per mensem

*§ **Allocation of Expenditure to Departments etc.** It frequently happens that expenses incurred indirectly over several departments in a large stores, or in an office working in several departments, have to be apportioned between them and charged to the respective accounts. The basis of apportionment varies in different institutions, but is mostly in reference to the benefits accruing to the several departments. The following examples would indicate some of the more usual methods.

EXAMPLES 30

1. From the following data about the income and expenditure of 133 Indian life assurance companies, determine the values of the various items corresponding to Re 1 premium.

(In thousand rupees)

Income	Rs	*Expenditure*	Rs
Premium	65827	Claims & Surrenders & Annuities etc.	28569
Interest, Rent & Dividends	15029	Expenses of Management	20692
Other Receipts	2587	Dividends	784
Total	Rs 834443	Depreciation, etc.	776
		Addition to Life Funds	31066
		Miscellaneous	1686
		Total	Rs 83443

[*Ans.* Prem.	Re 1 0 0	Claims etc.	0 7 0
Intt. etc.	0 3 8	Management Exps.	0 5 0
Or. receipts	0 0 8	Dividends	0 0 2
	Re 1 4 4	Depreciation etc.	0 0 2
		Life Funds addition	0 7 6
		Miscellaneous	0 0 5
			Re 1 4 3

N.B. The slight discrepancy in the total is due to approximation at various stages.

2. Allocate the following charges to the different departments of a big store in proportion to their respective turnovers :

Charges	Rs		*Departments*	*Turnover* Rs
General Management	66 480	*A.*	Tailoring & Clothes	283 650
Rent, Rates & Taxes	49 615	*B.*	Toys	105 900
Electricity	2 920	*C.*	Stationery	89 850
Depreciation of		*D.*	Toilet Goods	340 500
Furniture, Fixtures, etc.	13 780	*E.*	Travelling Goods	65 000
		F.	Jewellery	115 100

[*Ans.* *A*—28% ; *B*—11% ; *C*—9% ; *D*—34% ; *E*—6% ; *F*—12%.

3. It is decided to start a system of allocation of certain expenses of an Insurance Company transacting Fire, Marine, and Motor Car Insurance, on the basis of the last three years' average of the premium incomes of the three departments. From the following data find the proportion in which allocations are to be made on every voucher that requires to be treated in that manner.

Premium Incomes

Year	Fire Rs	Marine Rs	Motor Rs
1930	1 283 678	861 208	264 618
1931	1 613 913	693 122	288 917
1932	1 006 212	388 503	302 125

[*Ans.* @ 9as ; 5as and 2as in Re 1, respectively.

***4.** The Continental Motor Company maintains a chain of Service Stations in a Province. It is desired to charge certain expenses in the Branches Department at the Head Office to the different stations in proportion to their gross incomes and a uniform rate of interest at 2% p. a. on the value of the average monthly stock of parts and accessories at cost held at each station. Calculate the amount by which each station is to be debited for a month, the total allocable expenses being 45000 francs.

	Av. Stock of Parts at cost	Gr. Income
Station *A*	156 600 fr.	123 500 fr.
" *B*	86 400 "	96 000 "
" *C*	108 000 "	80 500 "

[*Ans.* *A*—18786 fr.; *B*—14544 fr.; *C*—12255 fr.

*5 A sales establishment is maintained jointly by a Cotton Mill and a Woollen Mill. It is arranged that the Mills should share the cost of the establishment in the ratio of the turnovers in Cotton and Woollen products during the season October-March, and 25% of the average monthly charges during the rest of the year will be borne by the Woollen Mill.

Determine the cost of the Establishment to the Mills from the following data:

	April-Septr. 1932 Rs	Oct.-March 1932-33 Rs
Cotton Goods Sales	168 500	112 000
Woollen " "	15 500	138 000
Average Monthly Cost	4 000	4 250

		Average Apr.-Sept.	Average Oct.-March
[*Ans.*	Woollen	Rs 1000	Rs 2346
	Cotton	Rs 3000	Rs 1904

§ Apportionment of Unexpired Credits etc. in accounts.

Illustrations

1. Insurance premium for 1 year paid up to March 25th 1937 is Rs 157 4as. Accounts are made up to December 31st 1936. What is the unexpired portion of the premium?

Decr. 31st to March 25th......84 days

$\therefore$ Unexpired premium $= \frac{84}{365}$ of Rs 157 4as

$= \text{Rs } \frac{13209}{365} =$ Rs 36 3as

2. Quarterly rent of Rs 463 8as is payable on the 1st April 1936. Accounts are made up to February 28th 1936. Find the rent accrued.

February 28th to 1st April......32 days

∴ Accrued rent $= \frac{32}{90}$ of Rs 463 8as

$=$ Rs 164 12as

3. Fire insurance premium for a year from 5th August 1940 amounts to Rs 1538 3as. Calculate the unexpired portion of the premium if the accounts are made up to 31st March 1941.

Date of Expiry ···	4. 8. 1941
" " Accounts	31. 3. 1941
Days unexpired	125

The unexpired portion of premium $= \frac{125}{365}$ of Rs 1538 3as.

Applying the 3rd, 10th & 10th Rule, the required premium is obtained as below :

$$\frac{2 \times 125 \times 1538.18}{10^3} = 384.55$$

	$\frac{1}{3}$	128.18
	$\frac{1}{10}$	12.82
	$\frac{1}{10}$	1.28
		526.83
Less correction		.05
		526.78

∴ Unexpired premium = Rs 526.78 = Rs 526 12as

N. B. The 3rd, 10th & 10th Rule is here applied to reduce the arithmetical work involved. Its use may be extended whenever 365 is a divisor.

EXAMPLES 31

1. A firm owns house properties which are let out on an annual rent of Rs 10500 on which the quarterly taxes payable amount to Rs 498 12as. The rent is payable on the 1st January every year, and taxes for the March quarter are payable on 31st March. Show the accrued rent and taxes to be shown in the books of the firm which are made up to 28th February every year. [*Ans.* Rent Rs 1750 ; Taxes Rs 332 8as

2. A floating fire policy on a jute godown is taken out on 18th November 1936 for a period of 3 calendar months at a cost of Rs 253. If the accounts are made up to 31st December 1936, find the unexpired portion of premium. [*Ans.* Rs 132

3. $3\frac{1}{2}$% Government Stock of Rs 30000 is sold on the 18th June 1936, the next half-yearly interest being payable on the 15th September 1936. Find the amount that the buyer will be called upon to pay as accrued interest on the holding. [*Ans.* Rs 273 5as

4. For Rs 450 a year a business firm contracts with a typewriter dealer to have all the office machines oiled and cleaned for 1 year. The agreement was entered into on 18th January 1936. Show how the cost of the contract will be shown in the firm's accounts closed on 31st March 1936. [*Ans.* Accrued Rs 90

5. A royalty of Rs 8000 a year or Rs 2 per ton of Kyanite raised is to be paid to the landlord for working a quarry of the mineral. The first annual instalment falls due on the 27th November. If the landlord's accounts are to be made up to 30th June 1936, how will the Royalty account be shown in the books ? [*Ans.* Minimum Royalty accrued Rs 4721

§ **Costing**. A more difficult type of problems arises in connection with the costing of manufactured articles. The different units produced, involving different processes of production, require difficult apportionment of the huge costs of large scale production and demand the assistance of trained experts. In the following pages, simpler cases of such methods are discussed, and the broad principles are set forth.

Costs are ascertained at different stages of production.

Raw materials directly concerned are valued at the factory and directly productive wages and expenses directly chargeable to the production are added to arrive at *Prime cost.*

Indirect charges incurred in running the factory, like factory rent and rates, power, supervision charges, repairs to and depreciation on plant and machinery, factory stores etc. are together known as *Works Oncost* or *Works Overhead* and added to the Prime Cost to obtain the *Works Cost.*

Other indirect charges incurred in connection with the final cost are grouped together under two heads. The first group may be described as Administration Expenses and will include rent and rates of the office premises, as distinguished from factory premises ; salaries to office staff ; Directors' fees, etc. ; discounts, bad debts, etc. ; depreciation on office furniture and fixtures, interest charges on capital, debentures etc. The second group may be described as Distribution Expenses and comprises all other expenses, including advertising, commission etc. incurred after production up to actual sale. These indirect charges, known as *Office Oncost* or *Office Overhead*, added to the *Works Cost* together form the *Total Cost*. The selling price is ascertained after allowing for anticipated profit.

The following chart may be usefully studied ; and the details in the Cost Summary outlined below may be noted as a practical illustration.

Direct Materials Direct Wages Chargeable Expenses	= Prime Cost + Works Oncost	= Works Cost + Office Oncost	= Total Cost + Profit	= Selling Price.

Cost Summary.

Description. Date

Style

Pattern No.

Date Manufacture Commenced...... Size

Job No......... Stock Order No.

Date Manufacture Finished...........

	Rs as p			United produced		Each	Rs as p
Direct Materials			+		=	Cost of materials	
Direct Labour			+		=	Cost of labour	
Chargeable Expenses							
Prime Cost	Rs						
Works Oncost			+		=	Works Oncost	
Total Works Cost		Rs	+		=	Works Cost	
Administration and Sales Expenses			+		=	Sales Expenses	
Total Cost			+		=	Total Cost	
Profit %			+		=	Profit	
Selling Price		Rs	+		=	Selling Price	

§ In this simple form the analysis serves the purpose where only a particular class and quality of an article is manufactured. It may easily be extended, by a system of apportionment of overheads, when different departments are engaged in producing each a particular type of articles. When shops are engaged in the production of different units of similar products, the methods become more complicated. Other considerations also will usually make particular costing methods highly specialised.

It sometimes happens that by-products are obtained in the course of manufacture of an article ; and sometimes the materials wasted during the production of the primary article are utilised in the production of other articles which are sold at a price. The obvious effect of these is to reduce the costs of the primary product to a corresponding extent.

*§ **Allocation of Expenses.** An efficient organization requires that the expenses incurred at various stages of production should be as carefully allocated as possible. For instance, depreciation, repairs to and maintenance of plant and machinery, will relate to the works only, as also carriage inwards, salaries of factory manager, labour, works expenses etc. Depreciation to buildings, rent, rates and taxes and similar items will be charged both to the works and the sales, usually on the basis of floor space occupied by each ; stationery, travelling expenses, electricity, water etc. are generally allocated according to actual consumption by a department.

There are, however, always some expenses which cannot be directly charged either to works or to sales ; and they are apportioned in accordance with the benefits accruing to a department by such expenditure. Departmental turnover provides a common basis ; while Rent is usually charged in proportion to the floor-space used. Expenses like advertisement, the cost of which has little relation to the cost of sales, is usually apportioned on the basis of the "capacity to bear" or "as much as the traffic will bear" principle.

§ **Standard Cost.** A valuable control over cost figures is obtained by maintaining a schedule of standard costs which are predetermined in reference to past experience under optimum conditions. The ratio of the Standard Cost of a unit to its current Actual Cost is known as the Efficiency Percentage.

Thus, if the Standard Cost (Labour) is Rs 10 000
and the Actual Cost recorded is Rs 12 000
the Efficiency Percentage of labour = 83⅓%.
Also, Labour Cost Ratio = 120%.

The causes of variation from the Standard cost may be classified under

(i) variation in price of raw material ;
(ii) variation due to waste of material ;
(iii) variation due to change in wages rate ;
(iv) variation due to inefficiency of labour or lost time due to strikes, lockouts, etc.

EXAMPLES 32

1. From the following figures for the cost of production of 10000 umbrellas, obtain the cost per unit.

	Rs	Per Unit
Raw materials	5600	
Wages for production	4700	
Prime Cost		?
Works Oncost :		
Factory Rent, Rates & Taxes	600	
Supervision charges	440	
Cost of stores	300	
Works Cost		?
Interest on Capital	200	
Selling Commission, Packing etc.	2400	
Total Cost		?

[*Ans.* P.C. Re 1·03 ; W.C. Re 1·164 ; T.C. Re 1·424.

2. The following is an analysis of normal costs of production of 460 bighas of sugarcane :

	Rs
Breaking soil, Harrowing, etc.	4400
Seeds	3260
Sowing, weeding, etc.	516
Irrigation	1215
Harvesting	5118
Carting	316
Supervision	150

Yield—92000 mds of cane.

Find the cost of cane per maund.

If, in a certain year of drought, irrigation expenses increase 120% above normal, what will be the effect of this increase on the cost of every 100 md ? [*Ans.* 2¾as ; Increase Re 1 9as 4p

3. The following figures relate to the working of a colliery :

	Rs
Opening Stock......2000 tons valued at	6 000
Closing Stock......7000 " " "	21 000
Coal despatched...30000 " " "	120 000
Total Expenditure during the period	70 000

Find cost (a) per ton raised, (b) per ton despatched, and (c) profit per ton raised.

[*Ans.* (a) Rs 2 ; (b) Rs 2 5as 4p ; (c) Re 1 13as 9p

***4.** From the following data relating to a colliery determine (i) the cost per ton at pithead and (ii) the cost per ton sold ; and also obtain the different items of cost as percentages of the total cost. Raisings during the period...6000 tons.

		Rs
Wages :	Underground	6000
	Aboveground	1500
Working expenses :	Stores	500
	Repairs, etc.	2000
	Power	750
	Timber	300
	Rent, Rates & Taxes	400
	Royalties	1800
	Depreciation of Plant	1200
Other charges :	Administration	1600
	Selling expenses	400
	Railway siding Expenses	150

per ton

[*Ans.* Pithead......Rs 2 as 3p per ton

Sold..........Rs

*5. Below are data of an electric oil mill. You are required to ascertain whether it is worth barrelling the oil for export @ Rs 29 7as per md., if the minimum profit expected is 12½%.

Raw materials......5000 md valued at Rs 800

Oil Expressed (crude) ... 57 md

Oil " (refined) ... 45 md

	Crushing	Refining	Barrelling (estimated)
	Rs	Rs	Rs
Wages	45	160	125
Power	185	50	*nil*
Stores	30	20	55
Repairs	15	5	15
Supervision	85	65	45

Estimated Cost of barrels Rs 38

Realised on sale of oil cake Rs 385

Realised on sale of seed-bags Rs 50

[*Ans.* C. P. in barrel Rs 29 per md

6. Ascertain the overhead expenses of each of 10000 cricket balls manufactured, from the data given below.

	Rs
Electric Power	6 000
Steam	3 500
Gas	2 800
Water	600
Repairs	980
Rent, Rates & Taxes	1 060
Factory Management	7 340

[*Ans.* Rs 2 3as 8p

7. A cost analysis gives the following percentages :

Selling Price	100
Prime Cost	32
Factory Oncost	28
Office Oncost	29
ofit	11

A sudden jump in the price of raw materials increases Prime Cost by 25%, which is sought to be set off by reductions in the estimated Profit and Office Oncost in the ratio of 3 : 1 ; other things remain unaltered. Find the adjusted percentages of profit and office oncost. [*Ans.* 5% and 27%

8. If the effect of the rise in the price of raw materials in the above example were passed on to the consumer, how will the price and the rate of profit be affected ?

[*Ans.* Increase 8% ; Decrease 8%. on original S. P.

9. If in example 7, a general depression lowers prime cost by 10%, factory oncost by 5% and office oncost by 2½%, how would the selling price be altered to assure a profit of $\frac{9}{10}$ths of that previously made ? [*Ans.* Decrease 6·325%

10. The cost of placing an article on the market per unit is analysed into the following percentage figures :

Raw materials...16 ; Labour...10 ; Power...15 ; Supervision ...18 ; Office Expenses...20 ; Distribution Expenses...21. Profit is estimated on this at 15%.

It was found necessary later to spend 20% of the estimated profit on increased advertisement to ensure a larger turnover. How does the rate of profit shrink, the selling price remaining unaltered ? [*Ans.* New profit 11·7%

11. If, in the above example, electrical rates are cheapened by 10% ; supervision charges are increased by 5% and the cost of raw materials by 10%, with a view to producing a better quality of goods, how would the selling price be altered, other things remaining the same ? [*Ans.* Increase 1%

12. The Standard Cost analysis gives the following percentages on the Total Cost : Prime Cost...40 ; Factory Oncost...38 ; Office Oncost...22. The actuals analysis showed Prime Cost to have gone down by 5% ; Factory Oncost and Office Oncost gone up by 5% and 2½% respectively. If the selling price was marked 10% above Standard Cost, how would the rate of profit be affected ? [*Ans.* Decrease ½%

13. The Income and Expenditure statement of a firm of brokers showed that the income from the Jute business in a year is Rs 20000 ; from Freight Rs 20000 and from Real Estate Rs 40000. The expenditures were as follows :

Rent, Rates and Taxes ...	Rs 4500
Stamps, Stationery and Ptg...	Rs 1400
Charges General ...	Rs 3215
Motor Car Up-keep ...	Rs 10845
Establishment ...	Rs 5040

If the expenses are apportioned in proportion to the income of the respective departments, find the percentage cost of earning the different incomes. [*Ans.* 31¼%.

14. The following figures relate to the manufacture of 6 gross bottles of *Chutney* :

Bottles	6 gross 3 doz	@ Rs 3 4as per gross
(Breakage)	3 doz	
Mangoes &c.		Rs 80
Spices		Rs 25
Sugar 2½ mds @		Rs 10 per md
Labour		Rs 55
Incidental Expenses		Rs 37 11as

A profit of at least 25% on costs is desired. Find the selling price per dozen. [*Ans.* Rs 4 3as 6p.

15. A publisher makes up the following account of the publication of 1100 copies of a book, sold at Rs 2 8as a copy.

Paper	110 reams	@ Rs 4 per ream
Printing		Rs 560
Binding		@ Rs 40 per 100 copies
Advertisement		Rs 160

100 copies were distributed free, and the rest were all sold out. Discount @ 25% was allowed on the sale of 800 copies through other booksellers ; and a royalty of 8as per copy was paid to the author on all sold copies. What is the publisher's profit ?

If a proportionate charge of Rs 125 on account of establishment &c. is allotted to this publication, how is the profit affected ?

[*Ans.* Loss Rs 100 ; Loss Rs 225

***16.** In a general stores, the different departments are charged to their share of the overhead charges on the basis of the turnover in the respective departments during the last accounting year ; and the ratio of this apportioned charge to the turnover determines the proportion in which cost prices are to be rated up.

Overhead Charges

Rent &c.	Rs 36450	Advts. &c.	12915
Electricity & Telephone	18404	Supervision	28216
		Miscellaneous	4015

Department	*Turnover*	*Department*	*Turnover*
Grocery	Rs 130 675	Boots & shoes	Rs 89 212
Furniture	603 812	Hardware	113 931
Electricals & Crockery	395 278	Order Department	472 224
Readymade Clothing, Hosiery, &c.	2163 421	Travelling Requisites	318 921
		Watches, Jewelleries &c.	712 526

Calculate the percentage rating up to obtain the departmental cost of stock valued at Rs 220 000 in the Travelling Department. If the establishment and other expenses of this department amount to Rs 2412, which is also to be charged directly to the goods in the department in proportion to the cost, find the percentage above cost at which the goods in the department should be marked in order to recover all charges.

If a profit of 20% on the all-in cost is desired, what should be the selling price of a suitcase costing Rs 25 ?

[*Ans.* 4% above cost ; Rs 31

***17.** A dealer makes the following purchases of rice at a country *hat* :

Quality	Mds.	Rate per md.
1	200	Rs 4 8as
2	800	Rs 4 0as
3	1200	Rs 3 4as
4	400	Rs 2 12as
5	160	Rs 2 8as
6	800	Rs 2 4as

The cost of carriage from the *hat* to his town godown works out at 3as 6p per md. He expects to clear his stock in 3 months ; and to make an all-over profit of 16⅔% on the first quality and 12½% on the rest. If the rent of the godown is Rs 113 per quarter, find the average selling price of the various grades of rice.

[*Ans*. (1) Rs 5 9as (2) Rs 4 12as 6p (3) Rs 3 15as
(4) Rs 3 6as (5) Rs 3 1a 6p (6) Rs 2 13as

§ **Depreciation**. The following are the principal methods of calculating depreciation.

A. Taking fixed periodical instalments on original cost, the asset being ultimately written off at the end of a pre-determined period of time. This is also known as the Straight Line Method.

Thus, if a building cost Rs 50000 to construct, and if it be desired to write off this value in 25 years' time, Rs 2000 is deducted from the outstanding value of the building every year.

B. Taking pre-determined percentages of diminishing balances. Frequently these percentages are low at the beginning and are higher during later years.

Thus, the rate on new machinery valued at Rs 20000 may be 2½% for the first year, 5% for the next year, and 10% per annum thereafter.

Machinery		Rs 20000
Less Depreciation @ 2½%	(1st yr.)	500
Reduced value of Machinery	(2nd yr.)	Rs 19500
Less Depreciation @ 5%	"	975
Reduced value of Machinery	(3rd yr.)	Rs 18525
Less Depreciation @ 10%	"	1853
Reduced value	(4th yr.)	Rs 16672
etc.	etc.	etc.

C. An asset is sometimes sought to be depreciated at a diminishing rate according to the Sum of Digits Method.

A boiler, an electrical saw or a power equipment, on which the depreciation is large during the early years of their life, may be treated in this way. The method is easily followed in the illustration below.

Illustration

A power lathe is purchased for Rs 2000. Prepare a Table of Depreciation assuming that it has a scrap value of Rs 200 at the end of 9 years. (By *digits* is meant the number of periods.)

Table

Year	Depreciation Rate	Depreciation Charge Rs	Depreciation Reserve Rs	Depreciated Value Rs
1	$\frac{9}{45}$	360	360	1640
2	$\frac{8}{45}$	320	380	1320
3	$\frac{7}{45}$	280	960	1040
4	$\frac{6}{45}$	240	1200	800
5	$\frac{5}{45}$	200	1400	600
6	$\frac{4}{45}$	160	1560	440
7	$\frac{3}{45}$	120	1680	320
8	$\frac{2}{45}$	80	1760	240
9	$\frac{1}{45}$	40	1800	200
45	1	1800		

The rates (fractions) have the common denominator of the sum $1+2+\cdots+9=45$. The numerators are the same *digits* taken in the reverse order.

D. Sometimes a rate of depreciation is ascertained by distributing the estimated loss in value in a pre-determined number of years. Thus, if a machinery now worth Rs 20000 is expected to realise Rs 5000 at the end of 10 years, this difference of Rs 15000 in value is spread over the years and written off on diminished balances.

Suppose, p is the new value of an asset, and r its residual value after n years, and d represents the depreciation per unit per annum. Then

New value	p
Depreciation 1st year	$p.d$
Reduced value 2nd year	$p.\ (1-d)$
Depreciation 2nd year	$p.d.(1-d)$
Reduced value 3rd year	$p.\ (1-d)^2$

Similarly, reduced value at end of n years $r = p(1-d)^n$

Whence, $$d = 1 - \left(\frac{r}{p}\right)^{\frac{1}{n}}$$

The computation is made from the relationship

$$n \log (1-d) = \log r - \log p$$

E*. The Annuity Method considers the purchase as an investment carrying interest during its life, at a previously determined rate, on diminishing balances.

If the rate of interest be i per unit per annum, p the price of the asset, r its scrap value, and D the equated annual depreciation over n years, the value to be depreciated is the compound amount of the value of the asset reduced by the scrap value.

$$\therefore\ D.s_{\overline{n}|} = p(1+i)^n - r, \text{ whence } D = \frac{p}{a_{\overline{n}|}} - \frac{r}{s_{\overline{n}|}}$$

Illustration

An asset costing £500 is to be written off in 3 years by equal instalments, reducing balances bearing interest at 5% per annum. Find the equated annual depreciation.

Here, $i = {}^{\cdot}05,\ p = £500,\ n = 3$ years.

$$D = \frac{£500}{a_{\overline{3}|}} = £183{}^{\cdot}605$$

$$= \underline{£183\ 12s\ 1d}$$

This method is generally used in depreciating leases, etc.

**F*. The Sinking Fund Method provides for the replacement of the asset at the end of a number of years by the accumulation at compound interest of a periodic payment. Interest is periodically added to the accumulated reserve. The asset account is left undisturbed till the end of the period, when it is credited with the total accumulation in the Fund.

** While the P & L account is debited annually with the amount of Depreciation, the asset account is credited with interest on the depreciated value. At the end of the period the asset account naturally stands at the scrap value.

If p = cost of the asset,

r = residual or scrap value of the asset after n years, and

i = rate of interest per annum per unit,

the first periodic payment into the Sinking Fund $=\frac{p-r}{s_{\overline{n|}}}$.

The subsequent periodical depreciation charges are each $=\frac{p-r}{s_{\overline{n|}}}$ + interest on the accumulated depreciation reserve.

Obviously, at the end of the period the Fund would stand at $p-r$.

Illustration

Five Service Lorries costing Rs 20 000 are to be scrapped at 10% of their cost at the end of 5 years. Determine the periodic depreciation charge by the Sinking Fund method on a 5% basis.

The first year's charge to depreciation

$$=\frac{90\% \text{ of Rs } 20000}{s_{\overline{5|}}}=\text{Rs}\,\frac{18000}{5\cdot52563}$$

Now, log 18000 = 4·255273

log 5·5256 = ·742379

log 3257·6 = 3·512894

∴ First year's charge	= Rs 3258 to the nearest rupee.	
The 2nd year's charge	= Rs 3258	
plus Intt. on 1st year's charge	= Rs 162 14as	
	Rs 3420 14as	
Accumulated Fund	=	Rs 6678 14as
3rd year's charge	= Rs 3258	
plus Intt. on accumulated fund	= Rs 333 15as	
	Rs 3591 15as	
Accumulated Fund	=	Rs 10270 13as
4th year's charge	= Rs 3258	
plus Intt. on accumulated fund	= Rs 513 9as	
	Rs 3771 9as	
Accumulated Fund	=	Rs 14042 6as
5th year's charge	= Rs 3258	
plus Intt. on accumulated fund	= Rs 702 2as	
	Rs 3960 2as	
Accumulated Fund		Rs 18002 8as

Note that there is practically an excess accumulation of Rs 2 8as necessitated by approximations made at the different stages.

EXAMPLES 33

1. Calculate annual depreciation on the Fixed Instalments basis on the following :

Machinery cost Rs 80000 to be written off in 10 years
Buildings cost Rs 38000 " " " 20 "
Motor Car cost Rs 3580 " " " 5 "

[*Ans.* Mach. Rs 8000 ; Bldg. Rs 1900 ; Motor Car Rs 716

2. A Reserve for Replacement and Obsolescence is created from the amounts written off in Depreciation at the following rates :

	Rs	Depreciation %
Additions to Plant and Machinery during the years	98600	5
Existing Plant and Machinery	496500	10
Motor Cars and Lorries	183600	25
Spare Parts, Tools and Accessories	55000	12½

Find the addition to the Reserve at the end of the year.

[*Ans.* Rs 107355

3. A printing press, which is being assessed for purposes of Income Tax, is allowed to debit the Profit and Loss account with depreciation on the following at prescribed rates noted against each :

	Value Rs	Rate of Depreciation %
Buildings	85200	2½
Machinery	70500	5
Electrical Equipments	18400	7½

Find the total amount of depreciation. [*Ans.* Rs 7035

4. A structure is erected on land taken on 10 years' lease at a cost of Rs 8200. It is estimated that at the expiry of the lease the structure will have a scrap value of Rs 2000. If the property is to be written off in 10 years' time, find the amount of depreciation written off annually, on the Fixed Instalments plan.

[*Ans.* Rs 620

***5.** Calculate the percentage charge of depreciation on machinery valued at Rs 40000 with a scrap value of Rs 4000 to be written off in 10 years on the basis of Diminishing Values.

[*Ans.* 20·567%

*6 Find, by the Annuity Method, the annual charge to depreciation at 4% of a Cinema Show House valued at Rs 50000 with an estimated scrap value of Rs 5000 at the end of 10 years. [*Ans.* Rs 5748

7. A machine becomes depreciated in such a way that the value of the machine at the end of any year is 90% of the value at the beginning of the year. The cost of the machine is £20 and it was sold eventually as waste metal for £7 10s. Obtain the number of years during which the machine was in use.— —I. Com. [*Ans.* 9¼ years app.

8. The machinery in a factory is valued at £24,537 and it is decided to reduce the estimated values, at the end of each year, by 18% of the value at the beginning of that year. When will the value be (a) £20000 and (b) one-tenth of its original value? —I. Com. [*Ans.* 1·03 years, 11·6 years.

*§ **Wasting Asset**. A quarry, mine, or a forest, has an exhaustible supply of stones, coal or timber, which is depleted as it is worked. Such an asset is known as a *wasting asset.* The value of the periodical *depletion* is determined by the value of the quantity of material removed in its relation to the estimated value of the product that will be extracted during the period of the lease.

When a wasting asset is transferred, its Capitalized value

$$= \frac{\text{Average annual operating profit}}{\dfrac{1}{s_{\overline{n}|}} + \text{a fair annual rate of income to proprietor per unit}}$$

where $s_{\overline{n}|}$ is the amount of an annuity of 1 during the period of depletion at a fair rate of interest.

It is customary to provide a fund for the replacement of the asset at its exhaustion in the form of a sinking fund.

Illustrations

1. 1750000 tons of coal are estimated to be recovered from a mine, which is valued at Rs 30 lakhs. Find the *depletion charge* per ton of coal mined.

$$\text{The depletion charge per ton} = \frac{\text{Rs 30 lakhs}}{17\tfrac{1}{2} \text{ lakhs}}$$

$$= \underline{\text{Re } 1\tfrac{5}{7}} \text{ per ton}$$

2. A timber tract yields an annual operating profit of Rs 10000. It is estimated that at the present rate of depletion, the asset will be exhausted in 8 years' time. The sinking fund accumulates at 4% p. a., and the proprietors get a dividend of 6% p.a. Find the capitalized value of the tract.

$$\text{Value} = \text{Rs} \frac{10000}{\frac{1}{s_{\overline{8|}}} + \cdot 06} = \underline{\text{Rs } 59337.}$$

*EXAMPLES 34

1. A tract of timber is valued at Rs 20000 and the footage is estimated at 15 million ft. Calculate the depletion charge for the year if the quantity removed during it is $1\frac{1}{4}$ million ft. [*Ans.* Rs 1667

2. Obtain the value of a coal mine which yields average annual profits amounting to Rs 21000 and is expected to be exhausted in 30 years' time. The dividend paid to the shareholders is 6% p.a. and the sinking fund is accumulated at 4% p.a. [*Ans.* Rs 269820

3. A stone quarry is estimated to yield 36 million cubic yards of gravel, and yield an average annual profit of Rs 14000. If dividends stand at the 6% level and the sinking fund accumulates at 5% p.a., what is the value of the rights, if the average monthly quantity of gravel quarried is 100000 c. yd. ? [*Ans.* Rs 186540

4. A machinery purchased for Rs 60000 is estimated to realise Rs 10000 after 15 years. If money is taken to earn 3% p.a. compound, find the equated value of the amount of depreciation spread over the period. [*Ans.* Rs 4488

5. Assuming that gold ornaments lose 12 per mille per annum in weight by continued use, what will be the estimated weight of an ornament weighing 1783 grains at the end of 10 years. ? [*Ans.* 1580 grains

6. A person using a motor car desires to set apart every month sufficient money to cover the cost of replacement of the car at the end of 6 years. If the original value of the car be Rs 4500 and its scrap value is estimated to be Rs 500, find the amount thus set aside monthly, assuming that interest may be earned at 2% p.a. on annual balances. [*Ans.* Rs 60 6as

7. The life of a hotel piano is taken to be 6 years. It cost Rs 3750 and is estimated to have a scrap value of Rs 250. Show the annual depreciation charge by the Sum of Digits method.

8. An electrical advertising sign is erected at a cost of Rs 8000. It is estimated to have a life of 3 years, when the lamps costing Rs 6000 will be scrapped. Write down the annual depreciation charges by the Sinking Fund Method, calculating interest at 3% p.a.

§ **Valuation of Shares.** It is always difficult to make a correct estimate of the intrinsic value of shares. The following illustration should be suggestive.

Balance Sheet of Premier Construction Co. Ltd.
as at 31st Decr. 1944

Liabilities		Rs	*Assets*	Rs
Capital 5000 Ord. shares			Building at cost	1,00,000
@ 100 fully paid up		5,00,000	Furniture " "	10,000
			Stock at market value	2,00,000
Reserve Fund		1,05,000		
Depreciation Funds :			Investments (cost)	1,50,000
Buildings	40,000		Sundry Debtors	3,50,000
Investments	15,000	55,000	Cash in hand	10,000
Sundry Creditors		50,000	Cash at Bank	60,000
Reserve for Bad Debts		20,000		
P & L a/c				
Bal. of 1943...	30,000			
Current year...	1,20,000	1,50,000		
		8,80,000		8,80,000

The following assumptions are made :—

(1) The book debts are all good.

(2) Prospects for the following year are good.

(3) Present valuation of buildings is Rs 2,50,000

(4) Other similar companies show a Profit of 10% on the current value of their shares.

Following are two alternative methods :

1. On the basis of yield.

		Rs
Book value of assets...		8,80,000
Less Non-trading Investments...		1,50,000
		7,30,000
Deduct—		
Deprcn. Bldgs...	40,000	
B/D Reserve...	20,000	60,000
Effective Capital		6,70,000
Earned Profit		1,20,000
Earning Capacity p.a.		18% app.
Similar shares yield p.a., say,		10%
Value on yield basis		
$= \text{Rs } 100 \times \frac{18}{10}$		Rs 180

2. On the basis of Capital Valuation.

		Rs
Book value of assets		8,80,000
Add Appreciation in value of Bldgs.		1,50,000
		10,30,000
Deduct—		
Deprcn. Investment	15,000	
B/D Reserve	20,000	35,000
Effective value of Assets		9,95,000
Less Creditors		50,000
Net assets available to Shareholders on break-up		9,45,000

∴ Break-up Value of each of 5000 shares Rs 189

SECTION FOURTEEN

FOREIGN EXCHANGE

§ The relationship between the monetary units of two countries is known as the rate of exchange. Thus, if value equivalent to 1 Rupee can be purchased in London for 1s $6\frac{31}{32}$d, the relationship is called a rate of exchange.

It is customary to relate the currency of a country to a certain definite weight of gold of a statutory fineness, on the basis of which is established the standard rate of exchange. This is called the *mint par of exchange.* In the present disorganized state of world finances, the pars of exchange have mostly been suspended. A British Sovereign* used to be made of gold $\frac{11}{12}$-ths fine (22 carat) and weigh 123·27447 grains. An Indian Rupee weighs 180 grains and contains 90 grains of pure silver. The gold coins of U.S.A., France, Germany and most other places used to be made of gold $\frac{9}{10}$-ths fine, before the War.

The course of exchange between two countries is determined by conditions of their parity values taken with the demand and supply of bills and remittances. The Indian Rupee is statutorily pegged to the British pound sterling—the par being Re 1 = 1s 6d —and accordingly follows the fluctuations thereof in exchange markets.

§ Direct rates of exchanges between financial centres are usually quoted on the money markets of different countries. The rate of exchange between two foreign countries is called a *Cross Rate.* It may sometimes be advantageous to make remittances through a third money market. Thus the direct rate between Calcutta and Paris on the 26th January 1937 was Rs 100 = 785 frs ; and on the same date Re 1 could be converted into 1s 6 $\frac{3}{32}$d. while £1 could buy 105·13 francs. If, then, Rs 100 were remitted on that date to Paris through London, it would buy, say $F = \frac{100 \times 18\frac{3}{32} \times 105{\cdot}13}{240}$ francs, neglecting incidental charges.

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* The value of a sovereign containing 113·0016 grs of pure gold may be obtained from the price per tola of pure gold, thus : $\underline{2}+\underline{5}+\underline{8}-\underline{6}$ app.

Now, $\log F = \log 579 + \log 10513 - \log 32 - \log 240$

$$\begin{array}{rll}
\log 579 = 2{\cdot}762679 & & \\
\log 10513 = \underline{4{\cdot}021707} & & 6{\cdot}784386 \\
\log 32 = 1{\cdot}505150 & & \\
\log 240 = \underline{2{\cdot}380211} & & \underline{3{\cdot}885361} \\
\text{Diffce.} & & 2{\cdot}899025 \\
& \text{Antilog} = 792{\cdot}55 & \\
\therefore & \text{Rs } 100 = \underline{792{\cdot}55 \text{ francs}} \text{ through London.} &
\end{array}$$

This is called *arbitration of exchange*: and the arbitrated value was Rs 100 = 792·55 frs, although on the same date the direct rate was Rs 100 = 785·00 frs. It is thus easily seen that it was cheaper to make a remittance through London to Paris than direct to Paris from Calcutta.

§ When more than one country is involved in the operation, the method is known as *compound arbitration.* The calculation is effected by means of the Chain Rule.

Illustration

Find the arbitrated exchange between Calcutta and Berlin from the following data, arranged in the form of a Chain.

$$\begin{array}{llll}
\text{Re } 1 & 18\text{d} & & \\
& 240\text{d} & 110 \text{ frs.} & \\
& & 100\ \text{''} & 12{\cdot}0 \text{ M}
\end{array}$$

$$\text{Then,} \qquad \text{Rs } 100 = \frac{18 \times 110 \times 12{\cdot}0}{240 \times 100} \times 100 \text{ M}$$

$$= \underline{99 \text{ M}}$$

In practice, the rate is obtained after allowing for stamp and other charges.

§ **Calculation of Rupee-Sterling Exchange.** Exchange Tables are used for purposes of conversion of one currency into another. A number of useful Tables have been appended for reference and study.

The present par value is Re 1 = 1s 6d ; and normally the rate should be fluctuating about this value. At this rate £1 = Rs 13⅓, and may be taken as Rs 10 + $\underline{3}$ for quick calculations with decimalized values. Also, Re 1 = £$\frac{3}{40}$, therefore, to convert Rupees into £, point off 1 place and deduct $\underline{4}$; convert into shillings and pence.

§ A day's exchange quotations may be studied with profit. The following points, among others, may be noted.

1. Currencies of foreign countries are quoted for a unit of the standard currency of the country of quotation.[*] In India, British exchange is quoted at so many pence and fractions for a rupee ; and the other centres are quoted at so much for Rs 100. In London and New York almost all financial centres of the globe are quoted for the £ sterling and the Dollar respectively. The rate on London in New York, however, is quoted at so many dollars to £1.

2. Remittances to foreign centres are made ordinarily by Telegraphic Transfers (T. T.'s) ; by Mail Transfers, Cheques and Demand Drafts ; by Time Bills, including *fine* trade bills ; for which different rates are quoted. The last named are bills arising out of genuine trade transactions between respectable parties.

3. When instructions are sent by a remitter's bank through his foreign agent to pay a specified sum to a stated person by cable, the transfer of funds is said to be by means of a T. T. This is the speediest, although the most costly way of settling payment. The rate sometimes includes the cost of cablegram, which is otherwise charged to the customer.

Cheques or Demand Drafts are issued by banks, against payment of the equivalent amount in local currency, and forwarded to the payee by mail. Foreign Mail Transfers may also be arranged directly through banks. The rate for such transfers is usually calculated from the T. T. rate by allowing off or on it interest for the time which must elapse between the date of the cheque and the date of payment. These are commonly known as "short bills".

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[*] A rate is favourable to a country whose currency has a larger purchasing power in terms of the foreign currency.

4. Trade Bills (Hundis in Indian inland trade) are bills of exchange arising out of trade transactions. When the parties to the bills are of good financial reputation, they are called *fine* bills. These are also used in the settlement of trade transactions between other parties, and are frequently guaranteed by banks. These are usually known as "long bills", and the tenure of the bills is known as "usance".

5. Another rate is also quoted in the Exchange Market known as the Bills Collection Rate (B. C. rate). It is an agreed rate for the day at which the exchange banks collect their dues on foreign bills between themselves.

6. Two rates are usually quoted for each class—the Buying and the Selling Rates for banks. For obvious reasons, the Bank gets more of foreign currency when buying than it is prepared to sell.

Thus, when a banker sells a Draft, he *adds* the interest on the T. T. rate at the rate prevailing in the relative *foreign* centre; and when he buys a cheque, he *deducts* interest at the same rate. Usually, an allowance of $\frac{1}{2}$ per mille for stamps, and an equal sum for contingencies; is also added to or deducted from the cheque quotation to arrive at a "*long*" quotation.

7. Exchange rates move by *points*, which are the lowest fractions of currency used commercially for this purpose. Thus, when the London exchange changes from 1s $6\frac{3}{32}$d to 1s $6\frac{5}{32}$d to Re 1; it is said to move 2 *points*, Indian quotations being made correct to the nearest 32-nd of a penny. The London-New York Cross Rate moves by points which are multiples of 32-nds of a cent.

§ Dealings in foreign exchange are made through Bill Brokers. When a bank sells a transfer, it usually covers itself by buying an equal amount of foreign currency; and when it buys a Bill *i.e.* foreign currency, it should cover itself by selling transfer of an equivalent amount. The basic rate ordinarily is the T. T. rate, and account has always to be taken of stamp duties of $\frac{1}{2}$‰ and usually incidental charges of a like amount. Account has also to be taken of the interest for the time taken for the transit of advice by mail.

§ The calculation of Long Rates between two financial centres is based on the T. T. rate between them. If a Calcutta banker is buying long bills on London, he ascertains how many pennies he gets in exchange for each rupee by T. T. To that he would *add* the interest that he might earn in London during the tenure of the bill and any allowance that he may choose for risk, stamps, etc. On the other hand, if a long bill is presented before a London banker, he would be prepared to pay for it a smaller amount than he could obtain by T. T. He would, therefore, *deduct* the relevant charges.

§ To find the value of a time bill on the basis of a demand rate, interest for the tenure and grace period at the current rate, along with stamp and incidental charges, has to be taken into account. If the quotation be *in a foreign centre on London*, these have to be *deducted*, for the foreign dealer will be prepared to pay less for £1 on a time bill than on a demand bill. If, however, the quotation be *on a foreign centre in London*, the London dealer will expect more foreign currency for a time bill than on a demand bill. He will *add* interest, stamp and other charges. The banker's formula is "Buy high, sell low."

Illustrations

1. The market quotation for New York in London is \$ 4·86$\frac{11}{16}$ – $\frac{13}{16}$ (T. T.). If the dealer desires a profit of $\frac{1}{16}$ cent, find how much he will charge a customer for transfer of \$ 100.

Dealer buys in the market @ 4·86$\frac{11}{16}$ dollars (Banks' Selling.); and retaining his margin of $\frac{1}{16}$ cent, he will sell @ 4·86$\frac{10}{16}$ dollars for every £1. ∴ Value of transfer = £20 11s.

2. A cheque for Rs 10000 on Calcutta is bought by a London Bank at 1s 6¼d. He sells a T. T. for the same amount @ 1s 6⅜d, 3 weeks later. Assuming that it takes 3 weeks for London Mails to be delivered in Calcutta, find his profit on the transaction.

Cost of cheque = £760 8s 4d

Value of T. T. = £765 12s 6d

Profit = £5 4s 2d in 3 weeks

$$\text{Profit per cent} = \frac{£5\ 4s\ 2d}{£760\ 8s\ 4d} \times \frac{365}{21} \times 100 \text{ per annum}$$

$$= \underline{11·9\%\ \text{p.a.}}$$

Note. More simply, the Bank earns ¼d for every 18¼d of its investment in 21 days.

$$\therefore \quad \text{His profit } \% = \frac{\frac{1}{4}}{18\frac{1}{4}} \times \frac{365}{21} \times 100 = 11·9\% \text{p. a.}$$

3. The T. T. rate Kobe-London is 2s 1¾d per yen. Find the demand rate when the London rate of discount is 3½%, assuming that Japanese Mails take 45 days to reach London.

T.T. rate	2s 1¾d
Add Intt. for 45 d/s @ 3½%	⅛d
∴ Demand Rate	2s 1⅞d

For every yen the dealer gets an immediate credit of 2s 1¾d in London. The payment on his Bill will fall due in London after 45 days when it reaches there by mail. Meantime, the amount at his credit earns interest *at the London rate* for this interval. He is therefore in a position to provide the added value 2s 1⅞d on Demand in London to every yen received.

4. The rate for cheques on London is \$ 4·865 – \$ 4·875 ; and the discount rate in London is 5%. Find the rate at which a New York banker will be prepared to buy a 3 months' bill on London. Provide for Stamps @ ½ per mille and Contingencies of an equal amount.

Cheque rate (buying) ···		\$ 4·865
Less Discount @ 5% for 3 months	\$ ·06081	
Stamp @ ½ p. mille	·00243	
Contingencies @ ½ p. mille	·00243	·06567
3 months' rate		\$ 4·79933
∴ 3 months' rate (commercial)···		\$ 4·79$\frac{15}{16}$.

5. **A London merchant has to pay 3000 francs in Paris at the end of 3 months. If the T.T. rates are 125·15 – 25, and discount rate in Paris is 5%, find the cost of a 3 months' draft, assuming that his banker makes a profit of 1 per mille. (Stamp 3d)**

The bank covers his Draft by transferring to Paris sufficient amount by T.T. @ 125·15 francs.

T. T. (selling) rate is	125·15 fr.
Less Bank's margin @ 1‰	·125
	125·025
Plus 3 months' interest @ 5%	1·563
Bank's rate	126·588 fr.
Cost of 3000 frs. @ 126·588...£23	14s
Add stamp	3d
Cost of remittance	£23 14s 3d

6. The day's rate in Calcutta on London is 1s $5\frac{31}{32}$d – 1s $6\frac{1}{32}$d for sight drafts. If the banker desires to make a profit of 1‰, find how much he will charge his customer for a remittance of £1000 by a sight draft. (Neglect charges)

Bank covers by buying @ 1s $6\frac{1}{32}$d	...	18·03125d
Less Profit @ 1‰	...	·01803
∴ Bank's rate for customer	...	18·01322d
i.e., to the next *lower* 32-nd of 1d		18d

∴ Banker charges @ 18d to Re 1

Rs 1000 × $\frac{240}{18}$ = Rs 13333 5as 3p

7. The demand rate of exchange (Banks' buying) on a certain day in New York is £1 = 4·48$\frac{3}{8}$ dollars. Discount rate in New York is 4% p. a.; Stamp Charges are @ ½ per mille and Commission is at ¼%. Find the 60-days' rate.

Demand rate		$ 4·48375
Deduct :		
Intt. @ 4% for (60 d/s + 3 d/s grace)	$ ·03095	
Stamp @ ½‰	·00224	
Commission @ ¼%	·01121	·04440
∴ 60-days rate		$ 4·43935

i.e. £1 = $ 4·43$\frac{15}{16}$

8. Calculate 3 months Bank's buying rate in London from the following data :

T. T. London—Calcutta	1s 6d	Buying rate
T. T. Calcutta—London	1s $6\frac{1}{32}$d	" "
Discount rate in London	$1\frac{1}{2}\%$	
Ditto in Calcutta	$1\frac{3}{4}\%$	

Allowance for risk and stamp is to be made at $\frac{1}{16}$d in the rate.

T. T. London—Calcutta Bank's buying rate 1s 6d	18 pence
Deduct 3 mos. intt. @ $1\frac{1}{2}\%$ …	·0675
Allowance for risk & stamp	·0625

London buying rate for 3 mos. Bills on Calcutta 17·87d

$= 1\text{s } 5\frac{27·8}{32}\text{d} = \underline{1\text{s } 5\frac{27}{32}\text{d}}$ approximated in favour of the banker.

*9. Calculate the three months' rate for buying bills in London and in Calcutta from the following data :

T. T. London—Calcutta …	1s 6d	Buying
T. T. Calcutta—London	1s $6\frac{1}{16}$d	"
Discount rate for 3 mos.		
Commercial Bills in London	$1\frac{1}{2}\%$	
Ditto in Calcutta	2 %	
Allowance for risk and stamps	$\frac{1}{16}$d in rate.	

(i) London Bank may buy T. T.	18d
Deduct Intt. *in Calcutta* 3 mos. @ 2%	·09
Allowance for risk &c.	·0625
	17·8475d

Bank's Buying rate $= 1\text{s } 5\frac{27·1}{32}\text{d}$

$= \underline{1\text{s } 5\frac{27}{32}\text{d}}$ approximated in favour of the banker.

(ii) Calcutta Bank may buy T. T.	18·0625d
Add Intt. *in London*	
3 mos. @ 1½%	·0677
Allowance	·0625
	18·1927d

Bank's buying rate $= 1\text{s } 6\frac{6{\cdot}2}{32}\text{d}$

i.e., = 1s 6$\frac{7}{32}$d approximated in favour of the banker.

***10.** Bar gold in London costs £8 8s per fine oz., which can be sold in New York at 35 Dollars. Freight charges are at $\frac{3}{20}$%, insurance is at ½‰, packing and trucking charges in London is at ⅛‰. If the time of transit between London and New York is 10 days and interest is @ 2½% p. a., find the gold export point.

Cost of 1 oz. fine gold ...		£8·40000
Charges ...	%	
Freight $\frac{3}{20}$% ...	·15	
Insurance ½‰ ...	·05	
Packing &c. ⅛‰ ...	·0125	
Intt. 10 days @ 2½%	·0685	
	·2810	
@ ·281% on £8·4		·02360
Cost per fine oz. in N. Y.		£8·42360
Selling Price thereof		$ 35·00

∴ Gold point, London—New York

$$£1 = \$\frac{35}{8{\cdot}4236} = \underline{\$4{\cdot}15\tfrac{1}{2}}.$$

***11.** On the 13th May 1940 a trader in London hands over the following bills to his banker for discount and credit of the proceeds to his account. The following are the day's market rates of discount :

60 d/s Bank Bills	1 %
3 m/s ditto	1⅛%
3 m/s Fine Trade Bills	2½%
6 m/s ditto	3 %

£150 accepted	13th March	@	60 days	by Lloyds Bank
£150 "	15th April	@	3 m/s	by Chartered Bank
£200 "	14th March	@	3 m/s	by Reliance Dealers, Ltd.
£150 "	18th March	@	6 m/s	by Indian Traders Syndicate

Ascertain the amount placed by the Bank to the trader's credit after charging for stamps @ $\frac{1}{2}$ per mille.

Zero-date 13. 5. 40.

Bill	Due Date	Period d/s	Discount %	Product (Bill × Period × Disct.)
£150	15.5	2	1	300
£150	18.7	66	$1\frac{1}{8}$	11137·5
£200	17.6	35	$2\frac{1}{2}$	17500
£150	21.9	131	3	58950
£650				87887·5

∴ Discount = Intt. @ 1% for 1 day on £87887·5 =	£2	8s	2d
Stamps		6s	6d
	£2	14s	8d
∴ Amount credited =	£647	5s	4d

*12. A merchant in Calcutta has on hand the following bills: £100 payable after 3 mos. accepted by Lloyds Bank; £200 payable after 2 mos. accepted by Barclays Bank; £100 payable after 4 mos. accepted by Acme Farmers, Ltd. and another for £400 payable after 3 mos. accepted by Sussex Granaries, Ltd. He desires to exchange these for a remittance of £800 to London by Telegraphic Transfer. Find how much more, if at all, he has to pay to his banker for the remittance. The day's quotations are:

		Banks Selling	Banks Buying
T. T. or O. D. on London		1s $5\frac{31}{32}$d	1s $6\frac{1}{32}$d
At sight on London (each month)		$\frac{1}{32}$d	1s $6\frac{3}{32}$d
Commercial Bills	3 mos.		1s $6\frac{7}{32}$d
Ditto	4 mos.	$\frac{1}{16}$d higher	1s $6\frac{9}{32}$d
Ditto	6 mos.	—	—

The Bank will credit merchant's account thus :

£100	...	@ 1s $6\frac{3}{32}$d		
£200	...	@ 1s $6\frac{3}{32}$d	...	Rs 3979 4as 6p
£100	...	@ 1s $6\frac{9}{32}$d	...	1312 13 3
£400	...	@ 1s $6\frac{7}{32}$d	...	5269 4 9
				Rs 10561 6as 6p

And it will debit his account with

£800	...	@ 1s $5\frac{31}{32}$	...	10685 3 6

Difference realisable from merchant Rs 124 13as

***13.** From the details below ascertain the amount a London banker would be prepared to pay for a sight draft on New York for \$ 1000, calculating profit @ $\frac{1}{4}$%.

T. T. London—New York	...	\$ 4·02$\frac{1}{2}$ – $\frac{5}{8}$
Rate of interest in London	...	2% p. a.
Ditto. in N. Y.	...	2$\frac{1}{2}$% p. a.
Mailing period : London—N. Y.		10 days

The banker may cover by selling T. T.

@ 4·02$\frac{5}{8}$ *i.e.*,		\$ 4·02625
add Intt. @ 2$\frac{1}{2}$% for 10 d/s ...		·00280
& Profit @ $\frac{1}{4}$%	...	·01007
		\$ 4·03912

∴ Banker quotes his customer the rate \$ 4·02$\frac{29}{32}$ to the nearest 32-nd *above* ; and pays $£\frac{1000}{4·03\frac{29}{32}} = £\frac{1000}{4·0390625}$

= £272 6s 10d

14. Find the arbitrated rate between Calcutta and Paris from the data below :

T. T. Calcutta—London	1s $6\frac{3}{32}$ – 1s $6\frac{1}{32}$d
T. T. London—Paris	178$\frac{1}{2}$ fr. – 176$\frac{1}{2}$ fr.
Brokerage at each centre	1‰
Incidental Expenses "	1‰

Applying the Chain Rule,

$$\begin{array}{lll} \text{Re} & \text{d} & \text{fr.} \\ 1 & 18\tfrac{1}{32} \text{ (selling rate)} & \\ & 240 & 176\tfrac{1}{2} \text{ (selling rate)} \end{array}$$

$$\text{Rs } 100 = 100 \times \frac{18\frac{1}{32} \times 176\frac{1}{2}}{240} \text{ francs}$$

$$= \frac{318251{\cdot}56}{240} \text{ fr.} = \underline{132{\cdot}6048} \text{ francs}$$

$$\begin{array}{lr} \text{Allow 2 brokerages London \& Paris} & {\cdot}2652 \\ \text{Incidental Expenses} & \underline{{\cdot}1326} \\ & \underline{\underline{132{\cdot}2070}} \text{ francs} \end{array}$$

15. The Bombay—New York T. T. rate is Rs 333½ – 330. The T. T. rates in London on the same date are 1s 6d to Re 1 and \$ 4·02½ – \$ 4·03½ to £1. Find the arbitrated reciprocal rate. Brokerage, etc. at each centre 1‰ and incidental expenses 1‰.

To find Arbitrated reciprocal rate,

$$\begin{array}{ccc} \$ & \text{d} & \text{Re} \\ 4{\cdot}03\tfrac{1}{2} & 240 & \\ & 18 & 1 \end{array}$$

$$\$\ 100 = \text{Rs } 100 \times \frac{240}{18 \times 4{\cdot}03\frac{1}{2}}$$

$$= \text{Rs} \frac{24000}{72{\cdot}57}$$

$$= \underline{\text{Rs } 330{\cdot}071}$$

$$\begin{array}{lr} \textit{Deduct } \text{2 Bkges. @ 1‰} & {\cdot}660 \\ \text{Incidental Exp.} & \underline{{\cdot}330} \\ \text{Arbitrated rate \$ 100} = & \underline{\underline{\text{Rs } 329{\cdot}081}} \end{array}$$

EXAMPLES 35

1. Obtain arbitrated exchanges from the following data :

   (i) London-Paris—108·00 fr. ; London-Calcutta—1s 6½d
   Find Calcutta-Paris rate. [*Ans.* Rs 100 = 832·50 fr.

   (ii) London-Berlin—12·36 M ; London-Calcutta—1s 6$\frac{3}{32}$d
   Find Calcutta-Berlin rate. [*Ans.* Rs 100 = 93·18 M.

(iii) London-Kobe—1s $1\frac{31}{32}$d ; London-Calcutta—1s $6\frac{5}{32}$d
Find Calcutta-Kobe rate. [*Ans.* Rs 100 = 129'98 yen

(iv) London-New York-$ $4'92\frac{13}{16}$; London-Calcutta-1s $5\frac{29}{32}$d
New York-Paris—22'80 fr. Find Calcutta-Paris rate.
[*Ans.* Rs 100 = 923'46 fr.

(v) London-Paris 92'80 fr ; London-New York $ 4'25¼
Find Paris—New York rate. [*Ans.* 4'58 cents per franc.

2. A firm of exporters of cotton to Japan agreed to receive payment in Japan at the fixed rate of Rs 100 = 134'26 yen. Determine the value of the bills drawn by them on the following sales :

	Yen
1st shipment......valued at Rs 13648 4as	[*Ans.* 18324'14
2nd " ...... " " " 18523 14as 9p	[*Ans.* 24870'21
3rd " ...... " " " 24932 13as 6p	[*Ans.* 33474'84

3. A merchant is New York buys goods in Geneva to the value of 4004 francs. Find the value of the goods in dollars, when the rate of exchange between London and New York is $ 4'865 to the £, and between London and Geneva 25'48 francs to the £.
—I. Com. [*Ans.* $ 764'50

4. The rate of exchange between Calcutta and Yokohama was Re 1 9½as per yen, and between London and Calcutta was 1s 5½d per rupee ; between Yokohama and New York $ 0'52 per yen. Calculate the arbitrated rate between London and New York.
—I. Com. [*Ans.* £1 = $ $4'51\frac{29}{32}$

5. A man received in 146 Dollars and gave English money at the rate of £1 for $5. He also received £38 17s 6d and gave U. S. money at the rate of £4'75 for £1. Find his gain % in each case, and calculate his total gain assuming the par of exchange to be £1 = $4'867.—I. Com. [*Ans.* 2'73% ; 2'46% ; $8'42

6. A man wishes to remit $ 15000 to New York. He can obtain a cable transfer at Rs 2'83 per dollar or a cheque at $0'36 per rupee. Which cost will be more, by what way and why ?
—I. com. [*Ans.* Cable transfer will cost Rs 783 5as 4p more.

7. A broker wishing to buy a bill for 25000 fl. drawn on Rotterdam borrowed £2500 from a bank for a week at 3¼%. He bought the bill at 10·53 exchange, sold it a week later at 10·515, and repaid the loan. What did he gain ? —I. Com.

[*Ans.* £1 16s 5½d

8. A man gave 25 twenty-franc pieces in exchange for 20 sovereigns. Calculate his gain in English money and in French money. What was his percentage gain ? —I. Com.

[*Ans.* 3s 6d ; 4·40 fr. ; ·88%

[*Hint.* Assume the old par £1 = 25·22 fr.]

9. I buy in London £1000 worth of Berlin bills at 20·53 RM to the £ and transmit them to Paris to be sold at 100 RM for 625·5 francs. In return I receive bills of two months' date at 125·22 francs to the £. Is the operation profitable ?—I. Com.

[*Ans.* Profit £15 16s 0½d

10. Is it profitable for a Bombay merchant to buy a Paris Hundi in order to pay a debt of 10000 francs, when the franc equals 10as 6p ; or to remit it through London, the course of exchange being 1s 3d for Re 1 and 25 fr. for £1 ? —I. Com.

[*Ans.* Profit Rs 162 8as

11. A man holds a Paris draft for 174000 francs which was drawn on 3rd March for 3 months. How much, in English money, will he receive for it on 1st May, if a banker discounts it at 3½% p. a. ? The rate of exchange is to be taken at £1 = 124·1 fr. —I. Com. [*Ans.* £1397 5s 1d

12. If the cheque exchange in London on Paris is 138·5 fr. to the £ and the rate of discount for a 3 months' bill on London is 4¾% p. a., what debt in Paris can be discharged by a person in London who holds a 3 months' bill on London for £750 ?

—I. Com. [*Ans.* 102641·48 fr.

13. If the exchange in Glasgow on Paris be 124·21, and the rate of banker's discount in Glasgow be 4% p. a., what debt can be discharged in Paris by a person in Glasgow who holds a three months' bill in Glasgow for £1000 ?—I. Com. [*Ans.* 122,967·90 fr.

14. Which is the more advantageous of the following methods which a London merchant might adopt to settle a debt in Paris? (a) Buying French 4 months' bills at 25·76 (b) Buying Vienna Bills at 24·30 and selling these in Paris at 1·05 (Brokerage 1%. and discount 3% p.a.) —I. Com. [*Ans.* First

15. Allowing for stamp @ ½ per mille and commission and contingencics at ½% at each foreign centre, calculate Calcutta-Paris rate from the following: Re 1 = 1s 6d; £1 = $4·80 and $1 = 48·00 fr. [*Ans.* Re 1 = 17·09 fr.

***16.** On 31st January 1920, London exchange quotations being, Copenhagen...22·03 – 22·07 and Berlin...290—297, at what rates would you have issued drafts so as to allow your bank a gross profit of ⅛% on both places? Rates to be quoted to the nearest manageable fraction, Copenhagen—nearest ¼ ore; Berlin —nearest 10 pfennige. (London Institute of Bankers)

[*Ans.* 22·00¼ kr; 289·6 M.

***17.** Gold in London is quoted in shillings and pence per ounce fine. In 1923, the price was called the American Parity price, *i.e.*, it was based on the rate of exchange for £1 gold in New York. Calculate the price of gold in London with exchange $ 4·68¼ ($1 = 23·22 grains gold; 1 oz = 480 grains gold).

(Lond. Institute of Bankers) [*Ans.* £4 8s 3d per oz.

***18.** The U. S. A. Dollar has been reduced in its gold content by 59·06% of its original value of 25·8 grains $\frac{9}{10}$ths fine. Calculate the premium in London price of fine gold at 146s per ounce, on the basis of the American parity £1 gold = $ 4·80.

[*Ans.* Parity price per fine oz. = 145s 10d. Premium = 2d.

***19.** From the following data calculate what rate should be given in New York for a 60 days' commercial bill on London for £120·155. Demand rate $ 4·68½, London Bank rate 3%, Stamp duty ½ per mille. (Allow 3 days' grace).

(London Institute of Bankers). [*Ans.* $ 4·65$\frac{27}{32}$

***20.** A customer offers you a sight draft drawn on a New York bank and asks for a draft on Milan for Lire 124774·8 in exchange. The rates current are : New York T.T. 4·85⅛, New York Cheque 4·85½ − ¾, Italy cheque 121½ − ¾. Calculate how many dollars you would require. Allow a profit for yourself of ⅛ cent in the New York rate and ¼ lira in the Italian rate. (Lond. Institute of Bankers). [*Ans.* Dollar draft = $ 5000

***21.** A day's quotation of selling rates in Calcutta on London is T. T. 1s 6¼d and Demand Draft is 1s 6$\frac{5}{16}$d. Allowing for a mailing period of 20 days, calculate the rate of interest represented by the "spread" (difference) between the two rates. [*Ans.* 6¼%

SECTION FIFTEEN

INDIAN INCOME TAX

§ Income tax is a tax on income. It has no direct reference to a person's savings, his investment or capital ; it is assessed on the income that these may earn.

All incomes, other than capital income (*e.g.* insurance money, legacies, lottery money, etc.), are chargeable to the tax, whatever be their source and wherever they may have been earned. In the case of business firms, taxable profits are ascertained on the principle that all expenses incurred for earning the income, and all losses actually borne by them in the normal course of business are chargeable to profits.

A loss incurred by an assessee in a year under any head may be set off against income under any other head in that year. It may also be carried forward, up to the sixth year following, to be set off against income under that head only.

Incomes of local bodies, charitable and religious institutions, etc. are exempted from the tax. Agricultural incomes are charged to a separate Agricultural Income Tax.

§ The Schedule of Indian Income Tax recognizes two different groups of assessees :

(i) an individual, an undivided Hindu Family, an association, a member of a registered firm, or an unregistered firm ; and

(ii) a Joint Stock Company and a registered firm.

There is a graduated scale of rates of Income and Super Tax applicable to the first group, and a fixed rate of 30 pies and three annas in the rupee as Income and Super Tax respectively on the taxable profit of a company, irrespective of its amount. There is at present also a *surcharge* on both Income and Super Tax as a War emergency measure.

§ Income Tax and Super Tax are deducted at source, before paying out salaries and dividing profits of unregistered firms, at rates applying to such incomes only. On dividends of Companies, and interest on Government and other securities and debentures, and profits of registered firms Income Tax only is deducted at the rate of 30 pies in the rupee plus the surcharge. The Super Tax that is payable by a company does not form part of the dividend declared and is not deducted therefrom. No Super Tax is chargeable to registered firms ; although it has to be separately assessed in the hands of the members.

§ The heads of Chargeable Income for purposes of Income and Super Tax are : (i) Salaries ; (ii) Interest on Securities ; (iii) Income from Property ; (iv) Profits and Gains of Business, Profession or Vocation ; and (v) Income from other sources.

Incomes under the several heads have to be shown by every assessee on the Annual Return. The object is to ascertain correctly the total chargeable "world" income and determine the correct amount of Income and Super Tax payable. Against the total sum thus payable is set off the actual amount of tax that may have been already recovered at source. If more has been recovered, the balance is refunded to the assessee. No part of a partner's share of the tax paid out of the profits of an unregistered firm, however, can be claimed as refund.

§ Without going into the details of filling up a Return, the following points may be usefully noted :

(i) Income from any assets transferred to his wife and minor children is to be treated as addition to a person's income.

(ii) Employer's contribution to an assessee's Provident Fund, and the interest accruing to the credit of an assessee's Provident Fund account, perquisites in addition to salary, etc. are to be shown as his income under Salaries.

(iii) The annual value of property, subject to certain deductions, or the rent received from such property, is to be shown under the head Property.

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[*] The Finance Act of 1945 makes a distinction between Earned and Unearned Income ; and provides for exemption of $\frac{1}{10}$th of the Earned Income (up to a limit of Rs 2000) from the incidence of Income Tax only. The exemption will not apply to incomes of companies or in respect of property or interest on securities. This provision, however, could not be taken account of in the following illustrations and examples.

In the case of an individual, the annual value of any property owned and occupied by him is to be shown at not more than 10% of the total gross income. [That is, ⅑th of the gross income from all other sources.]

(iv) Interest on a Tax-free security should be shown separately under the head Interest on Securities.

(v) The gross share of profit of a partner in an unregistered firm has to be shown under the head profession, etc. The relative income tax should not be shown under the head Taxed at Source.

(vi) Dividends received from companies free of tax have to be "grossed"[*] before inclusion under the head Other Sources. That is to say the corresponding gross income has to be ascertained and entered in the Return, which should also show the tax deducted at source.

§ The total gross income, which should include income derived from investments, etc. in other parts of the world, is chargeable to Income and Super Tax without deduction, in accordance with the rates shown in the Schedule.

While no relief is admissible on the total world income in respect of Super Tax, some relief is allowed by way of a rebate calculated at the *average rate of income tax* (with surcharge) on certain specified portions of the income and certain other specified expenses indicated below.

The average rate of income tax is ascertained by dividing the total income tax calculated on the gross Chargeable Income, by that Income.

The calculation both of Income Tax and the average rate of income tax involves certain complications, and are best looked up from the Income Tax Tables published by the Government of India.

The relief thus offered is now adjusted, along with taxes deducted at source, with the tax payable by an assessee. The amount of the relief is calculated on the total of the following items :—

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[*] At the present rate of 30 pies plus surcharge (24 pies), the "grossing" may be effected by multiplying net receipt by $1\frac{9}{23}$.

(*a*) Interest on tax-free securities :

(*b*) Income from an unregistered firm as share of profit ;

(*c*) Amount spent on provision for the assessee in his old age, or for the members of his family, in the shape of insurance premiums and contributions to recognized Provident Funds, etc.

The total amount under (*c*) shall not exceed $\frac{1}{6}$th of the assessee's total Chargeable Income or Rs 6000 (Rs 12000 in the case of a Hindu Undivided Family) whichever is less. No rebate is allowed for the assessment of Super Tax.

§ The Schedule of Rates shows that the first Rs 1500 of the Chargeable Income of an assessee of the first group is not chargeable to Income Tax, as the first Rs 25000 thereof is not chargeable to Super Tax. No tax is payable by a person with an income below Rs 2000 a year. There is also a marginal relief on incomes just above Rs 2000 by way of restricting the maximum tax payable by an assessee to half the amount of the excess of his income over Rs 2000.

§ A number of deductions are allowed off the gross income from property on the principle that reasonable expenses incurred solely for the purpose of earning the income reduces the available income. These are—

(i) $\frac{1}{6}$-th of the rateable value or rental for repairs ;

(ii) interest on money borrowed to finance acquisition or maintenance of the property ;

(iii) land revenue, ground rent, cost of fire insurance premium ;

(iv) collection charges, not exceeding 6% of the value ;

(v) proportionate allowances for vacancy.

§ Adjustments are necessary for ascertaining taxable profits from the Profit and Loss account in accordance with certain directions laid down in the Income Tax Manual. Capital expenses, provision for reserves, drawings, salaries, commission, etc. of partners, etc. which are ordinarily debited to the profit and loss and appropriation accounts are not allowed as a deduction from the taxable profit. Depreciation is allowed at certain specified rates, on the written down value basis. Losses carried over from past years are, however, admitted to be charged to profits with certain restrictions.

Schedule of Indian Income Tax

Total gross income Rs	te/Re 1	Max. Tax at the stage Rs as	Surcha
First 1500	*nil*	—	—
Next 3500	9p	164 1	6p
" 5000	15p	554 11	10p
" 5000	24p	1179 11	18p
Balance	30p	—	24p

The rate for Joint Stock Companies and registered firms is 30p on taxable profit plus surcharge at 2as in the rupee.

Note. No tax is payable on an income below Rs 2000. Tax on the margin may not exceed half of the excess of the income over Rs 2000.

Super Tax

Total gross income Rs	Rate/Re 1	Max. Tax at the stage Rs	Surcharge
First 25000	*nil*	—	
Next 10000	1a	625	1a
" 20000	2as	3125	2as
" 70000	3as	16250	2as 6p
" 75000	4as	35000	3as
" 150000	5as	81875	3as
" 160000	6as	141875	3as
Balance	7as	—	3as 6p

The rate for Joint Stock Companies is 3 annas in the rupee on taxable profit. This is known as Corporation Tax.

Note. Fractions of a rupee are ignored in writing down gross incomes; and taxes are calculated correct to the nearest anna.

Illustrations[**]

1. A person pays Rs 207 8as a year in income tax. What is his income?

Maximum tax @ 9p in Re is Rs 164 1a, the maximum tax at the next stage being Rs 554 11as. The balance of Rs 43 7as has therefore been calculated at the next higher rate 15p in the rupee. The corresponding amount = Rs 556. ∴ Total income = Rs 5556.

[**] No account has been taken of the *surcharge*, which is an emergency imposition.

2. Calculate monthly income tax of a person with a salary of Rs 200 p. m. and paying life insurance premium of Rs 150 p. a.

		Rate	Rs as
Total income ...	Rs 2400		
Tax on first ...	1500	nil	—
" " next	900	9p	42 3
Total	2400		42 3

Average rate of income tax

$= \frac{8100}{2400} = 3{\cdot}37$ p in Re 1

Relief :

Life Insce.	150	3·37p	2 10
Net annual tax			39 9
∴ Net monthly tax**			Rs 3 5as

3. Calculate income tax payable by a person with an income of Rs 2020 who spends Rs 120 annually on life insurance.

This is a marginal case.

∴ Income tax payable $= \frac{1}{2}$(Rs 2020 − Rs 2000) and = Rs 10

Average rate $= \frac{1920}{2020} = {\cdot}95$p

Relief on Insurance @·95p on Rs 120	9as
Net tax payable	Rs 9 7as

4. A person is in receipt of a salary of Rs 3000 p. m. and house allowance of Rs 500 p. m. He contributes Rs 5000 annually to a Provident Fund and spends another Rs 5000 annually on insurances on the life of his wife and himself. Make out a statement of income tax and super tax payable by him each month.

Income	Rs	Rate	I. Tax Rs as
Salary	42000		
First	1500	nil	
Next	3500	9p	
"	5000	15p	
"	5000	24p	1179 11
"	27000	30p	4218 12
Total	42000		5398 7

Average rate $= \frac{5398\frac{1}{2}}{42000} = 24{\cdot}68$p

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** Any extra deducted for proposes of approximation is provided for by a reduction in the amount deducted from the pay of the last month of the financial year, March.

Relief :				
Provident Fund	5000			
Life Insurance	5000			
Max. statutory limit	6000	24·68p	771	3
Net Income Tax p. a.			4627	4
Ditto p. m.			Rs 385	10 as

	Total Income	Rate	Super Tax	
First	Rs 25000	nil	——	
Next	10000	1a	625	0
"	7000	2a	875	0
Total	42000		1500	0
Super Tax p. m.			Rs 125	0
Monthly Income			Rs 3500	0
Deduct I. Tax		Rs 385 10as		
Super Tax		125	510	10as
Net payable each month			Rs 2989	6as

5. A person's income is made up as follows :

Salaries	...Rs 4800	Property......	Rs 840
Interest on Securities...	1200	Miscellaneous ..	520
Interest on Tax Free "	600	(including interest allowed by bank)	

Make out a statement of his total income and income tax, if he resides in the house owned by him, and pays life insurance premiums of Rs 2000 a year.

Sources of Income	Gross Income	I. Tax deducted at source	
	Rs	Rate	Amount
Salaries	4800		Rs 154 11as
Interest on Securities	1200	30p	" 187 8
Interest on Tax-free Securities	600	*nil*	*nil*
Property ...Rs 840 *less* ⅙-th for repairs... 140	700	"	"
Other Sources	520	"	"
Total gross income	7820	Total Rs 342 3as	

Income Tax payable Rs 384 6as

Average rate of tax 9·44p in Re 1

Relief :

Interest on Tax Free Securities	600	
L. Insce. Prem...Rs 2000.		
Maximum statutory allowance of $\frac{1}{6}$-th of total gross income	1303	
Total	1903	
Relief @ 9·44p in Re 1		Rs 93 9as
Income Tax payable		Rs 290 13as
Refund claimed		Rs 51 6as

6. Calculate income Tax payable by a Hindu Undivided Family from the following data :

	Rs		Rs
Salaries	6400	Premiums on insurance of lives of members	4400
Interest & Dividends	3200	Interest on mortgage of residential property	500
Professional Earnings	15800	Interest on mortgage of other properties	2400
House Property (residential)	5300	Rent collection charges	560
Properties	8400		

Statement

Sources of Income			I. Tax deducted at source		
			Rate	Amount	
			pies	Rs	as
Salaries		Rs 6400		273	7
Professional Earnings		15800	—	—	—
Interest & Dividends		3200	30	500	—
Properties	Rs 8400				
less $\frac{1}{6}$-th for repairs	1400				
Intt. on mortgage	2400				
Collection charges @ 6%	504				
	——	4096			—

			Tax at source
Property (residential)	Rs 5300		
less $\frac{1}{6}$-th for repairs	... 883		
Intt. on mortgage	... 500		
	Rs 3917		
Maximum statutory limit of $\frac{1}{10}$-th of total gross income		3277	—
Total gross income		Rs 32773 Total	Rs 773 7as
Income tax payable			3954 12

Average rate of Tax...23·172 in Re 1

Relief :

Life Insurance Premium	4400	
@ 23·172 in Re 1		531 0
Balance of I. Tax. payable		Rs 2650 5

7. A person drawing Rs 1550 a month has investments fetching Rs 8800 a year net and owns house property of annual value Rs 4120. He pays life insurance premia of Rs 4000 a year and contributes towards a recognised provident fund Rs 3480 a year. He also pays interest of Rs 2000 a year on a mortgage of the property, fire insurance premium of Rs 513 a year and collection charges amounting to Rs 400 a year. Make out a statement of income tax and super tax payable by him.

(i) From his salary of Rs 18600 is deducted income tax as below :

Tax on First Rs 15000 = Rs 1179 11as
" " Next 3600 = Rs 562 8
" " Rs 18600 = Rs 1742 3as

(ii) On the income from investments tax is deducted at source @ 30p on Rs 8800.

@ 24p	... Rs 1100
@ 6p	275
@ 30p	Rs 1375

(iii) The taxable value of property is obtained as below :

Gross value ...	Rs 4120
Less ⅙-th for Repairs	687
Interest on Mortgage	2000
Fire premium	513
Collection charges @ 6% (limit)	247
	Rs 673

		Rate	S. Tax
(iv) Total gross income	Rs 28073		
Super Tax on first	25000	nil	—
" " next	3073	1a	Rs 192 1a

(v) Relief on Rs 4679

@ 24p	Rs 584 14as
@ 3p	73 1 9p
	Rs 657 15as 9p
Less @ ·05p	1 3as 4p
	Rs 656 12as to nearest anna.

Income Tax Statement

Source of Income	Rs	Rate or Stage	I. Tax Rs	as	Rate or Stage	Super Tax Rs	as
Salary	18600	30p	1742	3	—		—
Property	673	—	—		—		—
Investment	8800	30p	1375	0			
Total Gr. Income	28073		3117	3			
Income Tax payable		30p	3941	2	1a	192	1
Average rate of I. Tax $=\frac{3941\frac{1}{8}}{28073}=26{\cdot}95$p							
Relief :							
L. Insce. * Rs 4000							
P. Fund " 3480							
Limit ⅙th of Gr. Income	4679	26·95	656	12			
Net Tax payable			Rs 3284	6		Rs 192	1a
Tax recovered			3117	3			
Balance payable			Rs 359	4as			

EXAMPLES** 36

1. A physician earns Rs 10000 a year from his practice, and has investments fetching a gross income of Rs 1500 a year. Calculate the income tax he is required to pay. [*Ans.* Rs 742 3as

2. The income of a person is made up as follows : property Rs 13000, dividend at 10 per cent on Rs 50000 shares, interest on 6 per cent Treasury Bonds of Rs 25000. He pays life insurance premiums of Rs 1500 a year. Calculate his net income after paying income tax and premium. [*Ans.* 17·11p ; Rs 16589 7as

3. A person, drawing a salary of Rs 625 a month, owns a house of annual value Rs 1200 in which he resides. He subscribes Rs 20 per mensem to a Family Pension Fund, and pays Rs 300 annually towards cost of life insurance. Calculate his net income after payment of the Fund subscription, insurance premium and income tax. [*Ans.* 9·78p ; Rs 6563 1a

4. The income of a person is Rs 6000 a year. He spends Rs 500 a year on life insurance, and subscribes Rs 35 per month to a Provident Fund and Rs 25 per month to a recognized Family Pension Fund. What will be his net income after making these payments including income tax ? [*Ans.* 7·75p ; Rs 4578 3as

5. A lawyer has an income of Rs 20000 per annum from the bar, and holds investments fetching an income of Rs 2000 a year, and owns house properties, all let out, of gross annual value Rs 2250. He has insured his life for Rs 75000 at Rs 41 8as per thousand. Calculate the income tax he is required to pay.

[*Ans.* 20·64p ; Rs 2231 13as

6. A person owns city properties of a gross annual value of Rs 20000, which is mortgaged for Rs 50000 at 12% p. a. Calculate the income tax he is required to pay, if he has no other source of income, and if he spends Rs 4000 per annum on life insurance.

[*Ans.* 11·48p ; Rs 531 12as

7. A widow holds Rs 40000 Government stock bearing interest at 6% p. a. If she has no other source of income,

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** The average rate of income tax is also shown in the answers. The answers do not take account of any surcharge or relief from Earned Income.

calculate the amount she can claim as refund from the income tax authorities. [*Ans.* Rs 332 13as

*8. A sum of Rs 12 7as is deducted from the monthly salary of a person as income tax. If he gets an increment of Rs 50 per mensem, determine the net value of the increment to him. [*Ans.* Rs 46 15as per mensem

*9. A levy of 25 per cent surcharge on income tax means a difference of Rs 125 annually in a person's net income. Find the gross income. [*Ans.* 9300

***10.** A partner in an unregistered partnership firm received Rs 6161 2as 6p in a year taxed at source as his half share of the profits. He holds the following investments : 10 shares of Rs 1000 each which paid a dividend of 25 p.c., 5 shares of Rs 250 each which paid no dividend during the year. He has insured his life for Rs 40000 for which he pays Rs 1000 annually in premium. Make out a statement of his case for assessment of income tax. [*Ans.* 10'24p ; I. T. Rs 434 8as

***11.** A person drawing a salary of Rs 240 a month has an income from investments in Rs 50000 shares paying a dividend of 10 per cent. Income tax is deducted at the usual rates from his salary and the dividends. Ascertain the amount he may claim from the income tax authorities as refund. [*Ans.* Rs 456 14as

***12.** A person has, in addition to his income of Rs 500 per mensem as salary, house property of a gross rateable value of Rs 20000. He pays an insurance premium of Rs 1000 a year, and subscribes Rs 25 a month to a recognised Family Pension Fund. Find the income tax he shall have to pay, if at all, direct to the income tax authorities in addition to what has already been deducted at source. [*Ans.* Rs 1999 2as

***13.** The gross rateable value of a person's house properties is Rs 24000. Allowances are admitted in respect of fire insurance premium of Rs 500, and collection charge up to 6% of the total annual value and ground rent Rs 860. The property is mortgaged for Rs 100000 at 9% p. a. If the proprietor has no other source of income, find how much of his claim of Rs 2000 for relief in respect of insurance premium can be admitted.

Determine the income tax paid, and his net income after meeting the foregoing expenses including income tax.

[*Ans.* Rs 1367 ; Rs 345 ; Rs 9855

*14. A person drawing a salary of Rs 125 per mensem is left by a rich relation Rs 75000 Treasury Bonds (6¼ per cent), and a house, in which he lives, of an annual value of Rs 2500. Ascertain the amount he has further to pay to the income tax authorities, if at all. [*Ans.* Refund Rs 434 15as

*15. The income of an undivided Hindu family is made up as follows : salary of one member Rs 900 per mensem, 10 p. c. dividend on Rs 20000 Preference Stock, 4 p. c. interest on Rs 50000 Government Stock, 5% Tax-free Rs 20000 Government stock. The house occupied by the family is owned by it, but is mortgaged for Rs 9000 at 12 per cent p. a. Life insurance premium on the life of a member amounts to Rs 800 a year. If the gross rateable value of the house be Rs 1200, calculate the net income of the family per mensem, after meeting the interest on the mortgage, the premium and the income tax. Determine whether any portion of the income tax deducted at the sources may be claimed as refund.

[*Ans.* 15·85p ; Rs 1063 11as ; Rs 123 10as

16. An unregistered firm of umbrella manufacturers starts with an opening stock of Rs 6000. It makes purchases during the year of the value of Rs 15000 and effects sales of the total value of Rs 25000, and closes with a stock of Rs 5000. The following actual expenses are allowed : manufacturing wages Rs 2300, rent, rates and taxes Rs 1200, salaries and wages Rs 1800, miscellaneous charges including advertisements etc. Rs 500. Find the taxable income of the business and the income tax paid. [*Ans.* Rs 3200 ; Rs 79 11as

*§ **Excess Profits Tax (E. P. T.).** This is a tax imposed for the present during the pendency of the War (1939—). The object is to augment public revenues by claiming a portion of the abnormal profits that may be earned under war conditions.

The rate is at present 87½% of the excess of the profit over a pre-determined *standard profit* ascertained over a *standard period.*

The standard period may be selected by the assessee from amongst the following with a view to claiming the largest normal profit that will not be touched by the E. P. T. This is also the standard profit, except in those cases to which the statutory percentages are applicable.

For Businesses

in existence before April 1936 :	*formed after March 31, 1936* :
Profit for 1936-37 or 1937-38 or average of profits during 1936-1940.	Profit at statutory percentages on average capital.

The profit is ascertained subject to certain adjustments, including one on the basis of statutory percentages on increase or decrease of capital.

For new businesses the *standard profit* is taken to be equal in amount to 8% p. a. in the case of companies, and 10% p. a. in other cases, on the average capital invested. (*Statutory percentages*)

The tax is to be levied on all excess profits earned after 1st September 1939, not below Rs 30000 a year (or a rateable portion thereof, if the chargeable accounting period is greater or less than a year). The tax is not payable on any income which is not chargeable to income tax, and the profit for the purposes of the tax is assessed as for income tax. The amount of excess profits tax payable, however, is allowed to be charged to taxable profits, before the assessment of Income Tax.

A Board of Referees is empowered to allow a standard profit in excess of that obtained under the rules, if they are satisfied that the ascertained standard profit is unduly low, subject to the restriction that in no case shall such increased standard profit exceed an amount sufficient to pay a dividend of 6% p. a. on the ordinary capital of a company, after meeting its obligations to preference shareholders etc. at stipulated rates.

A deficiency of profit in any chargeable accounting period, may be carried over to the next subsequent chargeable accounting period, and relief shall be granted therefor in accordance with certain rules.

SECTION SIXTEEN

TYPES, AVERAGES AND STATISTICS

§ An *arithmetical average* or *mean* of a number of similar quantities is obtained by dividing the total of such quantities by the number of items. The average ordinarily serves the following purposes :

(i) It gives a representative idea of the data which vary in value.

(ii) It thus gives a useful means of comparison of similar groups of data.

(iii) The average of a long series shows its normal value, and in simple cases, the chances are even that the excess or defect of a random item from the average value is balanced by defect or excess of equal or nearly equal values amongst the remaining items.

(iv) It is an arithmetical device summarising the total value of the quantities which can be directly obtained, if necessary, by multiplying the average by the number of items.

§ An easy method of calculating averages of numbers fairly distributed, is illustrated below :

	Diffce. from a trial number (50, say)	
Nos.	+	−
49·7		·3
51·3	1·3	
50·8	·8	
52·6	2·6	
48·8		1·2
51·9	1·9	
52·3	2·3	
48·9		1·1
Totals	8·9	2·6

+ *ive* diffce	8·9
− *ive* ”	2·6
Balance	6·3

No. of items = 8

∴ Diffce. of average = 6·3 ÷ 8 = ·7875

∴ Average = 50·7875.

This method will hold good whatever be the trial average chosen. If the balance of the difference be-*ive*, the difference of the average from the trial number will also be negative, and should be subtracted therefrom to obtain the correct average.

N. B. Cancellation of equal items in the two columns will simplify calculation.

§ **Weighted Arithmetic Average.** An arithmetic average does not always give a clear idea of the type of a group of observations. For example, if the wages in a factory are on three different scales—10as, 14as, Re 1 4as per diem, the average of these, 11¼as p. d., gives very little idea of the daily expenses on account of wages. An average obtained with the help of the following further data gives a very much more reliable measure :

100 men paid @ Re 1 4as per diem
300 ,, ,, ,, 14as ,, ,,
400 ,, ,, ,, 10as ,, ,,

$$\text{Average} = \frac{1 \times 20 + 3 \times 14 + 4 \times 10}{1+3+4}\text{as}$$
$$= \underline{12\tfrac{3}{4}\text{as.}}$$

The multipliers 1, 3 and 4 in the numerator are repeated in the denominator and are proportional to the actual number who are paid at these rates, and are known as *weights*. The average thus obtained is known as a *weighted average*.

It frequently happens that in estimating the value of a type, the population against each group is not possible of accurate determination. In such cases the experience of the computer about the relative population of the different groups helps him to give *weights* thereto on guess. The result thus obtained has a much greater value than that of a simple arithmetic average obtained from observed values,

Representing a type by m_r, and the relative weight by w_r, the weighted average would be $= \dfrac{\Sigma w_r . m_r}{\Sigma w_r}$. The values of w are usually estimated from random samples ; and it may be noted that although the weights might differ in value according to different estimates, the value of the weighted average is not materially affected so long as the number of observed types is fairly representative.

EXAMPLES 37

1. Obtain the average of the following numbers :

(i) 84, 86, 88, 85, 85, 86, 83, 88. [*Ans.* 85·625

(ii) 118, 119, 118, 117, 116, 114, 121, 120, 120. [*Ans* 118·1

(iii) 4365, 4103, 4415, 4386, 4410, 4390. [*Ans.* 4345 app.

2. Find the average number of $\frac{3}{4}$-in. bolts *used* in a workshop per diem during the week from the following data :

	No. issued.	No. wasted.	No. returned
Monday	543	7	16
Tuesday	568	3	14
Wednesday	672	9	20
Thursday	592	8	16
Friday	584	6	15
Saturday	248	2	4

[*Ans.* 515

2. Complete the following Table :

Average Prices of Securities

	$3\frac{1}{2}$% Stock	Ind. Iron	Howrah	Imp. Bank
Jan.	$93\frac{7}{16}$	$34\frac{5}{16}$	$56\frac{1}{4}$	1460
Feb.	$93\frac{9}{16}$	$33\frac{7}{8}$	$58\frac{1}{2}$	1484
March	$93\frac{1}{2}$	$32\frac{5}{8}$	$54\frac{3}{4}$	1440
April	$93\frac{9}{16}$	$33\frac{1}{8}$	$55\frac{1}{4}$	1480
May	$93\frac{5}{8}$	$33\frac{7}{8}$	$55\frac{1}{2}$	1500
June	$93\frac{11}{16}$	$33\frac{5}{8}$	$52\frac{7}{8}$	1490
July	94	$34\frac{1}{8}$	$54\frac{1}{4}$	1522
Aug.	$93\frac{13}{16}$	$34\frac{1}{4}$	$54\frac{5}{8}$	1510
Sept.	$94\frac{1}{16}$	$34\frac{3}{8}$	56	1520
Oct.	$94\frac{1}{8}$	$34\frac{1}{8}$	$55\frac{1}{2}$	1505
Nov.	$94\frac{1}{16}$	$34\frac{1}{2}$	$55\frac{3}{4}$	1498
Dec.	94	34	$55\frac{1}{2}$	1486
Average				

[*Ans.* $93\frac{3}{4}$; $33\frac{7}{8}$; $55\frac{3}{8}$; 1491

4. Three persons watched a crowd for obtaining an idea of the average age of persons, attaching different weights at their discretion. Fuid the average on the basis of each set of observations and prove that they all lead to very similar conclusions.

Age-group	Set No. 1. weights	Set No. 2 weights
10-20	3	10
20-30	10	32
30-40	5	24
40-50	2	6
50-60	1	5

Age-group	Set No. 3 weights
15-25	2
25-35	3
35-45	2
45-55	1

[*Ans.* 30 ; 30 ; 32

Note. The last group was obviously too small to be representative. Take the average age of an age-group at its *mid-value.*

5. The Engine Room consumed the following quantities of Crude oil at prices noted.

1944	Quantity	price p. gall.
March 3	8 gall.	13as 6p
	6 gall.	14as 7p
March 4	5 gall.	14as 7p
	10 gall.	15as 2p
March 5	2 gall.	15as 2p
	12 gall.	15as 11p
March 6	16 gall.	15as 7p
March 7	3 gall.	15as 7p
	17 gall.	15as 5p
March 8	6 gall.	15as 5p
	13 gall.	14as 10p

Find the average daily consumption and the weighted average cost per gallon for the week. [*Ans.* 16⅓ gall. ; 15as 1p

6. The costs of production of grey shirtings in a group of Mills are as follows :—

Mill	Production	Total Cost
A	38,763 yds	Rs 17,576
B	94,318 yds	Rs 41,318
C	31,514 yds	Rs 15,612
D	62,316 yds	Rs 29,312
E	1,19,625 yds	Rs 51,418
F	87,248 yds	Rs 42,525
G	75,618 yds	Rs 35,621

Find the average cost per yard for each of the mills and the average cost for the group of Mills. Answer to the nearest hundredth of an anna. [*N. B.* Use Logarithms.]

7. The weight of a body calculated as the average of seven different experiments is 53·735 grammes. The average of the first three experiments is 54·005 grammes. The fourth was greater than the fifth by ·004 grammes, while the average of the sixth and seventh was ·010 grammes less than the average of the first three. Find the weight of the body as obtained by the fourth experiment.

—P. S. C. [*Ans.* 53·072 gm

8. Twelve per cent of the men in one regiment, 33% of the men in a second regiment and 21% of the men in the two regiments taken together are less than 23 years old ; also the second regiment contains 600 men. How many men are there in the first regiment ? Test the accuracy of your result.

—P. S. C. [*Ans.* 800 men

9. During the year 1932 the bank rate was as follows :— 6% for 1 week ; 5% for 1 week ; 4½% for 1 week ; 4% for 17 weeks 3½% for 3 weeks ; 3% for 5 weeks ; and 4% for 24 weeks. What was the average rate during the year ? —I. Com. [*Ans.* 4% app.

10. The average dividend paid during 8 years by a company is 3·8125%. The dividends paid for the first 7 years were 3½, 3¼, 3½, 4, 4½ and 4 per cent respcetively. What was the dividend for the eighth year ? —I. Com. [*Ans.* 3½%

11. A trading concern in the first year made a profit of Rs 1875 on a turnover of Rs 23500 and in the second year a loss of Rs 710 was sustained on a turnover of Rs 22360 ; while in the third year a profit of Rs 935 was made on a turnover of Rs 24140. What was the average profit per cent on the 3 years' turnover ? —I. Com. [*Ans.* 3%

12. A person receives in April £31 17s 6d and every following month he receives £1 5s less than the previous month. Find out what he will receive for March and by how much per cent they fall short of the average monthly receipts for the year. —I. Com. [*Ans.* £18 2s 6d ; 27½%

§ **Statistics.** An array of figures relating to a particular subject is known as statistics. The statistics of the imports into a country during a certain period, for example, may contain figures during the twelve months of the year for each year showing the value, or the quantity of particular classes of goods, and would show to the trained person the increase or diminution in the value or the quantity of the imports, the directions of such movement, the particular class of commodities in greatest demand etc., and would suggest to him comparison with similar sets of figures for other years or of other countries, and help him in various ways in assessing the position of trade and industry in the land.

Statistics may relate to any subject which is capable of quantitative measurement—the birth-rate, the bank-rate, the movement of exchanges, cost of production figures and various other things. It is useful to train oneself in looking at such statistics so that they may *speak* to the reader. The following illustrations are meant for practice in reading the language of figures ; scientific methods of statistical investigation are beyond the scope of the present volume.

A common method for comparison of similar sets of figures is that of averages. While it is hardly manageable to compare the monthly or the weekly figures of exports of jute from Calcutta, say, with similar figures of another year, monthly, weekly or annual average figures tell their own stories.

The trend of movement of sets of figures is also studied in terms of percentages. An effective method of visualising these movements and studying the figures, otherwise, is by graphical methods, some easy varieties of which will be discussed in a subsequent Section.

Calculations of percentages etc. are usually made correct to the nearest integer, or to the nearest tenth in some cases, in obtaining which contracted methods have to be systematically used.

§ **Index Numbers.** A commonly used method of comparing data in regard to a group of values of the same subject is by reference to a set of proportional numbers in reference to a particular value in the group regarded as proportional to 100. These numbers are called Index Numbers. The illustrations given in the following examples will show with what ease the course of movement of the values may be studied with the help of Index Numbers.

EXAMPLES 38

1. Study the data given below :—

Paint Shipments
'000 gallons

Year	Jan.	Feb.	March	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
1940	358	362	370	378	390	401	398	380	384	382	389	400
1941	413	418	416	421	422	420	417	403	406	405	412	418
1942	501	515	532	529	524	527	525	547	538	526	518	522
1943	512	507	505	516	529	619	531	518	514	512	516	518
1944	518	522	526	530	536	540	542	546	545	543	548	550
1945	552	560	564	560	564	568	568	572	576	570	572	576

(i) What is the average shipment during the period in each of the months January to December ?

(ii) What is the average monthly shipment during the year for each of the 5 years ?

(iii) Do the figures for each month show steady improvement in the exports ?

(iv) Is the same rate maintained, month by month, during the quinquennium ?

(v) Is there a noticeable fluctuation in the shipments from month to month ?

(vi) In what months are the shipments usually large ?

2. Fill up the blanks in the following, where necessary : and make any observations that are justified by the figures.

A. *The Indian Bank Rate*

(Seasonal Averages)

Season	1921	1922	1923	1924	1925	1926	1927	1928
Jan-June	6·04	7·13	7·42	8·05	6·58	5·65	6·51	6·94
July-Dec.	5·11	4·51	4·50	5·92	4·70	4·70	4·96	5·46
Season	**1929**	**1930**	**1931**	**1932**	**1933**	**1934**	**1935**	**1936**
Jan-June	6·88	6·51	6·74	6·02	3·63	3·50	3·50	3·00
July-Dec.	5·79	5·28	7·35	4·03	3·50	3·50	3·42	3·00

B. *Structure of Indian Export Trade*

(*Percentages*)

Year	Cotton	Jute (manufacture)	Tea	Jute (raw)	Grains &c.	Seeds	Metals	Leather	Hides & Skins	Other items	Total
1928-9	20·20	17·24	8·06	9·80	10·21	8·97	2·70	2·82	2·90	17·10	100
1929-30	21·11	16·71	8·37	8·74	11·19	8·52	3·33	2·63	2·57	16·83	100
1930-1	21·19	14·46	10·68	5·84	13·55	8·10	3·60	2·90	2·48	17·20	100
1931-2	15·26	14·06	12·47	7·18	13·07	9·36	3·51	3·43	2·34	19·32	100
1932-3	15·63	16·40	12·96	7·35	12·14	8·54	3·54	3·60	2·09	17·75	100
1933-4	18·44	14·60	13·56	7·47	8·03	9·33	3·75	3·61	2·91	18·30	100
1934-5	23·15	14·20	13·32	7·19	7·83	6·97	3·91	3·63	2·07	17·73	100
1935-6	21·47	14·63	12·35	8·54	7·73	6·43	4·82	3·51	2·57	17·95	100

C. *Sale of Textiles in a District Town*

	Average 1919-24	Average 1924-29	Average 1929-34	1935
	Rs	Rs	Rs	Rs
Indian made	318 614	1 093 105	1 262 638	1 311 612
British ,,	1 295 103	616 213	209 801	189 114
Japanese ,,	406 708	711 305	1 311 306	1 412 618

What observations would you offer on the steady displacement of British goods from the market ?

D. *Imports of Soap into British India during November*

	1934	% Total	1935	% Total	1936	% Total
	Rs		Rs		Rs	
Household	285 037		24 019		14 999	
Toilet	226 898		244 159		160 969	
Others	14 379		13 324		11 219	
Total	526 314		281 502		187 187	
Share U. K.	445 788		212 258		121 836	
Other countries	80 526		69 244		65 351	

Also make observations on the movement of the different values by means of Index Numbers based on total of November 1934 = 100.

E. *Index numbers of wholesale Prices in Tokyo.* (*October 1900-100*)

	Year	General Average	Rice	Raw Silk	Coal
January	1936	192	249	104	288
February	1936	191	251	95	288
March	1936	191	253	92	288
April	1936	192	257	91	288
May	1936	192	262	86	288
June	1936	194	271	84	288
July	1936	197	269	90	288
August	1936	201	275	94	288
September	1936	201	270	90	289
October	1936	200	250	94	290
October	1935	194	266	113	288

F. *Exports from Calcutta*

Year	Row Jute '000 tons	Index Number (100 = av. of 1915-20)	Gunny Bags millions	Index Number (100 = av. of 1915-20)	Hessians million yards.	Index Number (100 = av. of 1915-20)
1915-16	600		724		1192	
1916-17	540		835		1213	
1917-18	378		758		120	
1918-19	398		583		1103	
1919-20	592		343		1275	
1920-21	472		584		1353	
1921-22	468		387		1121	
1922-23	578		344		1254	
1923-24	660		414		1349	
1924-25	696		425		1456	
1925-26	647		425		1461	
1926-27	708		449		1503	
1927-28	892		463		1553	
1928-29	898		498		1568	
1929-30	807		522		1651	
1930-31	620		434		1271	
1931-32	587		389		1021	
1932-33	563		415		1012	
1933-34	748		402		1053	
1934-35	752		423		1063	

G. *Investment of Indian Insurance Companies.*

Particulars	1929	1932	1935	1937
Mortgages	3·68	3·59	4·01	3·8
Policy Loans	7·16	9·31	8·84	8·6
Loans on Stocks & Shares	1 75	1·12	0·60	0·3
Indian Govt. Securities	52·04	52·08	50·54	50·8
Municipal and Port Trust Debentures	14·46	13·67	10·25	9·4
Shares of Indian Companies	3·29	4·12	6·12	7·4
Land and Buildings	5·00	4·91	5·26	5·9
Deposits, Cash &c.	3·80	2·48	4·56	4·0
Capital Expenses	0·45	0·51	0·47	—
Miscellaneous	5·89	6·47	7·55	3·0
Foreign Securities	2·48	1·47	1·31	1·1
Total	100	100	100	100

3. Express the following items of expenditure in 1932 as percentages on the total cost, and compare their growth by index numbers, taking 1927 as the base period.

000's omitted

	1927 Rs	1928 Rs	1929 Rs	1930 Rs	1931 Rs	1932 Rs
Direct charges :						
Raw Materials (incldg. carriage)	184	213	212	227	206	180
Labour	94	96	93	92	89	88
Electricity	134	140	142	146	138	129
Miscellaneous charges	53	58	56	60	61	69
Indirect charges :						
Management exps.	46	48	53	58	58	56
Depreciation of plant etc.	64	68	68	70	70	69
Other charges	35	38	39	41	40	37

4. The following figures show the way sales have moved during the last five years with increase in the cost of advertisements, and corresponding costs of distribution.

Fill up the blank spaces, and make your observations.

	Sales 000's omitted Rs	Advt. Rs	% Sales	Distribution cost Rs	% Sales
1932	53	2 112		8 115	
1933	68	4 028		12 513	
1934	94	8 015		16 218	
1935	115	11 320		18 330	
1936	120	14 525		19 824	
Totals					
Averages					

5. Study the following analysis of costs of manufacture in a Soap factory.

Analysis of Production

	1930	1931	1932	1933	1934	1935
Units Produced (thousands)	3456	3865	4418	4964	5838	6434
Raw Materials (units)	8001	8912	9953	10957	13532	14156
Labour (Thousand Rs)	5257	5908	6423	7136	8316	90031
Fuel &c. (tons)	2083	2513	2836	3206	3708	4335
Supervision (Rs)	15180	17302	21005	23182	24103	24212
Other Charges (Rs)	18231	21514	25338	25816	26206	26413

Make observations on the following points :

(i) **The rate of growth of production.**

(ii) **The changes in the ratio of the use of raw materials to the number of units produced. Do the figures suggest a reduction in wastage with growth in production ? Is this reduction steady throughout the period ?**

(iii) **How does the cost of labour increase with production ?**

(iv) **Does the cost of fuel bear a definite proportion to the volume of production ?**

(v) **Do the changes in the cost of supervision agree with your ideas about economies resulting on large scale production ?**

(vi) **Do you notice any definite relationship between any of the other items and other charges ?**

6. Study the following analysis of the cost of living figures for an average working class family at Ahmedabad.

Annual Income in Rupees	Yearly consumption expenditure	Percentage of consumption expenditure on						
		Food	Rent	Furniture	Fuel & light	Total	Clothing	Miscellaneous
	Rs. As.							
Less than 240	249 5	49.2	13.6	0.6	8.4	22.6	10.2	18.0
240-360	309 5	48.0	14.2	0.4	7.7	22.3	10.0	19.7
360-480	394 10	48.6	13.4	0.4	7.1	20.9	9.1	21.4
480-600	476 8	49.3	11.6	0.4	6.7	18.7	8.9	23.1
600-720	534 5	50.7	9.5	0.3	6.4	16.2	9.3	23.8
720-840	626 15	51.4	8.9	0.3	6.6	15.8	9.2	23.6
840-960	710 5	49.7	9.9	0.2	6.2	16.3	8.7	25.3
960-1080	792 3	48.4	7.0	0.7	6.0	13.7	8.9	29.0

SECTION SEVENTEEN

INTERPOLATION AND POSSIBLE ERRORS IN CALCULATION

§ **Interpolation.** It is frequently necessary to find intermediate values from statistical or other tables. It will be sufficient for our purposes to find first approximations of such interpolated values. (*Straight line interpolation*)

Illustrations

1. It is noted from the adjoining Table of Interest that the interest on Rs 100 for 115 days @ 2% is Re ·6302 and @ 1½% it is Rs ·47265. We are to obtain the interest on Rs 100 for 115 days @ 1¾%.

Interest on Rs 100 for 115 days

Rate % p. a.	Interest Re
1 %	·31510
1½%	·47265
2 %	·63020
2½%	·78775

The difference in interest for a difference of ½% in this case = Re ·15755

Difference for ¼% = Re ·078775

Add interest @ 1½% = Re ·47265

∴ Interest @ 1¾% = $\underline{\text{Re ·551425}}$

2. It is observed from a Table of Squares that $53^2 = 2809$ and $54^2 = 2916$. To find a rough value of $\sqrt{2904}$.

It is noted that the difference between 54^2 and $53^2 = 107$, and that between 54^2 and $2904 = 12$. ∴ $\sqrt{2904} = 54 - \frac{12}{107} = \underline{\underline{53·9}}$ app.

3. Find log 953·4, given log 9·53 = ·97909 and log 9·54 = ·97955

log 9·54 = ·97955
log 9·53 = ·97909

Diffce. 10 → 46
∴ " 1 → 4·6
And " 4 → 18

Then, log 953 = 2·97909
Diffce. ·4 → 18

∴ log 953·4 = $\underline{\underline{2·97927}}$

§ It is sometimes necessary to obtain similar values at a stage beyond the range of the data. Such a method is known as **Extrapolation.**

Thus, in the last illustration above

$$\begin{array}{lrr} & \log 9{\cdot}54 & = {\cdot}97955 \\ \text{Diffce.} & \ldots 2 \rightarrow & 9 \\ \hline \therefore & \log 9{\cdot}542 & = {\cdot}97964 \end{array}$$

§ These interpolated values are safe when the law relating the two variables is linear. They are also usually dependable in other cases when the given tables are close enough. With statistical data, interpolated values may be obtained graphically, on the assumption that, within small ranges, a line curve may be regarded as straight. It is beyond the scope of this volume, however, to estimate the value of errors in such cases.

§ **Double Interpolation.** It may at times be necessary to study the changes in value of three or more variables, in which case the method followed in the illustrations below should be suggestive.

Illustrations

1. A large industrial establishment supplies free foodgrains to its labour population, consisting of males and females, adults and children. Form the following data estimate the cost of supporting a population of 38573.

Population	Foodgrains (tons)	Cost (thousand Rs)
32 734	18 341	3681
36 252	18 652	4287
39 269	18 928	4563
40 287	18 862	4486
42 394	20 006	4779

Interpolating first for foodgrains,

$$\begin{array}{lrcrlrcr} & \text{Pop.} & & \text{Tons} & & \text{Pop.} & & \text{Tons} \\ & 39269 & & 18928 & & 39269 & & 18928 \\ & \underline{36252} & & \underline{18652} & \textit{Less} & \underline{696} & \rightarrow & \underline{64} \\ \text{Diffce.} & 3017 & \rightarrow & 276 & & 38573 & & 18864 \\ & 696 & \rightarrow & \dfrac{276}{3017} \times 696 & & & & \\ & & & = 64 & & & & \end{array}$$

Interplolating again for cost,

	Tons		Cost	Tons		Cost
	18928		4563	18862		4486
	18862		4486	2	→	2
Diffce.	66	→	77	18864		4488
	2	→	2			

∴ Estimated cost = **Rs 44,88,000**

Note. In the absence of any knowledge of the stucture of the population (males, females &c.) it would not be safe to interpolate directly from population to cost, which would naturally depend upon the ration quota of each individual group.

The first interpolation, therefore, gives an estimate of foodgrains necessary to support the given population, and the second gives the estimated cost directly.

2. The following Table gives the horse power (H. P.) at which an engine must work in order to draw a train of a certain weight at a certain speed :—

Horse Power at the level

Weight of Train	200 tons	250 tons	300 tons
Speed of train in ft. per sec.	H. P.	H. P.	H. P.
30	110	135	160
40	145	180	215
50	180	225	270

At what H.P. would you expect an engine to work if it is drawing a train

(*a*) of weight 240 tons at 44 ft. per second ;
(*b*) of weight 260 tons at 36 ft. per second ?

Interpolating first for weights :

Speed 40 ft. p. sec.			*Speed 30 ft. p. sec.*		
(i) For	200 tons ···	145 H.P.	(i)	250 tons	135 H.P.
	250 " ···	180 ···		300 ···	160 ···
Diffce.	50 " ···	35 ···	Diffce.	50	25 ···
∴ ···	10 " →	7 ···	∴	10 ··· →	5 ···
∴ For	240 " ···	173 ···	∴ For	260 ···	140 ···

Speed 50 ft. p. sec.

(ii) For	200 tons ···	180 H.P.
	250 „···	225 ···
Diffce.	50 „ ···	45 ···
	10 „ →	9 ···
∴ For	240 „ ···	216 ···

Speed 40 ft. p. sec.

(ii)	250 tons	180 H.P.
	300 ···	215 ...
Diffce.	50 ···	35 ···
∴	10 →	7 ···
∴ For	260 ···	187 ···

Re-arranging,

(i) For 240 tons train	40 ft. p. sec. requires	173	H.P.
	50 ··· ···	216	···
Diffce.	10 ··· ···	43	
∴	4 ··· →	17	···
∴	44 ··· ···	190	···

(ii) For 260 tons train	30 ft. p. sec. requires	140	H.P.
	40 ··· ···	187	···
Diffce	10 ··· ···	47	···
∴	4 → ···	19	···
∴	36 ··· ···	168	···

N. B. This is a case for double interpolation. This method may be usefully applied when the two sets of relationships between the data are linear, as in this case.

On the graph paper the three different sets of data will furnish three straight lines, one for each tabulated weight. From the graph the distance between the 200 ton and 250 ton lines at the height 44 ft. per second is interpolated to obtain the H.P. at 240 tons. That is the 44 ft./sec line is drawn between the 200 tons and 240 tons line; and the intercept between them is divided in the ratio 4 : 1.

EXAMPLES 39

1. Following is a Table of Yields of 4% Irredeemable Stock :

Price	100	99	98	97	96	95
Yield	4·00	4·04	4·08	4·12	4·17	4·21

Find approximate yield of the stock @ 97¼, @ 98½ and @ 99⅛. [*Ans.* 4·11% ; 4·06% ; 4·035%

2. An experiment in the reduction of rates gave the following figures of receipts :

Rate%	2¼	2⅛	1⅞	1¾
Recpts '000 Rs	158	159	163	168

What will be the estimated receipt (i) @ Rs 2 1a, (ii) @ Re 1 15as and (iii) @ Re 1 11as per cent ?

[*Ans.* Rs : 160000 ; 162000 ; 170500

3. The following table shows the corresponding values of number of units produced and the total cost of production.

Production (units)	Total Cost Rs
15725	78 628
18813	90 110
23602	110 643
29815	125 513

Find the likely cost of production of 16000, 21000 and 26000 units. [*Ans.* Rs 79465 ; Rs 99487 ; Rs 116382

4. From the following record of experiments regarding the grant of return concession fares over a certain railway in ten consecutive years, find the possible effect of fixing the concession fare at 1⅜th or 1⅔th of a single fare.

Year	Return Fare	No. of Passengers
1930	1¾ single fare	83 600
1931	ditto	84 050
1932	ditto	82 890
1933	1⅔ "	87 815
1934	ditto	89 602
1935	ditto	91 205
1936	1½ "	93 816
1937	ditto	94 112
1938	ditto	92 606
1939	ditto	95 180

Note. Take the number of passengers availing of the concession at a rate to be the arithmetical average of the corresponding numbers over the period during which the concession was offered.

5. (i) Given log 34938 = 4·5432980 and log 34939 = 4·5433105, find log 34·9388. [*Ans.* 1·5433080

(ii) Given log 12842 = 4·1086327 ; log 12843 = 4·1086665 find the antilog of 1·1086616. [*Ans.* 12·8429

(iii) Find the 5th-root of 349388 correct to the third place of decimals from the above data. —I. Com. [*Ans.* 12·843

6. The relation between the H. P. of a motor and the capacity of an electric pump is as follows :—

H. P.	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	3	4
'000 Gallons/Hour	5·23	9·51	14·75	19·25	26·34	36·28

Estimate the capacity for a motor of (i) $1\frac{1}{8}$; (ii) $2\frac{3}{4}$; (iii) $3\frac{5}{16}$ H. P. [*Ans.* 10820 ; 24570 ; 29440 gall/hour

7. The table below shows the changes in the value of (i) Gross sales, (ii) Gross profit and (iii) Net profit.

'000 Rupees

G. Sales	218	315	552	560	500
G. Profit	45·1	60·6	88·4	86·3	80·8
N. Profit	12·6	25·2	35·6	36·1	30·4

If in the next accounting period the gross sales amount to (i) 4,83,700, (ii) 5,57,300, what should the estimated Profits be ?

Draw Line-charts and compare the values thus obtained.

§ **Possible Errors in Calculation.** The number of significant digits obtained in an approximation indicates the degree of accuracy of the result. If it is correct to *n* significant figures, it is true within $\frac{1}{10^{n-1}}$ of the real value.*

The student, however, must be warned against the fictitious sense of security sought to be obtained by carrying the work of computation to an unwarranted number of significant figures.

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* If there are 3 significant figures, the number will be of value $(100a+10b+c)$ times the unit of c. The maxium possible error is $\pm\frac{1}{2}$ of the same unit. ∴ Maximum relative error $= \frac{\pm\frac{1}{2}}{100a+10b+c}$. The largest value of this occurs when the denominator is smallest *i.e.* 100, when it $= \pm\frac{1}{200}$. Thus, the result is correct to within $\frac{1}{100}$-th of the true value.

Thus all the figures obtained from the product 726 × 10200 are not equally reliable. As a matter of fact the first figure 7 only is accurate ; and the best value that can be obtained from the data, which are assumed to be approximate (the first to the nearest unit and the second to the nearest hundred) can be correct to only 2 significant figures. The work can be contracted accordingly to obtain the product $= \underline{74 \times 10^5}$ nearly.

The extra digit, it may be noted, will tend greatly to reduce the probable error, which is normally much smaller than the maximum possible error.

§ It should be noted that the sum of a number of items of varying degrees of approximation can have no greater degree of accuracy than that of the most faulty item. Thus the sum below can be correct only to the nearest 100.

$$\begin{array}{r} 5368{\cdot}3 \\ 6800 \\ 15343{\cdot}61 \\ 128{\cdot}57 \\ 9096{\cdot}88 \\ \hline 36737{\cdot}33 \end{array}$$

The best result justified by the data, of which obviously 6800 is likely to be the most faulty item, is 36700. The rejection of the inaccurate digits, however, should be reserved till the last stage.

The assessment of the degree of accuracy in a subtraction operation is similar, but is somewhat complicated in multiplication and division.

Below are given a number of illustrative methods to determine the accurate digits of a result justified by the data, *which are assumed to be only approximately correct.* It should be remembered that the possible errors indicated are cumulative, when more than one arithmetical operation is involved.

§ When a number like 9998 is written to be approximately equal to 10000, the deviation 2 from the true value is known as the *absolute error.* Although there is the same absolute error in taking 100 for 98, it is easily seen that the two errors cannot be compared ; and that the first approximation is very much better than the second.

A safe assessment of the error is obtained from the evaluation of the *relative error* $= \frac{\text{absolute error}}{\text{true value}}$, which is ordinarily expressed as a percentage. In the first instance, the relative error is ·02% and in the second, which is evidently more faulty, the error is 2% nearly.

Illustrations

1. Find the limits within which the value $251 \times 23 = 5773$ lies ; and find the accurate figures of the product.

The limits of value of 251 are $251 \pm \frac{1}{2}$, and that of 23 are $23 \pm \frac{1}{2}$. ∴ Maximum possible value of product $= 251\frac{1}{2} \times 23\frac{1}{2}$ or $5910\frac{1}{4}$; and the minimum possible value $= 250\frac{1}{2} \times 22\frac{1}{2}$ or $5636\frac{1}{4}$. ∴ The real value of the product may lie anywhere between $5910\frac{1}{4}$ and $5636\frac{1}{4}$; *i.e.* in the product the first figure only is reliable. A result correct to 2 significant figures in this case is the best warranted by the data. As has already been observed, this extra digit, the second, reduces the *probable* error to a considerable extent.

∴ The best value of the product justified by the data is 5800 app., which is correct to within $\frac{1}{10}$-th of the true value.

2. Find the limits within which the value $\frac{25}{\cdot 023}$ lie ; and find the accurate figures of the quotient.

The limits of the value $\frac{25 \pm \cdot 5}{\cdot 023 \pm \cdot 0005}$. The maximum possible value $= \frac{25 \cdot 5}{\cdot 0225}$ or 1133·3... and the minimum possible value $= \frac{24 \cdot 5}{\cdot 0235}$ or 1042·1... And, $\frac{25}{\cdot 023} = 1086 \cdot 9...$

∴ The only accurate digit of the result is 1 ; and the best value warranted by the data is 1100 correct to 2 signficant figures, which is correct to within $\frac{1}{10}$-th of the true value.

* § **Possible error in Multiplication.** It is possible to ascertain by inspection the possible relative error in the product

and thus determine beforehand the possible number of accurate digits in the product, and contract the work of multiplication accordingly.

If m and n are approximate values, the product $m \times n$ lies between $(m \pm x)(n \pm y)$, where x and y are small quantities in reference to m and n respectively.

$$\text{The largest possible absolute error} = (m+x)(n+y) - mn = xy + (my + nx)$$

$$\therefore \quad \text{Largest possible relative error is nearly} = \frac{xy + (my + nx)}{m.n} = \frac{x}{m}.\frac{y}{n} + \left(\frac{x}{m} + \frac{y}{n}\right)$$

If $\frac{x}{m}$ and $\frac{y}{n}$ are of the same order of smallness $\frac{x}{m}.\frac{y}{n}$ will be negligible in value ; and the possible relative error per cent will be roughly $= 100\left(\frac{x}{m} + \frac{y}{n}\right)$.

If either, say $\frac{x}{m}$, is of a lower order than the other, a rough value of the error $= 100.\frac{y}{n}$ per cent.

If this percentage error is referred to the number of significant digits in the product, the number of accurate digits can be determined in advance, and the work of multiplication contracted accordingly.

It may be easily verified that the largest value of $\frac{x}{m} = \frac{1}{2M}$, M representing the number formed by the significant digits of m.

Also, the order of $\frac{x}{m}$ is 10^{-N}, N representing the number of significant digits of m.

Illustration

Consider : $5863 \times {\cdot}00253$.

Here, $m = 5863 \qquad x = \pm{\cdot}5$

$n = {\cdot}00253 \qquad y = \pm{\cdot}000005$.

Evidently, $\frac{y}{n}$ is of the order 10^{-3} and $\frac{x}{m}$ is of the order 10^{-4}.

$\therefore$ Roughly, the largest possible relative error is

$$=100\times\frac{y}{n}=2‰.$$

In the product $5863\times\cdot00253$, the number of significant digits is 7 ; and an error of 2‰ therein may vitiate 4 of them and has a chance of affecting the 5th digit from the end.

$\therefore$ There can be only two accurate digits justified by the data and the result can at best be obtained correct to 3 significant figures.

By contracted multiplication the product is nearly $=\underline{\underline{14\cdot8}}$.

*§ **Possible error in Division.** If m and n are approximate quantities the ratio $\frac{m}{n}$ lies between $\frac{m\pm x}{n\pm y}$, as before.

The absolute error $=\frac{m\pm x}{n\pm y}-\frac{m}{n}$, and the relative error per cent

$$=100\times\frac{\frac{x}{m}\mp\frac{y}{n}}{1\pm\frac{y}{n}}, \text{ of which the maximum value }=\frac{100\left(\frac{x}{m}+\frac{y}{n}\right)}{1-\frac{y}{n}}.$$

If $\frac{y}{n}$ is of a higher order than $\frac{x}{m}$, the possible maximum value of the relative error is nearly $100\cdot\frac{x}{m}$.

If $\frac{x}{m}$ is of a higher order of smallness than $\frac{y}{n}$, the percentage error is roughly $=\frac{100y}{n}$, neglecting $\left(\frac{y}{n}\right)^2$ &c.,

Illustrations

1. Find the correct figures of $\frac{25}{\cdot023}$.

The maximum possible value of relative error %

$$=100\times\frac{\frac{1}{50}+\frac{1}{46}}{1-\frac{1}{46}}=\frac{\cdot02+\cdot02}{1-\cdot02}\times100=4\%\text{ nearly.}$$

The number of integral places in the quotient is 4, by inspection, the first digit whereof is 1. An error of 4% may affect

3 of them. ∴ The first figure of the quotient only is reliable; and the result may be obtained as 1100 correct to 2 significant figures or within $\frac{1}{10}$-th of its real value.

2. Find the accurate digits of $\frac{1441}{\cdot 012}$.

Here, $\frac{y}{n} = \frac{1}{24}$, and $\frac{x}{m} = \frac{1}{2882}$

∴ $\frac{x}{m}$ is of a higher order of smallness. The maximum possible error in the value $= \frac{100}{24}\% = 4\%$ nearly.

The integral portion of the result consists of 6 digits beginning with 1, of which 5 digits may become affected by the error. ∴ The first digit only is accurate, and the result may be obtained at best as 120000 correct to 2 significant figures.

It may be similarly seen that the largest possible error in $\frac{14 \cdot 41}{1 \cdot 2}$ is also 4%, and the dependable figures are 12 and the value ⇒ 12.

3. Obtain the value of $\frac{136}{627}$ to the greatest degree of accuracy justified by the data.

Here, $\frac{x}{m} = \frac{1}{272} = \cdot 004$ and $\frac{y}{n} = \frac{1}{1254} = \cdot 0008$

∴ Maximum value of error possible in the computation $= 100 \times \cdot 004\%$ nearly *i.e.* $= 4‰$. By inspection, the result is ·2… Hence an error of 4 per mille might vitiate all but the first digit of the answer. ∴ The best value of the result warranted by the data will be correct to 2 significant figures, and is seen to be = ·22 app.

*§ Possible error in the evaluation of

$$\prod_{r=1}^{n}\left(\frac{p_r}{q_r}\right) = \frac{p_1 \times p_2 \times p_3 \ldots \times p_n}{q_1 \times q_2 \times q_3 \ldots \times q_n}.$$

The limits of value of $\prod\left(\frac{p_r}{q_r}\right) = \frac{(p_1 \pm x_1)(p_2 \pm x_2) \cdot (p_n \pm x_n)}{(q_1 \pm y_1)(q_2 \pm y_2) \ldots (q_n \pm y_n)}$

$$= \prod\left(\frac{p_r}{q_r}\right) \times \frac{\left(1 \pm \frac{x_1}{p_1}\right)\left(1 \pm \frac{x_2}{p_2}\right) \cdots \left(1 \pm \frac{x_n}{p_n}\right)}{\left(1 \pm \frac{y_1}{q_1}\right)\left(1 \pm \frac{y_2}{y_2}\right) \cdots \left(1 \pm \frac{y_n}{q_n}\right)}$$

The maximum relative error per cent

$$= 100 \times \left[\Pi \left(\frac{1+\frac{x_r}{p_r}}{1-\frac{y_r}{q_r}} \right) - 1 \right], \text{ and is roughly}$$

$$= 100 \times \left[\frac{1+\Sigma\frac{x_r}{p_r}}{1-\Sigma\frac{y_r}{q_r}} - 1 \right], \text{ provided squares and}$$

higher powers of $\frac{x_r}{p_r}$ and $\frac{y_r}{q_r}$ can be neglected.

$$= 100 \left[\frac{\Sigma\frac{x_r}{p_r}+\Sigma\frac{y_r}{q_r}}{1-\Sigma\frac{y_r}{q^r}} \right]$$

Illustration

Evaluate $\frac{534 \times \cdot 00846 \times \cdot 6138}{646 \times \cdot 00983 \times \cdot 5243}$ to as many significant figures as are justified by the data.

Here, $\frac{x_1}{p_1} = \frac{1}{1068} = \cdot 001$; $\frac{y_1}{q_1} = \cdot 0008$

$\frac{x_2}{p_2} = \frac{1}{1692} = \cdot 0006$; $\frac{y_2}{q_2} = \cdot 0005$

$\frac{x_3}{p_3} = \frac{1}{12276} = \cdot 00008$; $\frac{y_3}{q_3} = \cdot 0001$

etc etc.

∴ The largest possible relative error is roughly

$$= 100 \times \frac{\frac{x_1}{p_1}+\frac{x_2}{p_2}+\frac{y_1}{q_1}+\frac{y_2}{q_2}}{1-\frac{y_1}{q_1}-\frac{y_2}{q_2}}\,\% = \frac{\cdot 3}{\cdot 999}\,\% = 3‰$$

By inspection, the first significant figure of the final result will be after the decimal point. An error of 3‰ may affect the 3rd decimal place of the result, which may therefore be obtained correct to 3 decimal places. The actual value, obtained by the help of logarithms, is found to be $=\cdot\underline{827}$ app. which is correct to within $\frac{1}{100}$-th of its own value.

§ Possible error in Square Root.

The square rooot of N lies between $\sqrt{N+x}$ and $\sqrt{N-x}$, where x is the possible small error in N.

The largest possible error in the square root is then

$$= \pm \frac{\sqrt{N+x}-\sqrt{N-x}}{2} = \pm \frac{\sqrt{N}}{2}\left[\sqrt{1+\frac{x}{N}}-\sqrt{1-\frac{x}{N}}\right]$$

$$= \pm \frac{\sqrt{N}}{2}\left[\left(1+\frac{1}{2}\cdot\frac{x}{N}+\cdots\cdots\right)-\left(1-\frac{1}{2}\cdot\frac{x}{N}+\cdots\cdots\right)\right]$$

$$= \pm \frac{\sqrt{N}}{2}\cdot\frac{x}{N} + \text{smaller quantities} \overset{-}{\rightarrow} \pm \frac{x}{2\sqrt{N}}.$$

Evidently, the possible error of the square root is much smaller than that of the original number.

§ Possible error in the Square of a number.

The square of N lies between $(N+x)^2$ and $(N-x)^2$, where x is the possible error in N.

$$\therefore \quad \text{Largest possible error in } N^2 = \pm \frac{(N+x)^2-(N-x)^2}{2}$$

$$= \pm 2N.x.$$

Illustration

Ascertain the accurate digits in $(1300)^2$.

Largest possible error $= 2.1300.50 = 130000$.

$\therefore$ The value 1690000 is correct only in the million place, being $= \underline{2000000}$ app.

§ Errors in a series of observations or in different stages in computation may however be cumulative or compensating. In actual calculation due attention has to be paid to this fact in gauging the degree of accuracy of the final result. When a large number of observations are made of the same subject or similar data, it is likely that excesses in assessment would be set off by defects in other cases, and in the arithmetic average of such numbers the likely error is usually small. If the errors, however, are known to trend in the same direction, the average will be a defective assessment of the data.

The possible absolute error of the arithmetic average is equal to the average possible error of the items in the series.

The possible error is, however, hardly the likely error ; or better, the *probable* error is very much smaller than the possible error. For, in a series of a large number of similar items, particular items differ from the average usually by compensating errors, some being greater and some less than this value. If the *possible* error is estimated to be ε in the *arithmetic average* of a series of items numbering n, the *probable* error is known to be $= \frac{\varepsilon}{\sqrt{n}}$.

§ It is also known that (i) the best estimate of the *absolute error* of the total of a number of items = average absolute error of the items × square root of the number of items.

(ii) the best estimate of the *relative error* of the total = *absolute error* as above ÷ the estimated total.

If the total of say 9 items is known to be 146000 correct to the nearest thousand, it may be assumed that each of the items comprising the total may have an error ranging between 0 and 499. An estimate of the likely error of any item is naturally the arithmetic average of numbers 1 to 499, which = 249·5, or 250, say.

The best estimate of the absolute error of the total, then, $= 250\sqrt{9} = 750$.

The best estimate of the relative error $= \frac{750}{146000}$, which $\Rightarrow$ ·00514.

The estimate 146000 may then be taken to be 5·14‰ correct.

Also, the *probable error* of the average obtained from these data $= \frac{\text{av. error}}{\sqrt{\text{number of items}}} = \frac{250}{\sqrt{9}} = 83{\cdot}\dot{3}$.

EXAMPLES 40*

1. The following estimates are correct to the number of significant figures indicated. Write down the limits within which the correct values lie, and verify that the relative error is the same for all numbers with the same significant digits.

Rs 81400 ; ·0814 Kg ; 8·14 miles ; £81·4 × 10^6.

2. In converting 8352 m and 83·52 m, the metre is taken to be 39·4 in. How far can the result thus obtained be trusted in each case, assuming that a better value of the metre = 39·3708 in.?

3. In obtaining the area of a circle of diameter 2400 yds, the value of π is taken to be $\frac{22}{7}$. What are the dependable figures of the result, assuming that a better value of $\pi = 3·14159265$?

4. The receiver of a bankrupt estate declares a dividend of 3as 2·5 pies in the rupee. Find to what extent a creditor for Rs 240000 may be benefited, if the rate is worked correct to the nearest thousandth of a pie.

5. An Invoice clerk uses a Table of Nine Values for the rate 1s $5\frac{1}{4}$d per lb decimalised correct to 3 decimal places. Find how far his results may be thrown out in finding the value of 20 lb, 200 lb, 2000 lb and 20,000 lb. How many figures obtained from the Table can be relied upon in finding the cost of 3581 lb. ?

6. To how many decimal places after the pence-figure, should the rate of dividend be calculated, if the largest claim on the estate of a bankrupt is £4806 14s ?

7. If the result be desired correct to the nearest anna, find to how many places a Table of Nine Values should be worked out to give correct values for quantities of 3 integral figures.

Verify your work with a rate of Re 1 5as 6p per lb.

8. The average rate of income tax in the case of a person is 18·53 pies. He claims relief of income tax on Rs 6000 paid towards the cost of life insurance. Would a greater degree of accuracy in the rate of tax result in any better value of the extent of the relief, income tax being calculated correct to the nearest anna ?

9. An Insurance company holds Rs 45 lakhs $3\frac{1}{2}$ per cent stock at $95\frac{3}{8}$. If the yield is calculated correct to 2 decimal places, how far can the annual dividend thereon be depended upon, if obtained from the yield thus obtained ?

SECTION EIGHTEEN

ALGEBRAIC METHODS

A relationship between variables may be best followed, if expressed by means of algebraic symbols. The following illustrations may be studied for guidance in the treatment of some typical problems. A simple graph in most cases would enable the different related values to be followed visually.

Illustrations

1. Two mixtures of wine and water are mixed to contain wine and water in the proportion 2 : 5. If the corresponding proportions in the first two were 2 : 3 and 1 : 6, find the proportion in which the two have been mixed.

If x parts of mixture I is taken with y parts of Mixture II, the total quantity of wine in the resulting mixture $(x+y)$ $=\frac{2}{5}x+\frac{1}{7}y$, *i.e.* wine $=\dfrac{\frac{2}{5}x+\frac{1}{7}y}{x+y}$ of the total, and is given to be $=\frac{2}{7}$.

$$\therefore \quad \frac{4x}{5}=y \quad \text{or} \quad x:y=\underline{5:4.}$$

2. In washing lead ore it loses in weight, while the percentage of lead is increased. Write down a relationship between the percentage of lead, before and after washing.

If the percentages of lead before and after washing are x and y, respectively, the weight of the ore is w and the quantity washed out is z, we have

$$x\% \text{ of } w = y\% \text{ of } (w-z)$$

$$\textit{i.e.} \quad \frac{y}{x}=\frac{w}{w-z}$$

$$\text{or,} \quad \frac{x}{y}=1-\frac{z}{w}.$$

Ex. If 240 tons of crude ore be reduced to 192 tons after washing, the change in the lead content is given by

$$\frac{x}{y}=1-\frac{48}{240}=\frac{4}{5}.$$

3. A river-side brickfield can be used for 5 months to produce 2 lakh bricks a month or for 4 months at $2\frac{1}{4}$ lakhs a month. Assuming that there is a steady deposit of silt on the site, find for how long it may be possible to carry on production at the rate of 3 lakh bricks a month.

Assume that the initial deposit of silt is x and the rate of fresh deposit is y every month. If the quantity of earth required to make a lakh bricks is z, we have

$$2.z.5 = x + 5y\ ; \qquad 2\tfrac{1}{4}.z.4 = x + 4y$$

$$i.e. \quad x + 5y - 10z = 0 \qquad \text{and} \quad x + 4y - 9z = 0.$$

$$\text{Whence,} \quad \frac{x}{-45+40} = \frac{y}{-10+9} = \frac{z}{4-5}\ ; \text{ or } x = 5z,\ y = z.$$

If n be the reqd. no. of months,

$$3.\ z.\ n = x + n.y = 5z + nz$$

$$i.e. \quad 3n = 5 + n, \qquad \text{whence } n = 2\tfrac{1}{2} \text{ mos.}$$

N. B. As the silting is supposed to be continuous, theoretically the equations would be $10z = x(1+i)^5$ and $9z = x(1+i)^4$; where i is the rate of growth per month per unit.

$$\text{Whence } \frac{10}{9} = 1 + i\ ; \qquad \text{or, } \ i = \frac{1}{9}.$$

$$\therefore \quad 3nz = x(1+i)^n = 5z(1+i)^n \text{ app.}\ ; \quad i.e. \ 3n = 5(1+\tfrac{1}{9})^n$$

$$\log 3 + \log n = \log 5 + n \log \tfrac{10}{9}.$$

$$\cdot 47712 + \log n = \cdot 69897 + n\ [\,1 - \cdot 95424]$$

$$\log n = \cdot 22185 + n \times \cdot 04576$$

This equation is to be solved by the method of successive approximation ; $n \rightrightarrows 2\frac{1}{2}$ mos.

4. Two samples of milk are found to contain 90% and 92% of water. Pure milk contains 89% of water. By how much should a mixture of the first two be evaporated to reduce the water content to its standard value ?

Suppose, x parts of the first quality is mixed with y parts of the second ; and the whole is reduced to r times its volume by evaporation. We shall then have.

$$r.\frac{90x+92y}{100(x+y)}=\frac{89}{100}$$

$$i.e.\quad r=\frac{89(x+y)}{90x+92y}=\frac{89}{90}\cdot\frac{1+\dfrac{x}{y}}{1{\cdot}0\dot{2}+\dfrac{x}{y}}$$

Ex. If the proportion in the mixture be 4 : 5, $r=\frac{89}{90}\cdot\frac{1{\cdot}8}{1{\cdot}82}=\frac{89}{91}$; *i.e.* only $\frac{2}{91}$ part of the mixture requires to be evaporated.

*5. At a general meeting of shareholders voting is comulative. Each shareholder is entitled to cast a number of votes equal to the product of the number of shares held by him and the number of directors to be elected. A shareholder may cast all his votes in favour of one or more directors. Find the least number of shares required to elect a specified number of directors from amongst a group.

Say, the total number of shares $= n$
the total number of directors $= N$
the number of directors desired to be returned $= D$
least number of shares required to return D $= s$

Total no. of votes that may be cast $= n.N$

No. of votes that may be cast by s $= s.N$

No. of votes in favour of each of D $= \frac{s.N}{D}$

No. of votes outstanding $= N(n-s)$

No. of votes in favour of each of $(N-D)+1=\frac{N(n-s)}{(N+1)-D}$

If all of the directors D have to be elected,

$$\frac{s.N}{D}>\frac{N(n-s)}{(N+1)-D}$$

$$i.e.\quad \frac{s}{D} > \frac{n-s}{(N+1)-D}\,;\quad i.e.\quad s(N+1) > n.D$$

$$\therefore \quad \text{Least value of } s = \frac{n.D}{N+1} + 1$$

Ex. If the total number of shares = 10,000, and the minority desire to elect 4 out of 9 directors, the least number of shares necessary $\frac{10000.4}{10} + 1 = \underline{4001}.$

6. The cost of printing a book is made up of a fixed charge and a variable charge depending on the number of copies printed. A publisher desires to clear 20% p.a. on his outlay, and estimates that 500 copies of a book, priced Rs 2 each, will be sold out in 3 months' time ; 1000 copies will take 9 months to be cleared. What profit does he make if 4830 copies are printed and sold out in 50 months ?

S. P. of 500 copies = Rs 1000 and returns a profit @ 20% p.a. for 3 months *i.e.* @ 5%.

$$\therefore \quad \text{C. P. of 500 copies} = \text{Rs } \frac{1000 \times 100}{105}, \text{ and } = 500x + c,$$

x representing the variable charge per copy, and c the fixed charge.

Similarly, for 1000 copies sold out in 9 months

$$\frac{2000 \times 100}{115} = 1000x + c.$$

$$\therefore \quad 500x = 10^5\left[\frac{2}{115} - \frac{1}{105}\right] = \frac{95}{115 \times 105} \times 10^5$$

$$x = \frac{760}{483}\,;\ \text{and } c = \frac{8 \times 10^4}{483}.$$

$\therefore$ Cost of publication of 4830 copies

$$= \text{Rs } 4830 \times \frac{760}{483} + \frac{8 \times 10^4}{483}$$

$$= \text{Rs } (7600 + 166) = \text{Rs } 7766 \text{ app.}$$

Realises on sale Rs 9660, and profit = Rs 1894.

$$\therefore \quad \text{Average Return \% p.a.} = \frac{1894}{7766} \times \frac{12}{50} \times 100 = 5\tfrac{6}{7}\% \text{ app.}$$

Obviously, the publisher took an unfavourable decision in printing 4830 copies. A relationship between the number of copies (n) and the time of offtake (m months), may be written down on the basis of a 20% return p.a.

$$\frac{2n \times 100}{100 + \frac{m}{12} \times 20} = n.x + c$$

$$i.e. \quad \frac{2n}{1 + \frac{m}{60}} = n.\frac{760}{483} + 8.\frac{10^4}{483},$$

$$\text{or,} \quad \frac{966n}{1 + \frac{m}{60}} = 760n + 8 \times 10^4,$$

$$\text{or,} \quad n\left(206 - \frac{38m}{3}\right) = \frac{4000}{3}.(60 + m)$$

The following values may be tabulated :

m.........3 ; 6 ; 9 ; 12 ; 15 ;......
n......500 ; 677 ; 1000 ; 1778 ; 6250 ;......

These give a series of values of the number of copies that may be printed to ensure a return of 20% p.a. on the basis of estimated periods of offtake. It will be noted that if $m > 16\frac{1}{2}$, the equation becomes impossible, and profit cannot be maintained at the 20% p. a. level.

7. War conditions make the cost of Direct Materials and Distribution both liable to sudden fluctuations. Obtain an expression for the percentage increase in the final cost.

Representing cost of Direct Materials by M and Distribution by D, if the respective rises be x and y per centum, the increase in the final cost $= \frac{x.M + y.D}{M + D}\% = x + \frac{D}{M + D}(y - x)$ per centum.

Ex. If Distribution originally represented 40% of final cost, the increase $\% = x + \frac{2}{5}(y - x)$. If, again, $x = 5$ and $y = 20$, say, the value will be 11%. Here also the corresponding values may be tabulated for different values of x and y.

*8. B is a by-product in the manufacture of a synthetic dye A. For every 15 parts by weight of A, 100 parts by weight of B is produced, and the respective costs are estimated to be in the ratio 9 : 4. The manufacturer desires to make an all-round profit of 20% ; but cannot fix a price higher than Rs 7 8as per cwt. for *B*. How should the price of *A* be fixed, if the cost per cwt. of *B* ranges between Rs 6 and Rs 6 8as ?

	A	*B*
Quantity, say,	15 cwt	100 cwt
Cost Price, say, is	$9k$ p. cwt	$4k$ p. cwt
Total cost is	$135k$	$400k$
S. Price, say, is	Rs x p. cwt	Rs 7/8 p. cwt
Realised	$15x$	750

Total cost *A* & *B*	$535k$	Actual sale Proceeds *A* & *B*
Profit @ 20%	$107k$	$= 15x + 750$
Sale Proceeds	$642k$	$= 642k$
$\therefore$ $15x = 642k - 750$		$\therefore$ $x = 42{\cdot}8k - 50$

$\therefore$ When cost price of *B*, $4k$, = Rs 6 ; 6·25 ; 6·5 per cwt

Sale price of *A*, x = Rs 14·2 ; 16·875 ; 19·55. per cwt

*9. *A* and *B* are joint products, 40 parts of the first being obtained to every 100 parts of the second. The manufacturer finds that there is a market for *B* at Rs 8 per ton ; and *A* can be placed on the market even at Re 1 8as per cwt. If the costs of manufacture be in the proportion 7 : 3, and the estimated profit is 40% overall, establish a relationship between cost and selling prices of *A*.

	A	B
Cost of Production, say is	$7x$ p. ton	$3x$ p. ton
Quantity produced, say is	40 tons	100 tons
Total cost	$280x$	$300x$
Total cost *A* & *B* ...	$580x$	
Profit ...	$232x$	
Total to realise	$812x$	
S. P. say is	y p. ton	Rs 8 p. ton
Realised	$40y$	800

Total realised $= 40y + 800$

$\therefore$ $812x = 40y + 800$

or $y = 20{\cdot}3x - 20.$

Assuming that B is not being sold at a loss, $3x < 8$ or $x < 2\frac{2}{3}$, the following values may be tabulated.

$x = 1,\ 1\cdot5,\ 2,\ 2\cdot5$
C.P. of $A = 7,\ 10\cdot5,\ 14,\ 17\cdot5$
S.P. of $A = \cdot3,\ 10\cdot45,\ 20\cdot6,\ 30\cdot75$

Assuming, again, that A is not being sold below cost, it can be placed on the market not below Rs 20 10as p. ton if the cost be Rs 14 p. ton; and at a price not below Rs 30 12as p. ton, if the cost be Rs 17 8as per ton.

***10.** A milliner desires to turn over her stock half-yealy. She marks up her goods 50% above cost. In 5 months her sales realise R; and in the following month she desires to put up the balance of stock on sale at considerably reduced prices. If on the whole she desires to clear 25%, find how she marks down her goods at the sale. Assume that the entire stock is cleared at the sale.

Say, her stock is.........S		To realise	$\frac{5}{4}S$
and she sells in 5 mos...$\frac{2}{3}R$		*Realised*	R
Balance stock at cost...$S - \frac{2}{3}R$		Balance...$\frac{5}{4}S - R$	

List price of balance stock $= \frac{3}{2}[S - \frac{2}{3}R] = \frac{3}{2}S - R$

Selling price $= \frac{5}{4}S - R$

$\therefore$ Reduction $= \frac{1}{4}\cdot S$

$\therefore$ Rate of reduction on marked price

$$= \frac{\frac{1}{4}\cdot S}{\frac{3}{2}\cdot S - R} = \frac{S}{6S - 4R}$$

$$= \frac{S/R}{6.\ S/R - 4} = \frac{m}{6m - 4}\ ,$$

where, $m = \dfrac{S}{R}$ or, $= \dfrac{2}{3}\cdot\dfrac{S}{S'}$, where S' represents at cost the value of stock sold in the first five months.

Thus, (i) if she clears $\frac{1}{3}$rd of her stock during the first five months, $m = \frac{2}{3}\cdot3 = 2$, and reduction below marked price

$$= \frac{2}{6\times2-4} = \frac{1}{4} = 25\%$$

(ii) If she clears $\frac{1}{2}$ her stock in the same time,

$$m = \tfrac{2}{3}\cdot2 = \tfrac{4}{3}, \text{ and reduction} = \frac{\frac{4}{3}}{\frac{24}{3} - 4} = \frac{1}{3} = 33\tfrac{1}{3}\%$$

(iii) If she clears $\frac{3}{4}$-ths of her stock thus,

$$m=\frac{2}{3}.\frac{4}{3}=\frac{8}{9} \text{ and reduction}=\frac{\frac{8}{9}}{\frac{48}{9}-4}=\frac{2}{3}$$

(iv) If she desires to know when she might put up her goods at a sale at a reduction of 50% below list,

$$\frac{1}{2}=\frac{m}{6m-4} \text{ or } m=1\ ; \quad i.e.\ \ S=\tfrac{2}{3}S'$$

∴ When she has sold $\frac{2}{3}$rd of her stock she may decide on the sale.

***11.** *B* offers to join *A* in partnership with a capital of Rs 5000. *B* assures *A* that if *A* brings a capital of Rs 20000 to the business the profit may be expected to be at least Rs 6000 a year. He proposes that profits will be shared in proportion to capital, after charging interest at 4% p. a. thereon and allowing *B* a salary of Rs 300 p. m. Assuming that the proposal is sound, examine how the growth of profits will affect the partners. Also, what salary may *A* propose to *B* so that they may divide profits equally on the basis of a profit of Rs 6000 ?

Let the profit be	P in a year.
Intt. on capital	1000
B's salary	3600
Divisible profit	$P-4600$

A gets $\frac{4}{5}(P-4600)+\qquad 800=\frac{4}{5}P-2880$

B gets $\frac{1}{5}(P-4600)+3600+200=\frac{1}{5}P+2880.$

Ratio of incomes from business of A and B

$$=\frac{\frac{4}{5}P-2880}{\frac{1}{5}P+2880}=\frac{4P-14400}{P+14400}$$

A gets more than B, so long as the ratio is >1

$$i.e.\ \ 4P-14400>P+14400$$

$$\text{or}\quad 3P>28800$$

$$\text{or}\quad P>9600.$$

This shows that A cannot expect to get a larger share of the profit so long as the total profit does not exceed Rs 9600 a year.

Also, if the profit is near about Rs 6000, the ratio $=\frac{8}{17}$; that is A will get about $\frac{1}{2}$ of what B gets.

(ii) If S be the salary to B proposed by A the divisible

$$\text{profit} = 5000 - S$$

A gets on the whole $\frac{4}{5}(5000 - S) + 800$

$= 4800 - \frac{4}{5}S$, and

B gets on the whole $1200 + \frac{4}{5}S$

If these two are to be equal,

$4800 - \frac{4}{5}S = 1200 + \frac{4}{5}S$, or, $3600 = \frac{8}{5}S$; and $S =$ Rs 2250

In this case the ratio $= \dfrac{4P - 9000}{P + 9000}$,

and A will begin to get a total share of profit larger than B's after the profit has crossed the Rs 6000 level.

*12. A dealer in building materials is allowed the following discounts: 10% prompt cash on Hardware; 20% 1 month on Fittings; 20% 2 months on Paints. His sales are all for cash at list prices. If money be worth 6% p. a. find his gross profit on turnover, and how it varies with changes in the proportion of sales in different groups.

Let H represent sales in hardware
F " " " fittings
P " " " paints
and T " " total turnover $(H = + F + P)$

$$\text{Total discount} = \frac{H}{10} + \frac{F}{5} + \frac{P}{5} = \frac{1}{10}(H + 2F + 2P)$$

$$= \frac{1}{10}(T + F + P)$$

$$\therefore \quad \text{Margin \% on turnover} = \frac{10(T + F + P)}{T}$$

Value of the time accommodation

$= \frac{1}{2}\%$ on $(F + 2P)$

$\therefore$ Total gross profit % on turnover

$$= \frac{10(T + F + P)}{T} + \frac{F + 2P}{2T}$$

$$= 10 + \frac{21}{2} \cdot \frac{F}{T} + 11 \cdot \frac{P}{T}$$

Ex. Thus, if fittings and paints respectively represent $\frac{3}{16}$th and $\frac{1}{5}$th of the total turnover, gross profit % $= 10 + \frac{21}{5} \cdot \frac{3}{16} + 11 \cdot \frac{1}{5}$ $= \underline{13 \cdot 6.}$

§ Indeterminate Equations. When the number of equations is less than the number of variables, the solution is indeterminate. It is necessary in such cases to assume suitable empirical facts to obtain a set of possible values.

Illustrations

1. A number of electric light points are to be fitted with 50 and 20 watt lamps, so that the total consmption does not exceed 390 watts per hour. How many of each kind are used ?

Say, x lamps each of 50 watts and y lamps each of 20 watts are used. From data,

$$x \times 50 + y \times 20 = 390, \text{ i.e. } 5x + 2y = 39.$$

Since, for obvious reasons, x and y have both to be positive integers, the following are the possible values.

$$\text{When, } \left.\begin{array}{l} x = 1, \ \ 3, 5, 7 \\ y = 17, 12, 7, 2 \end{array}\right\}$$

The largest number of points that may thus be fitted = 18.

2. A railway wagon is loaded with First and Third class bricks to its full capacity of 40000. The freight charged is Rs 25. Apportion the charge as a rate per thousand between the two qualities, if the prices of the two qualities are Rs 21 and Rs 12 respectively.

Say, there are x-thousand First and y-thousand Third class bricks. Assume that the apportioned rates are Rs t per thousand for First class and Rs $r.\,t.$ per thousand of Third class bricks, r being < 1. The data give, $x + y = 40$; $x.t + r.t.y = 25$. Eliminating x, $t.y.(1 - r) = 5(8t - 5)$. $\because$ $r < 1$, $t >$ Re $\frac{5}{8}$ *i.e.* 10as.

A reasonable value of r may be taken to be $\frac{1}{3}/\frac{2}{7}$, or $\frac{7}{6}$ on the basis of their respective values. In that case, $t.y = \frac{35}{3}(8t - 5)$,

$$\text{or } y = \frac{35}{3}\left(8 - \frac{5}{t}\right).$$

The following values may now be tabulated :

$t =$ 10as ;	11as ;	12as ;	13as :	14as ;	Re 1 &c.
$y =$ 0 ;	8·5 ;	15·6 ;	21·5 ;	26·7 ;	35 &c.
$x =$ 40 ;	31·7 ;	14·4 ;	18·5 ;	13·3 ;	5 &c.
$v.t. =$ $5\frac{5}{7}$as ;	$6\frac{2}{7}$as ;	$6\frac{6}{7}$as ;	$7\frac{3}{7}$as ;	8as ;	$9\frac{1}{7}$as &c.

N. B. These values may be usefully displayed on a graph paper.

***§ Method of Least Squares.** It sometimes happens that two variables are related to each other by means of equations which are not fully compatible. A method of obtaining plausible values of the variable is by solving *normal equations* obtained from the data in the manner indicated below. These values, it may be noted, give errors in the relationships, the sum of the squares of which is the minimum.

Illustration

Two types of toys are produed in different quantities, and the total costs noted. Find the cost of production of 1000 units of each from the following data :

Type A No.	Type B No.	Total cost
3000	5000	Rs 530
5000	6000	„ 610
5000	8000	„ 850
6000	5000	„ 560

Taking the cost of a unit of Type A to be Re x and that of Type B to be Re y, the following relationships are obtained :

$$3x + 5y = \cdot 53$$
$$5x + 6y = \cdot 61$$
$$5x + 8y = \cdot 85$$
$$6x + 5y = \cdot 56$$

The *normal equation for x* is obtained by adding up the equations after multiplying each by the co-efficient of x in it. The *normal equation for y* is similarly obtained.

Thus,

$$\begin{array}{rr}
9x+\ 15y=\ 1{\cdot}59 & 15x+\ 25y=\ 2{\cdot}65 \\
25x+\ 30y=\ 3{\cdot}05 & 30x+\ 36y=\ 3{\cdot}66 \\
25x+\ 40y=\ 4{\cdot}25 & 40x+\ 64y=\ 6{\cdot}80 \\
36x+\ 30y=\ 3{\cdot}36 & 30x+\ 25y=\ 2{\cdot}80 \\
\hline
95x+115y=12{\cdot}25 & 115x+150y=15{\cdot}91
\end{array}$$

are the two normal equations. These are reduced to

$$\begin{array}{l} \\ \text{and} \end{array}\quad \left.\begin{array}{r} 19x+23y=2{\cdot}45 \\ 23x+30y=3{\cdot}182 \end{array}\right\}\quad \begin{array}{l} \text{Whence } x={\cdot}00766 \\ \text{and}\qquad y={\cdot}1002 \end{array}$$

That is, the cost of 1000 units of Type A = Rs 7·66 and of Type B = Rs 100·2.

SECTION NINETEEN

GRAPHICS

§ A graph is a mathematical device for the visualisation of the relationship between two sets of data. Where an exact relationship between the two is not known, statistical methods are employed to graduate the data and establish relationships between the variables to which the data would closely conform. Such relationships are also studied in reference to graphs.

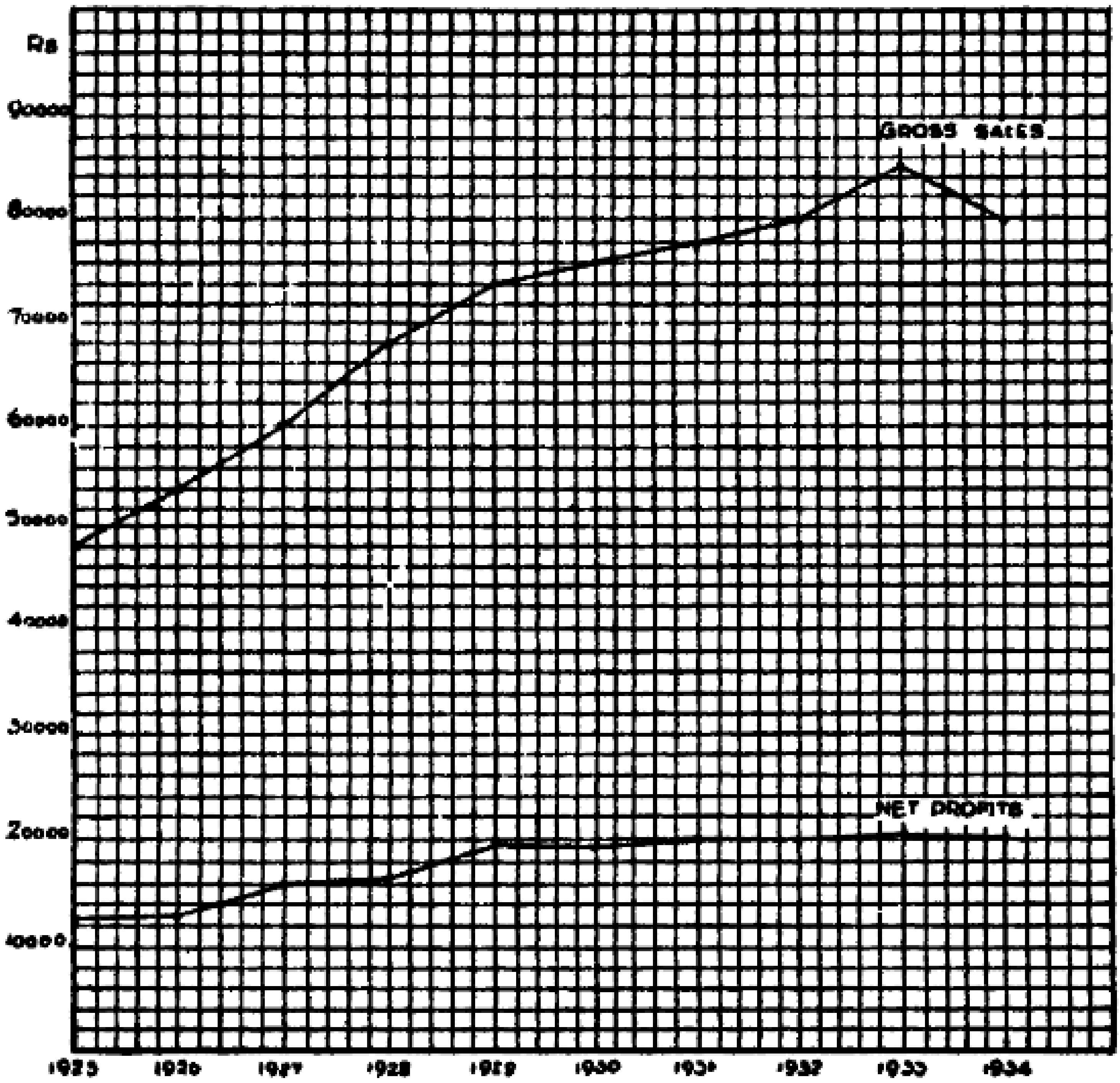
§ Two perpendicular straight lines drawn on a graph paper are called the co-ordinate axes. The horizontal axis is customarily called the x-axis, and the vertical one the y-axis. When one of the variables is time, it is customary to represent it on the x-axis, magnitudes being shown on the y-axis. Suitable scales are selected for representation on the axes—the relationship between a side of a square on the graph and the value represented thereby being shown on the graph paper, as in the illustrations.

Sets of data are now represented by points obtained on the graph in reference to the scales thereon ; and the points are ultimately joined to form a broken, or continuous curve, which gives a graphic repesentation of the facts.

§ Below is a graphical representation of the gross sales and the corresponding net profits of a manufacturer of rubber goods during 10 accounting years.

Year	Gross Sales Rs	Net Profits Rs
1925	48000	12500
1926	54000	13000
1927	60000	16000
1928	68000	16500
1929	74000	19500
1930	76000	19500
1931	78000	20000
1932	80000	20000
1933	85000	20500
1934	80000	20250

The scales selected are 5 sides of a small square = 1 year on the x-axis, and Rs 10000 on the y-axis ; the origin being taken as 1925 on the x-axis and 0 on the y-axis.



GROSS SALES & NET PROFITS

The points are plotted—one set for the gross sales and the other for the net profits. The two lines are also drawn, showing a visual relation between sales and profits.

The following observations may be made in reference to the graph thus obtained :

(*a*) that net profits increase with sales ;

(*b*) that an increase in sales is not necessarily followed by a proportionate increase in net profits ;

(*c*) that the peak value or maximum value of net profits corresponds to gross sales of Rs 85000 ;

(*d*) that commensurate with gross sales, the maximum net profits is Rs 20250, when the gross sales stand at Rs 80000, which is then the most desirable (*optimum*) figure for sales in existing circumstances.

More things can be read on the graph by those who are able to judge.

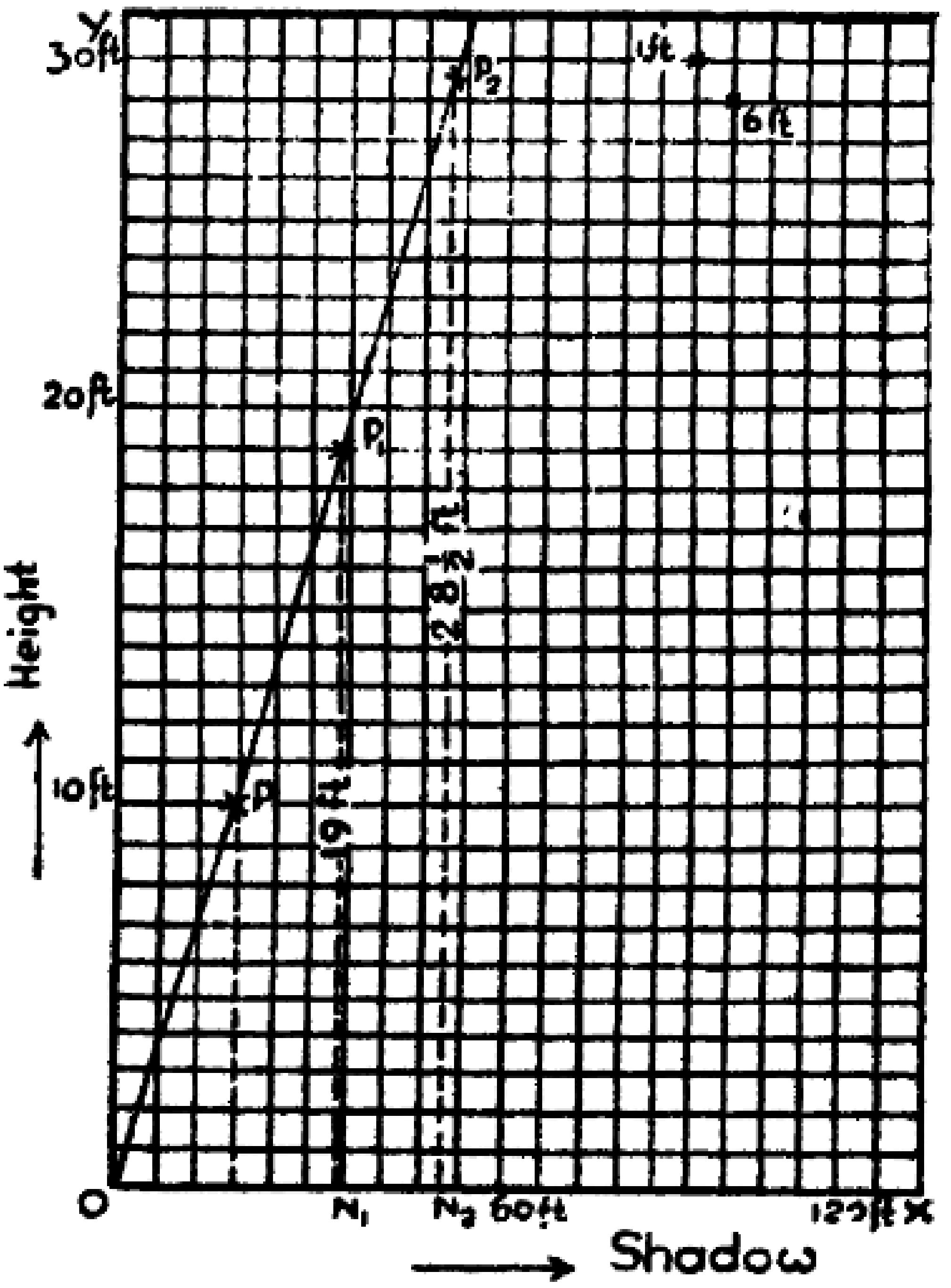
§ **The use of graphs is becoming more and more common in a large number of mathematical problems. Without going over the field covered by specialised courses of study of subjects like Statistics, Engineering, Electricity etc., the following illustrations may be usefully studied to suggest larger application of graphs in everyday work.**

§ **Calculation Graphs.** The properties of similar triangles and of conic sections provide some useful methods of simplifying calculation with the help of graphs. The following illustrations are based mainly on the theorems of similar triangles, and the method may be extended to the use of the properties of conic sections.

Illustrations

1. A pole 10 ft high casts a shadow 18 ft long. At the same instant the shadows cast by two walls are measured to be 34 ft and 51 ft. 4 in. Find the height of the walls.

It is noted that heights are proportional to the lengths of the shadows cast.



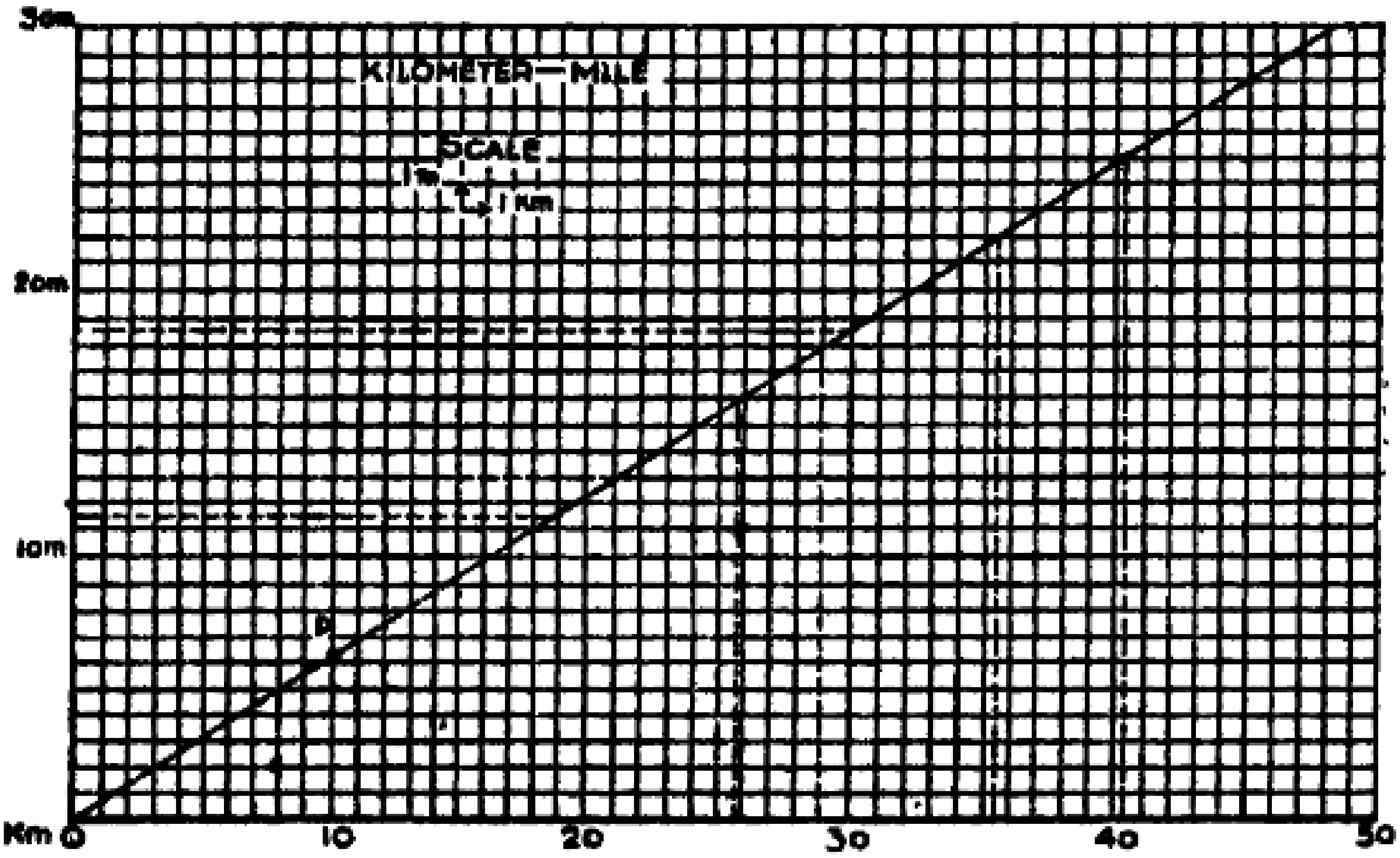
Scales are now chosen as indicated in the graph, so as to cover the given range. (Side of a small square = 1 ft on vertical axis = 6 ft on horizontal axis.)

P is now plotted to indicate shadow 18 ft and height 10 ft. OP is joined and produced.

P_1 and P_2 are now obtained on OP to correspond to shadows 34 ft and 51 ft 4 in respectively.

The ordinates of P_1 and P_2—19 ft and 28 ft 6 in—give the required heights.

2. Draw a graph to convert Kilometres into miles and *vice versa*, given 1 Km = ·621 mile. Read off equivalents of 25·7 Km ; 29 Km ; 35·4 Km ; 40·3 Km ; 11·4 miles ; 13 m ; 18·5 m ; 23·4 m ; 25·6 m.



With a view to get a fair value for plotting, it is noted that 10 Km = 6·21 miles.

Scales are now chosen as in the graph to cover the range of values.

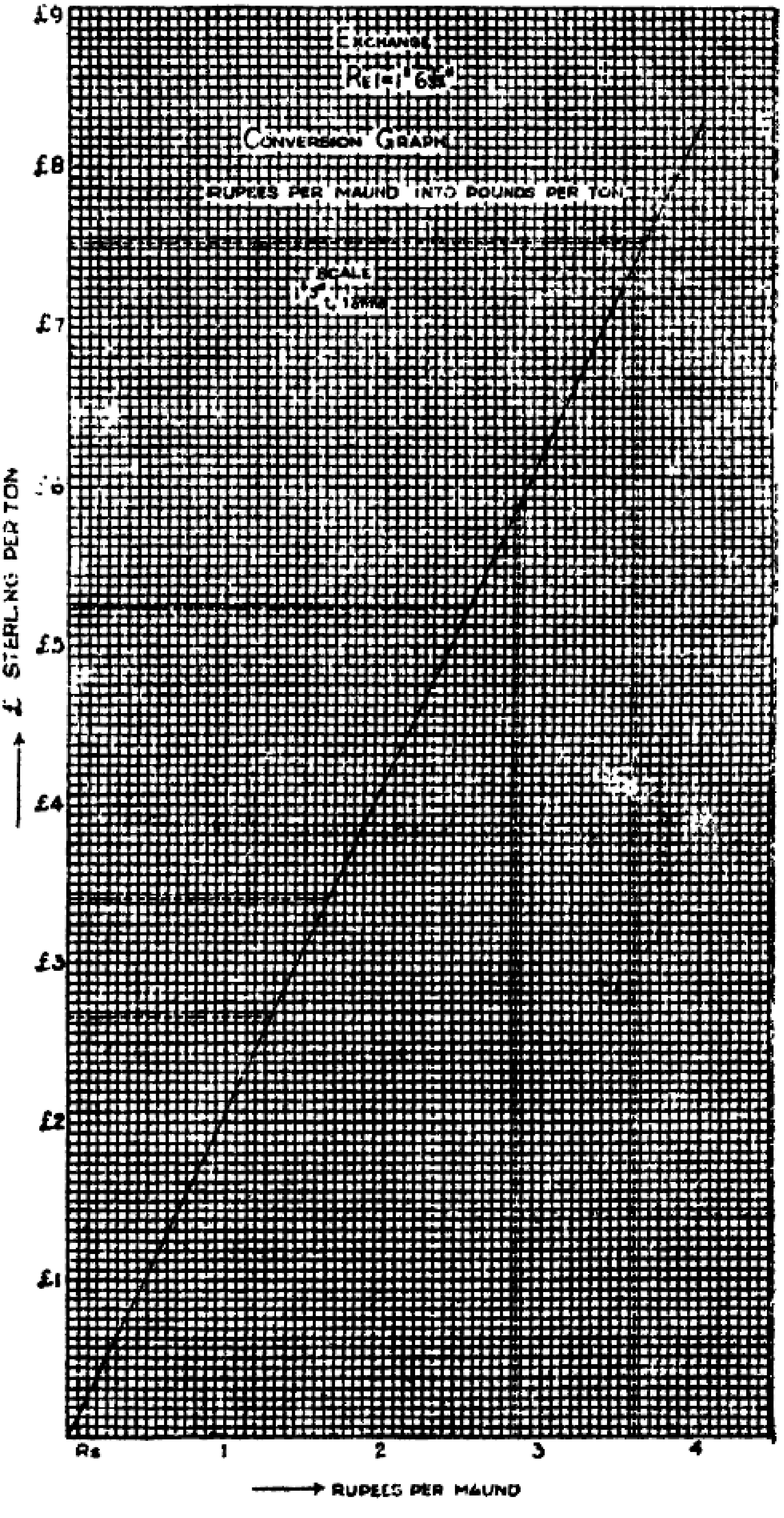
P is now plotted with an abscissa corresponding to 10 Km, and ordinate to 6·21 miles. It is now joined to the origin and produced.

From the graph the following are read off :

25·7 Km = 16m 29 Km = 18 m 35·4 Km = 22 m &c. &c.
11·4 m = 19 Km 13 m = 21 Km 18·5 m = 30 Km &c. &c.

3. Draw a graph to convert Rs per Md. into £ sterling per ton, and *vice versa*. Given 1 ton = 27$\frac{2}{9}$ md and Re 1 = 1s 6$\frac{7}{32}$d.

The multiplier to convert *Rs-per-md* into *Sterling-per-ton* is obtained by the Chain Rule to be 2·07.



Apparently, the *£-per-ton* rate requires the longer range, and is represented along the vertical axis, the *Rs-per-md* rate being measured horizontally.

Appropriate scales are chosen on a sixteenths section paper, starting with zero values at the origin. Now 2·07 is represented by 33·12 sides of a small square. The point (1,2·07) is now plotted and joined to the origin by a straight line.

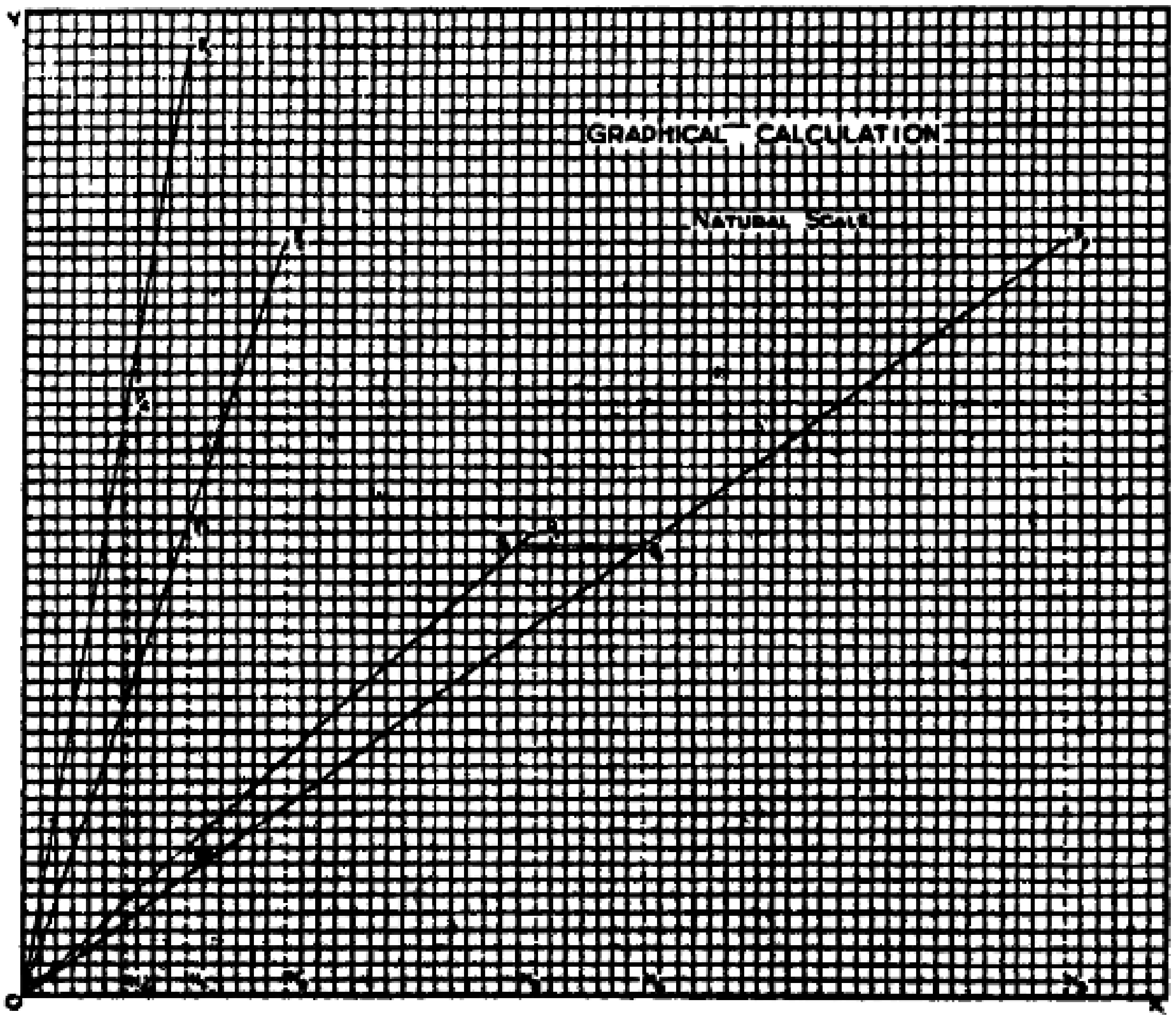
It is now possible to read off equivalent rates from a reference to this straight line.

Thus, a rate Rs 2 5as per md corresponds to £4 15s per ton and,
£2 13s 1½d per ton „ „ Re 1 4as 9p per md.

4. Find roughly the values of :

(i) $2{\cdot}864 \times 1{\cdot}605$. (ii) $3{\cdot}604 \div 4{\cdot}812$

(iii) $\dfrac{4{\cdot}6 \times 3{\cdot}8}{6{\cdot}4}$ *(iv) $\dfrac{4{\cdot}6 \times 3{\cdot}8 \times 3{\cdot}1}{6{\cdot}4 \times 2{\cdot}8}$



These values can be readily obtained from the property of similar triangles. (i) It is to be noted that the required value should satisfy the relationship $\dfrac{2{\cdot}864}{1} = \dfrac{?}{1{\cdot}605}$.

Plot the point P_1 (1, 2·9); join OP_1 and produce it. Take $ON_2 = 1·6$, then the corresponding ordinate of P_2 on the line = ?

$$\therefore \quad ? = \underline{4·62}.$$

(ii) Similarly, $\frac{4·8}{1} = \frac{3·6}{?}$.

Plot P_3 (1, 4·8). Join OP_3; and on the line find P_4 with an ordinate 3·6. Then the abscissa ON_4 is ?, and = ·62.

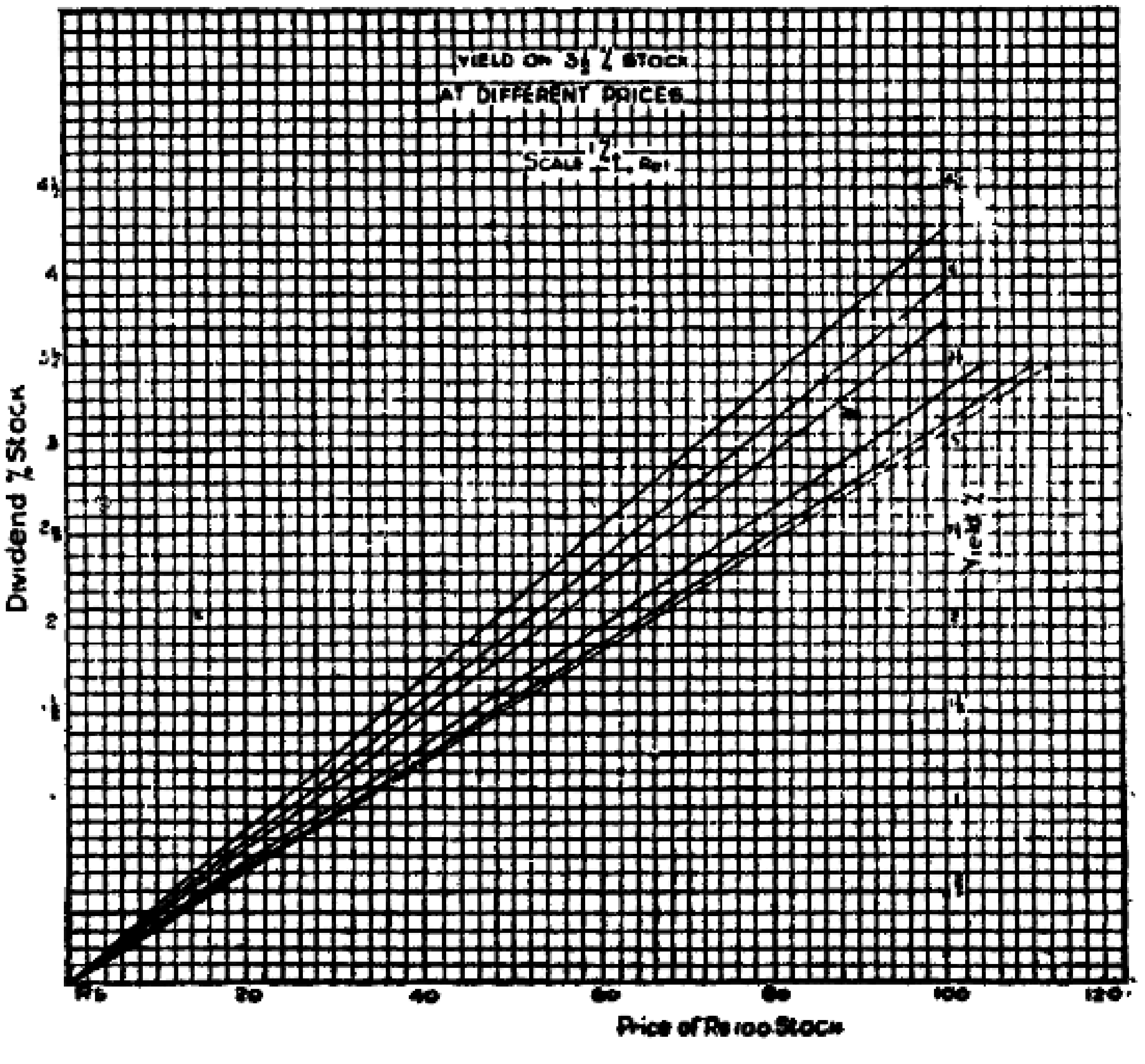
(iii) Also, $\frac{4·6}{6·4} = \frac{?}{3·8}$. Plot P_5 (6·4, 4·6). Join OP_3 and find P_6 on the line with an abscissa 3·8.

$$\text{Then } ? = \text{ordinate of } P_6 = \underline{2·73}.$$

(iv) To obtain the value of this, first find out value P_6N_6 as in (iii). Plot P_7 (3·1, 2·8); and join OP_7.

Draw P_6P_8 parallel to the x-axis to meet the line in P_8. The required value ? = abscissa of $P_8 = \underline{3}$.

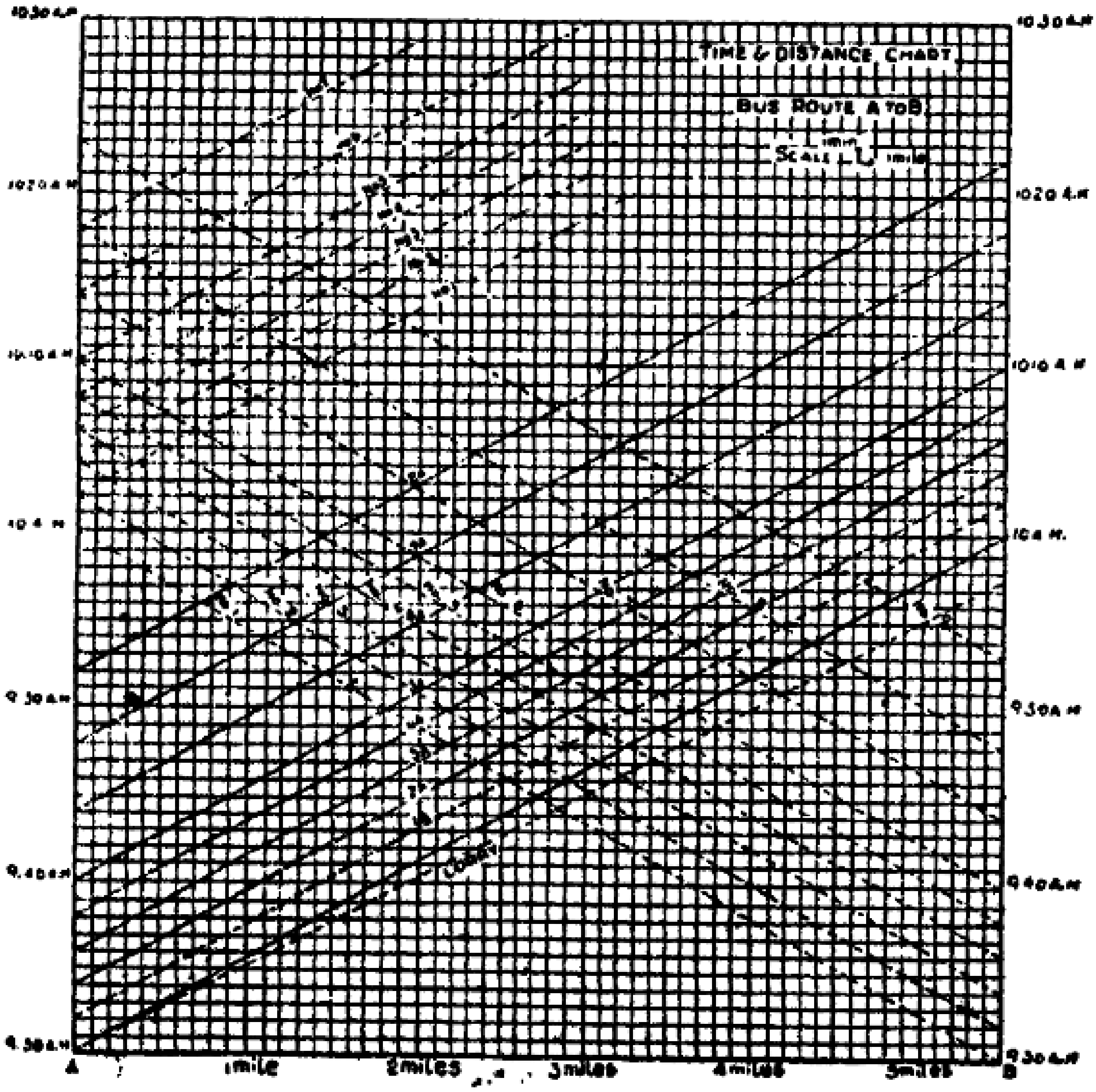
5. Chart the yield on $3\frac{1}{2}\%$ Irredeemable stock at various prices up to 120.



It will be noted from the graph that besides the axes of x and y the ordinate line at 100 (the Yield-line) is thickened to indicate the percentage yield. The prices are plotted on the $3\frac{1}{2}\%$ Dividend-line and joined to the origin ahd produced, if necessary, to meet the Yield-line. The following yields may be ascertained from the graph:

Price ...	82	88	94	104	110	112
Yield ...	4·26%	3·98%	3·72%	3·35%	3·18%	3·12%

6. From 9-30 A.M. there is a 2 minutes' service of buses between A and B, a distance of $5\frac{1}{2}$ miles. At 9-40 A.M., the interval is increased to 4 minutes and after 9-46 A.M. the service runs every 5 minutes. The average speed of the buses is 11 miles an hour. Draw a chart to show when and where the buses to and from A meet. If a lorry also starts from A at 9-30 A.M. at an average speed of 13 miles an hour, find how many buses it meets before it reaches B. Indicate on the chart the return trips from A at the same intervals as at start.



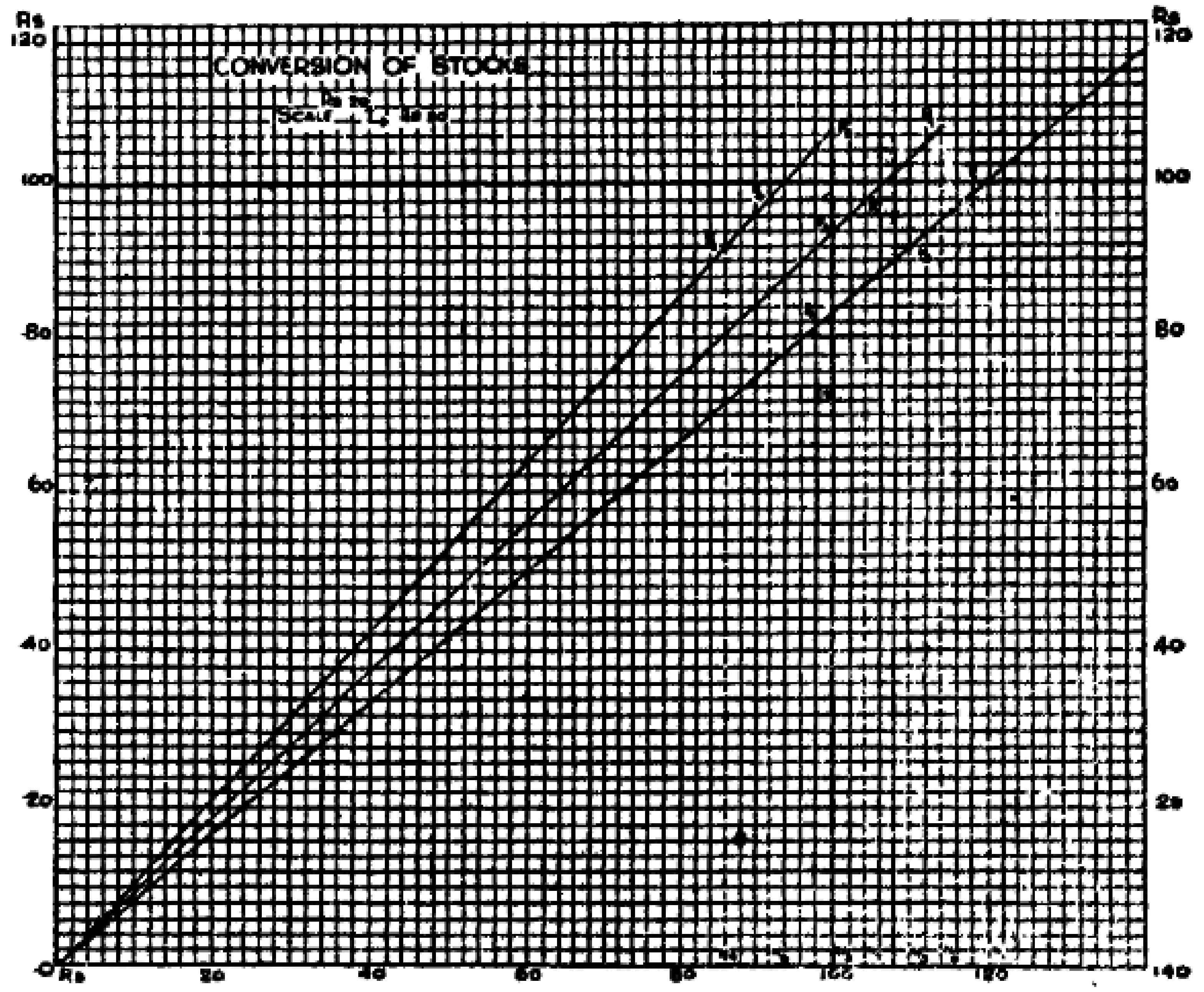
The fixing of the scales in this case does not present any difficulty. As the average speed of the buses is the same, the routes are indicated by two sets of parallel lines at the given intervals. The abscissæ and the ordinates respectively of the points of intersection give the distances from *A* and the times at which the buses meet.

The lorry has a quicker speed, and its route is represented by the lowest line from *A*. It is seen that it arrives ot *B* at 9 hrs. $57\frac{2}{3}$ min.

It should be noted how the stoppages at *A* and *B* are charted by an equivalent rise in the vertical scale from which the next journey commences.

7. A person invests Rs 10000 in the 5 per cents at 120 and sells out on the stock falling to 110. With the proceeds he purchases Municipal Stock at 86, and sells out the entire holding at 92. He re-invests the proceeds in the 4 per cents at 104, and sells out again at 114.

Find the profit or loss on his original investment at the end of each transaction.



In this case, the same scale is to be chosen on both axes, as stock and cash will both have to be represented on either axis.

Starting with an investment of Rs 100 invested at 120, the point P_1 is obtained, OP_1 is joined, intersecting the 100-line at P_2. P_2N_2 now represents the stock purchased, since

$$\frac{P_2N_2}{\text{Rs } 100}=\frac{\text{Rs } 100 \text{ stock}}{\text{Rs } 120}$$

P_2N_2 stock is now to be sold at 110 to realise Rs $\frac{P_2N_2 \times 110}{100}$ cash *i.e.* P_3N_3, which = Rs 91·7.

Loss on this transaction is @ 8·3% ; *i.e.* = Rs 830.

Again, P_3N_8 is invested at 86 ; and P_3P_4 is drawn parallel to the x-axis so that $ON_4 = 86$. P_5N_2 is now the new stock; P_5 being obtained by producing OP_4 to meet the 100-line.

This is sold at 92 ($P_6N_6 = 92$), and the proceeds re-invested at 104 ($P_7N_7 = 104$). OP_7 is joined meeting the 100-line at P_8. P_8N_2 is now the new stock, which is sold at 112 ($ON_9 = 112$). The corresponding ordinate P_9N_9 now represents the final realisation.

The height of P_9 above the hundred-line = 7 and represents a profit of 7% on original investment.

Final profit on Rs 10000 = Rs 700.

§ Figures in the following cases may be drawn by the student.

1. Find a mean proportional between 2·2 and 3·1.

Take $AB = 3{\cdot}1$ and $BC = 2{\cdot}2$ in the same straight line. Construct a semicircle on AC. Produce the ordinate through B to meet the circle at P.

$\because$ $PB^2 = AB.BC$, PB measures the mean proportional between AB and BC. $PB = \underline{2{\cdot}6}$.

2. Find the square root of 3·8.

Describe a semicircle as above, AB being $= 3{\cdot}8$ and BC being $= 1$. Draw the ordinate BP as above.

Required square root $= PB = \underline{1{\cdot}95}$.

3. Draw a graph to represent the yield on stocks and shares at different prices.

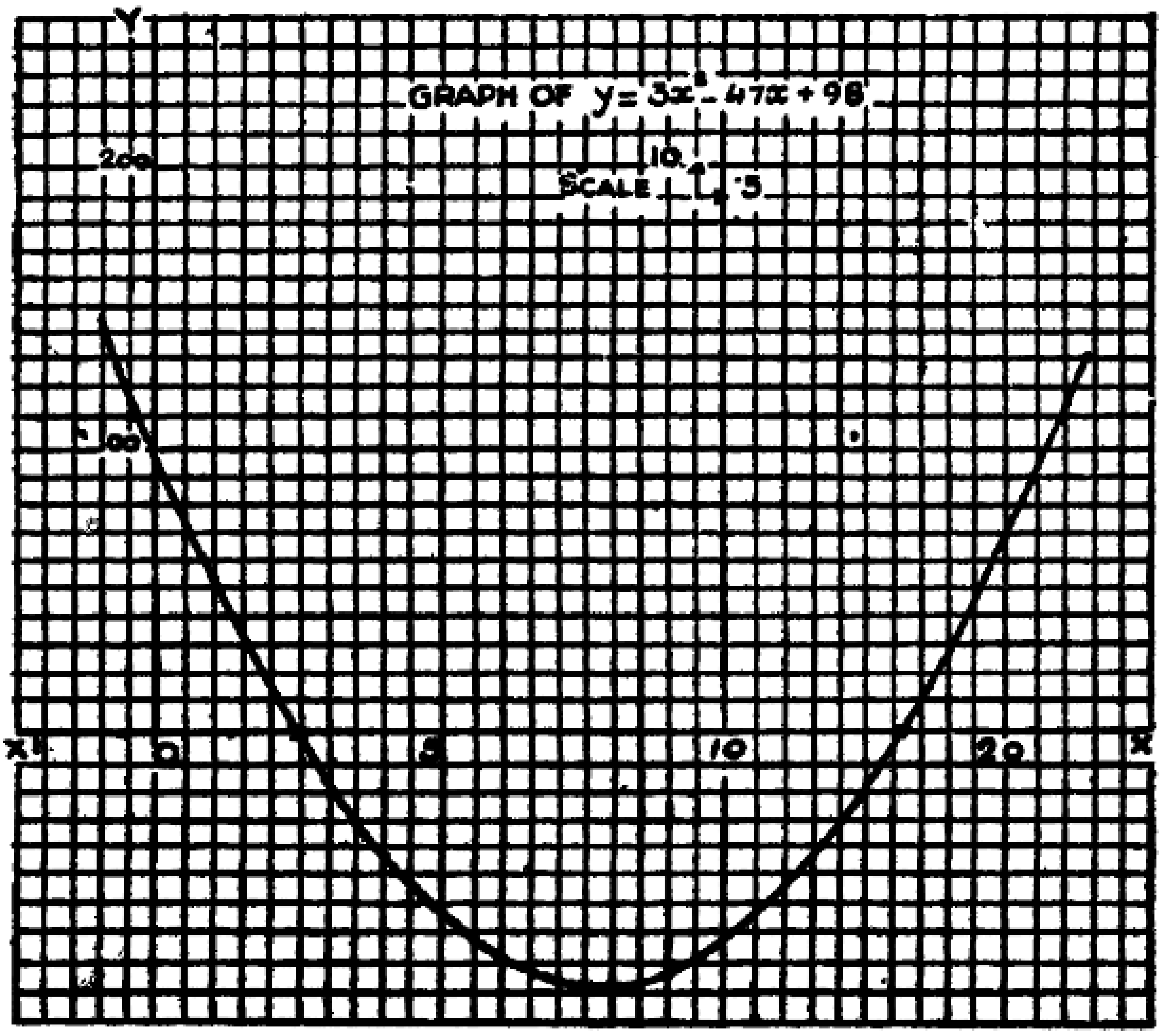
The yield-line may be thickened and scaled to represent yield percentages. The y-axis may be differently scaled in Rupees, Annas to indicate dividends. The following may be drawn on the graph and the respective yields read off.

Dividend	Re 1	Re 1 8as	2¾%
Paid-up Value of Shares or Stock	Rs 20	Rs 40	Rs 100
Price	Rs 32 12as	Rs 46	Rs 92
Yield%	3·075	3·32	2·955

§ **Algebraic Graphs.** To find the roots of the equation $ax^2+bx+c=0$, the graph $y=ax^2+bx+c$ is plotted. The roots are given by the abscissæ of points where the curve meets the x-axis; *i.e.*, where $y=0$. Simultaneous equations of two variables can be solved by noting the co-ordinates of the points of intersection of the two curves representing the two equations.

Illustration

To find the roots of the equation $3x^2-47x+98=0$



The following values of x and y are tabulated for

$$y = 3x^2 - 47x + 98$$

$x = 0, \ 1, \ 2, \ 3, \ 4;$

$y = 98, \ 53, \ 16, \ -16, \ -42;$

$x = 7, \ 8, \ 9, \ 10, \ 12, \ 13, \ 14, \ 16.$

$y = -84, \ -86, \ -82, \ -72, \ -34, \ -6, \ 28, \ 114.$

The points are plotted and the curve through them is drawn intersecting the x-axis at 2·5 and 13·2. These are evidently the values which make $y = 0$, and are thus the approximate roots of the given equation.

§ A continuous curve represents a relationship between x and y, which is of the form $y = f(x)$ or $f(x, y) = 0$. At any point on the curve *the rate of increase* in the value of y corresponding to a small increase in the value of x, may always be obtained by drawing a tangent to the curve at the point and reading its gradient to the x-axis from the graph.

When the data give points which do not lie on a continuous curve, a continuous curve may in most cases be drawn to fit the data, so that the plotted points are fairly distributed on and about the curve.

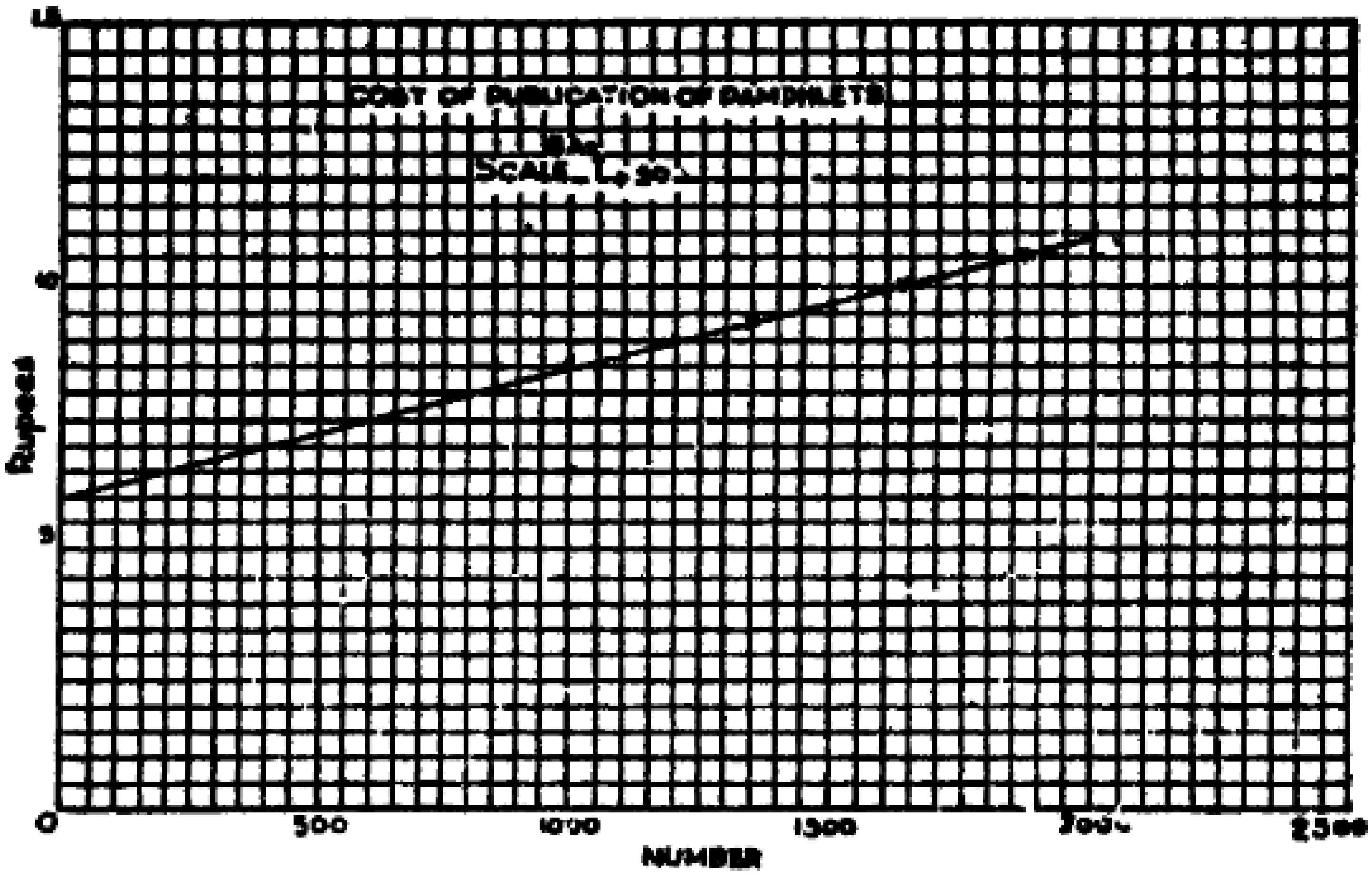
Following is an illustration of such *curve-fitting* with the help of the eyes only. Statistical methods, of course, provide effective methods for tackling data of all kinds.

Illustrations

The following figures represent the number of copies of pamphlets of the same size and the cost of publication. Plot the points, and from the data find a probable relationship between the number of copies printed and the cost of publication.

No. of pamphlets	Cost
1000	Rs 8 10as
1200	Rs 9 2as
1400	Rs 9 7as
1500	Rs 9 11as
1600	Rs 9 15as
1800	Rs 10 9as
2000	Rs 11 2as.

It is noted that the points lie nearly on a straight line, which is drawn so that the points may lie close about it. The line



makes an intercept representing Rs 6 on the y-axis, and has a gradient $= \frac{1}{400}$.

The required relationship between the cost of publication (y) and the number of copies (x) is then $y = \frac{1}{400} \cdot x + 6$.

This graph may now be used directly to read off the probable cost of publication of intermediate numbers of copies. Thus :

1350	copies	will cost	Rs 9	6as
1625	„	„	Rs 10	1as
1860	„	„	Rs 10	10as
	&c.	&c.		

§ **Statistical Graphs.** The chief use of graphs in business is the representation of statistical facts. Suitable scales are chosen to represent data along the two axes, so that the range of data may be fully covered in the space. It is not necessary to start from the origin as zero, which may be pointed to indicate a value about the minimum of the range. Where time is one of the variables, it is customary to represent it on the x-axis.

Such graphs provide a very convenient representation of facts, visualising them effectively for purposes of comparison. They are used to study the effect of one set of factors on another, indicate the trend of movement of values, to observe maxima and minima values, and for a thousand other purposes.

§ Determination of Laws from Observation. Graphics also provide a very useful method of determining relationships between two variables, which may otherwise be difficult of attainment. The scientific method of obtaining such relationships is by the methods of Statistics. Graphics provide a rough and ready method.

The data are plotted on a suitable scale, and a continuous curve is passed through the path indicated by the points, so that the points lie evenly about the line and do not all trend away therefrom.

Fairly accurate results are obtained when the lie of the points indicates a straight line. The equation to such a line is obtained by noting the intercept of the line on the y-axis, symbolised by c, and the slope or gradient of the line, symbolised by m. The relationship between x and y is then $y=mx+c$.

It will be noted that the value of m can be easily obtained at any convenient point on the line.

The distances between the plotted points and the corresponding points on the line may be regarded as errors of observation ; and usually they may be ignored in establishing an algebraic relationship between the variables.

When the curve represents an equation of second or higher degree, the usual theorems of conic sections may sometimes lead to the formulation of a function of x and y. The results obtained, however, may frequently be of doubtful value, unless special care is exercised in fitting the curves.

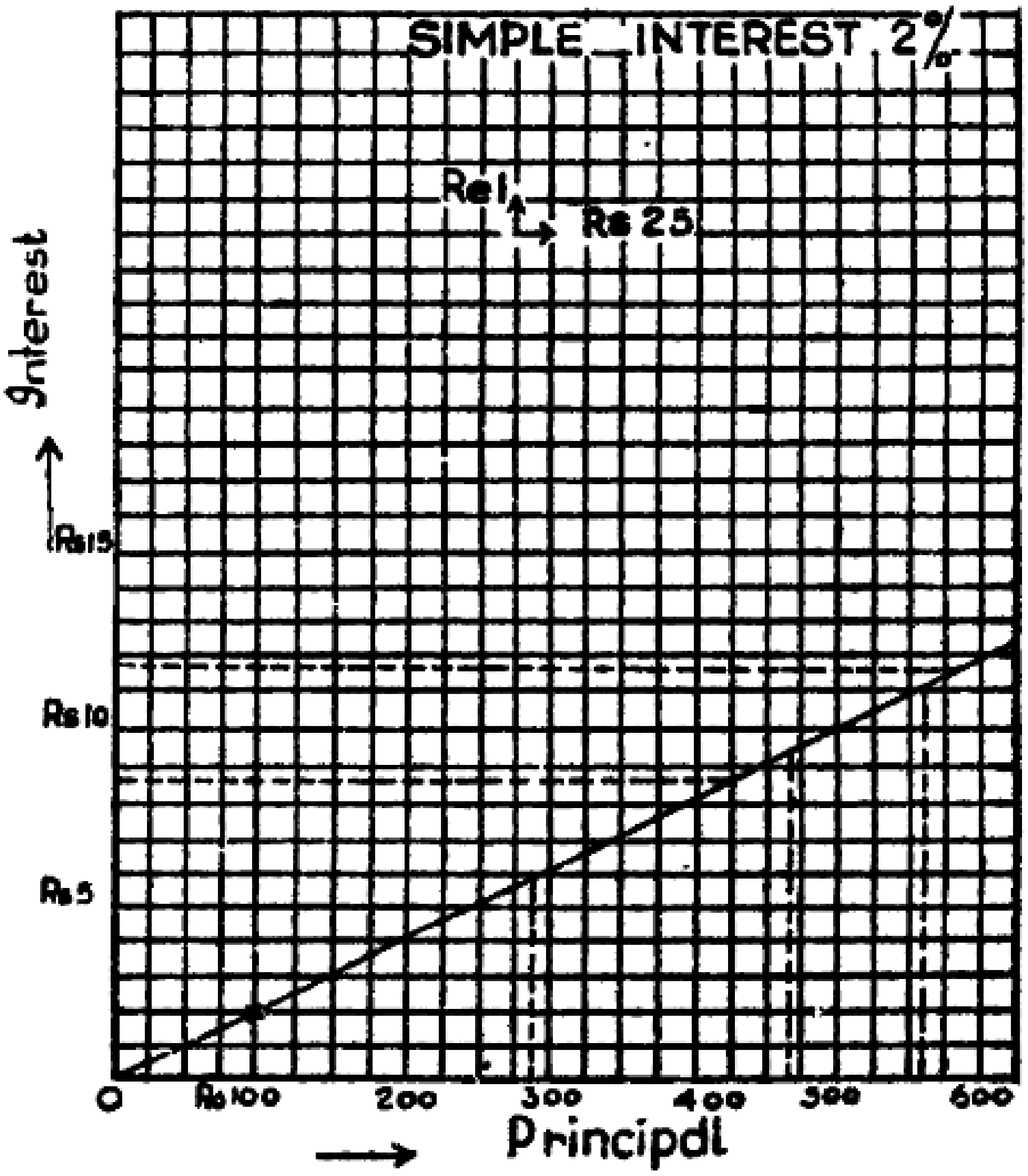
§ Interpolation At any point of a series of closely graduated data, a fairly good value may be obtained by what is called straight-line interpolation. Arithmetically, the range between two successive values is divided proportionately to fit in with the requirement, as has been shown in a previous Section. Graphically, the continuous curve is drawn through the points given by the data, or as close to them as possible; and the required interpolated value is obtained by inspection to correspond to a given value of the abscissa or the ordinate, as the case may be.

The following illustrations will suggest methods that may be usefully followed.

Illustrations

1. Draw a graph to represent simple interest for 1 year at 2% per annum for amounts up to Rs 600.

Choose scales as in figure below. Draw the straight line through the origin and the point (100, 2).



The following may be easily read off.

Interest for 1 year on Rs 465 is Rs 9 5as; on Rs 560 is Rs 11 2as.

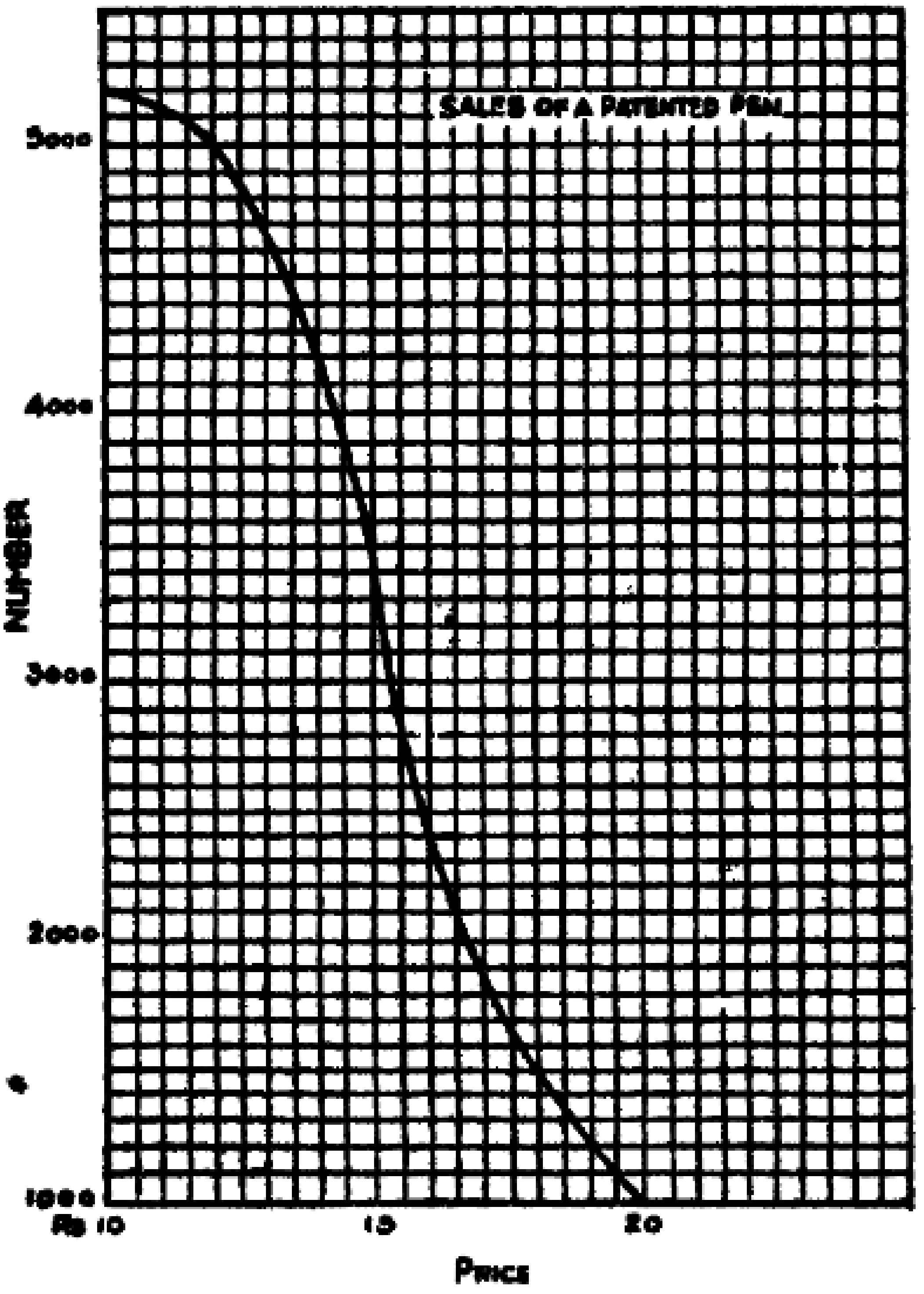
Principal earning interest in 1 year of Rs 8 8as is Rs 425; of Rs 11 12as is Rs 590 etc.

2. A patented Fountain Pen is placed on the market under conditions noted below :

No. of Pens sold	1000	1500	3500	5000	5200
Corresponding Prices	Rs 20	Rs 18	Rs 15	Rs 12	Rs 10

Find by inspection the largest number of Pens that may be sold, and the corresponding price, by interpolation.

It is observed that the turnover at a price is represented by the area of the rectangle contained by the abscissa and ordinate at the corresponding point on the graph starting at price 0. It may then be noticed by inspection that a price about Rs 13 would give the maximum turnover.

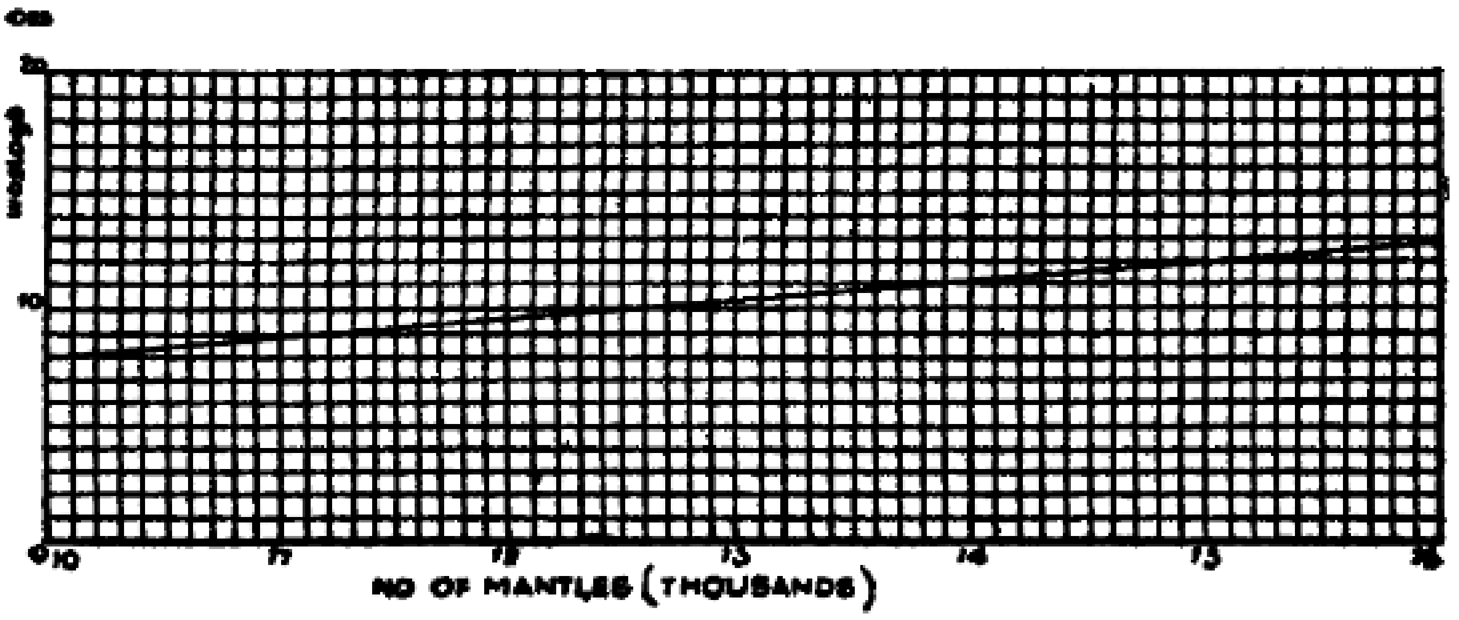


3. In the manufacture of gas mantles, the factory superintendent makes note of the following facts :

	Jan.	Feb.	Mar.	April	May
No. of mantles turned out (*thousands*)	15·0	13·6	12·0	12·8	14·6

Wasted materials in final stage (*oz.*)	12·0	10·9	9·5	10·2	11·8

Find a possible relationship between the output and the wastage; and estimate the possible wastage when production reaches 16000.



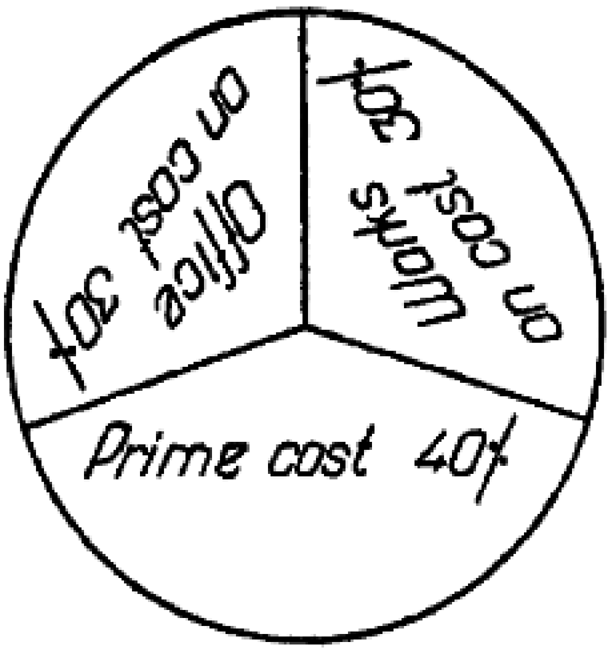
It will be observed from the lie of the plotted points that a straight line may fit the data. Here, c is obviously $=0$, since when $x=0$, $y=0$, and $m=\frac{4·8}{6}=·8$.

The relationship between the variables way well be represented by the equation $y=·8x$

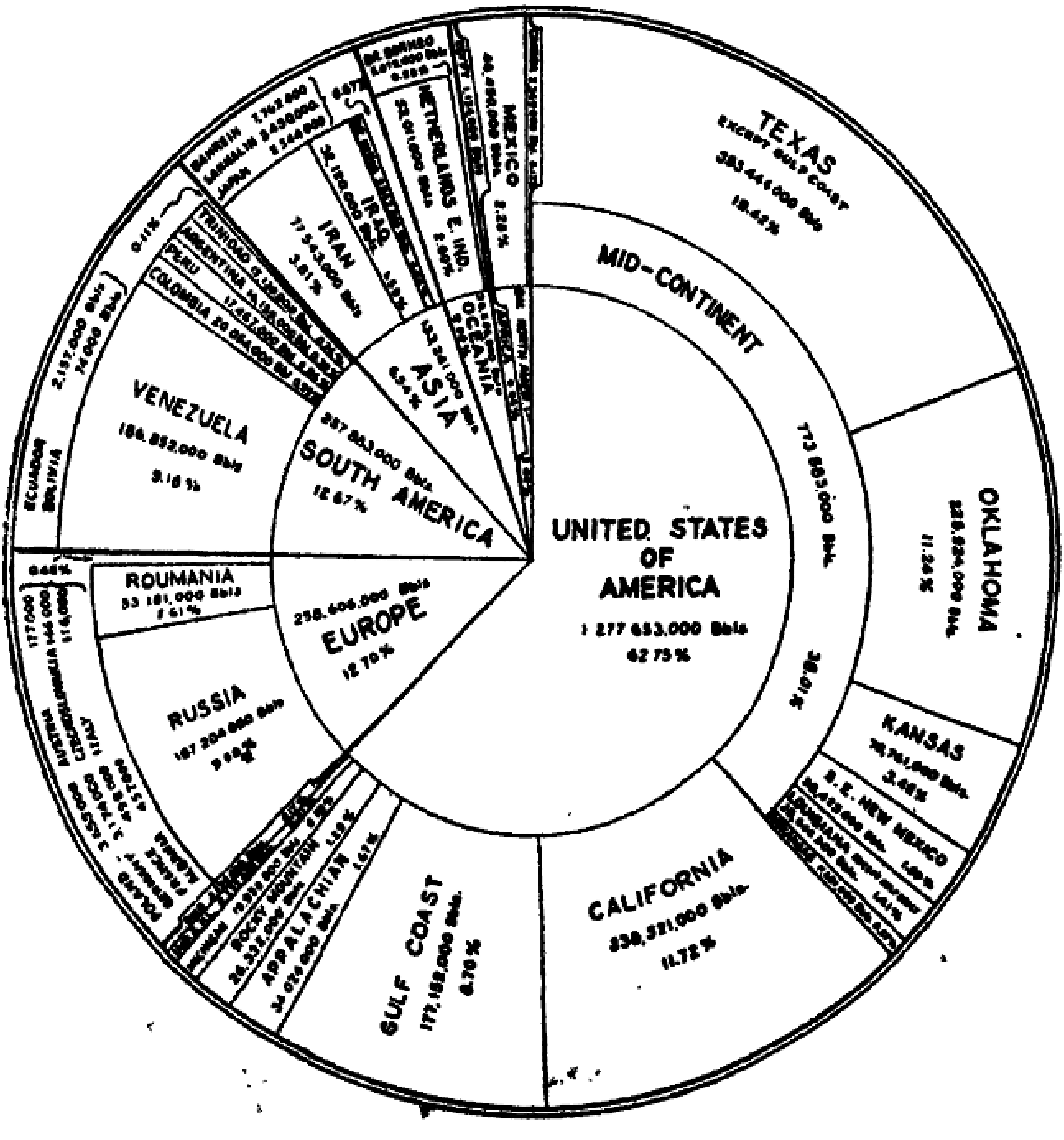
The trend of the line shows that the wastage of materials when production is 16000 = 12·8 oz.

§ **Business Charts.** Charts are growing in use in business, and are capable of representing facts and explaining them in a forceful manner. It is not possible to discuss within the scope of this book even the more important devices now commonly used in this connection. A brief description of some of the useful methods, that are easy to manage, is given in the following pages. The limits of space make it impossible to discuss all the important conclusions that the illustrations in the following pages lead to in regard to the respective data. The student will do well to make his inferences and have them verified by reference to his teacher. The data in the Sections on Statistics, Types and Averages &c. in this book should provide useful materials for practice in the methods of Graphics.

§ The *Pie-Chart*, in spite of its technical defects, has its use in visualising the weight of items in a composite quantity. It consists of a circle, broken up radially into portions to represent, in proportion to their respective areas, the value of the items under investigation. Thus in the adjoining figure, representing cost of production, different sections indicate the Prime Cost = 40%; Works Oncost = 30%; Office On cost = 30%.



Distribution of Petroleum Production



It should be remembered that the areas of sectors are proportional to the angles at the centre. The bottom chart on the last page taken from The Oil is an excellent illustration of complex data.

§ *Pictograms or Figure Charts.* In comparing the size of a series of similar items the device of using geometrical figures or pictures of varying sizes is commonly used. Although such charts are of little scientific use, they serve to make a direct appeal to the layman. Thus the production of wool in various parts of the globe may be represented by sheets of different sizes, or by bales of different sizes. It should be carefully noted that the relative dimensions do not indicate relative sizes directly.

Thus, a ⊙ with the radius doubled covers an area 4 times as great :
a □ " a side doubled " " " " " " " :
any picture with its linear dimensions doubled " " " " ;

In the representation of solid figures, doubling the dimensions would amount to representation of data eight times as big.

§ The *Bar-Chart* is quite a useful method of presenting facts reliably. It consists of a number of areas on the graph paper, usually made up in Indian ink or differently hatched, having the appearance of solid bars. Below are given a few illustrations which explain themselves. They should be carefully studied and the details mastered.

A bar chart, to be effective, should invariably start from zero. The bars should be of uniform width and equally spaced. If time is a variable, it should be shown on the horizontal axis.

A compound bar chart may usefully be made up in colours, to be more attractive.

Bar charts may be used to illustrate simple facts, in which each item recorded is self-sufficient. The following, among others, are specially adapted to representation by bar charts :

(i) Stock Records : A control over stocks may be arranged, by representing stocks by plain bars, and inking up portions thereof from the top or bottom to indicate sales or deliveries, as and when necessary.

(ii) Periodical or seasonal fluctuations in production or deliveries.

(iii) Income and Outgo by items, or by periods, arranged along opposite sides of axes.

(iv) Exports and Imports.

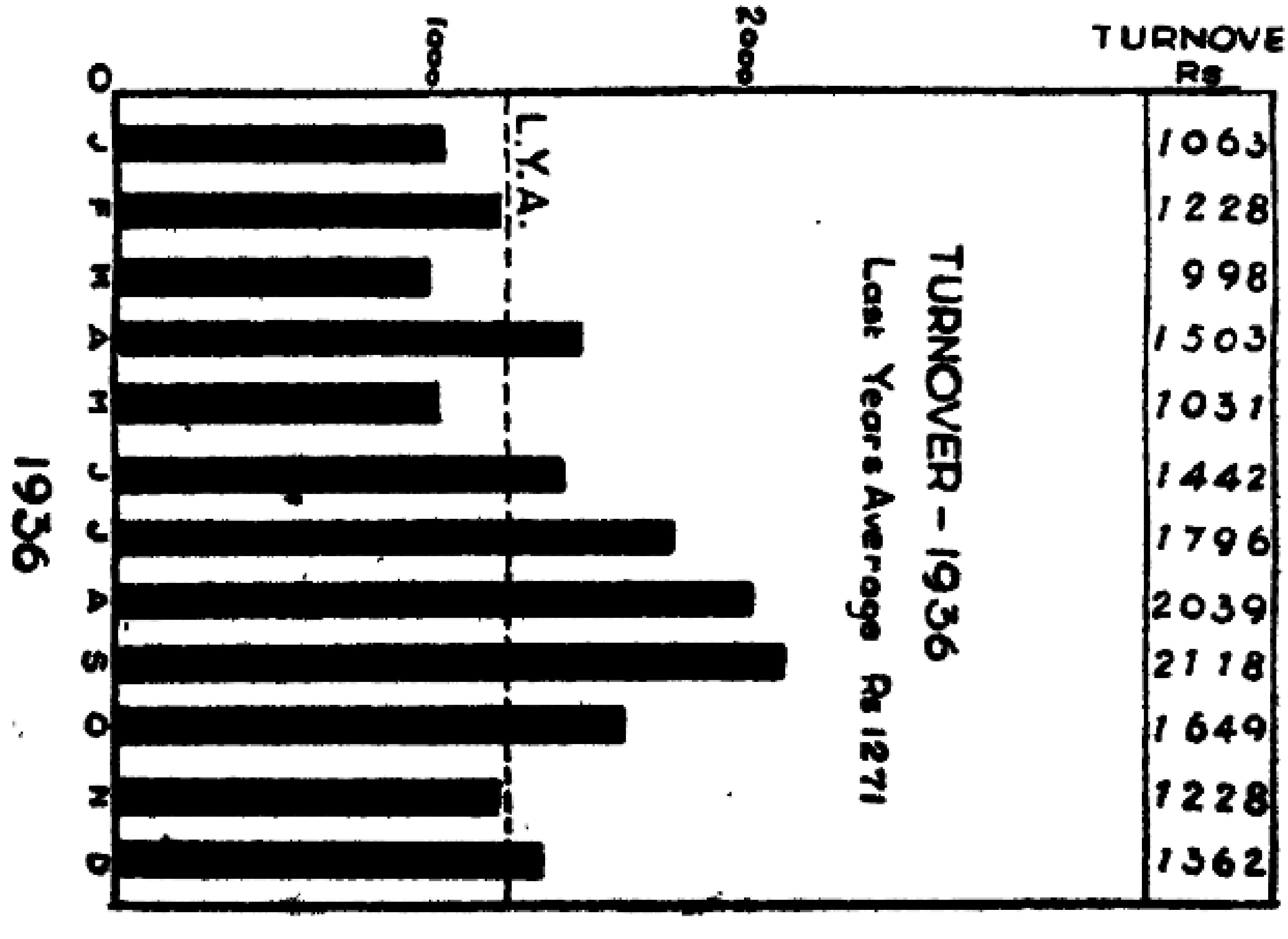
(v) Profit and Loss &c. &c.

It should be remembered that for correct representation of facts, bars should be drawn on suitable scales, with the zero-line shewn.

When an item is made up of several parts, a composite bar diagram gives a very effective representation.

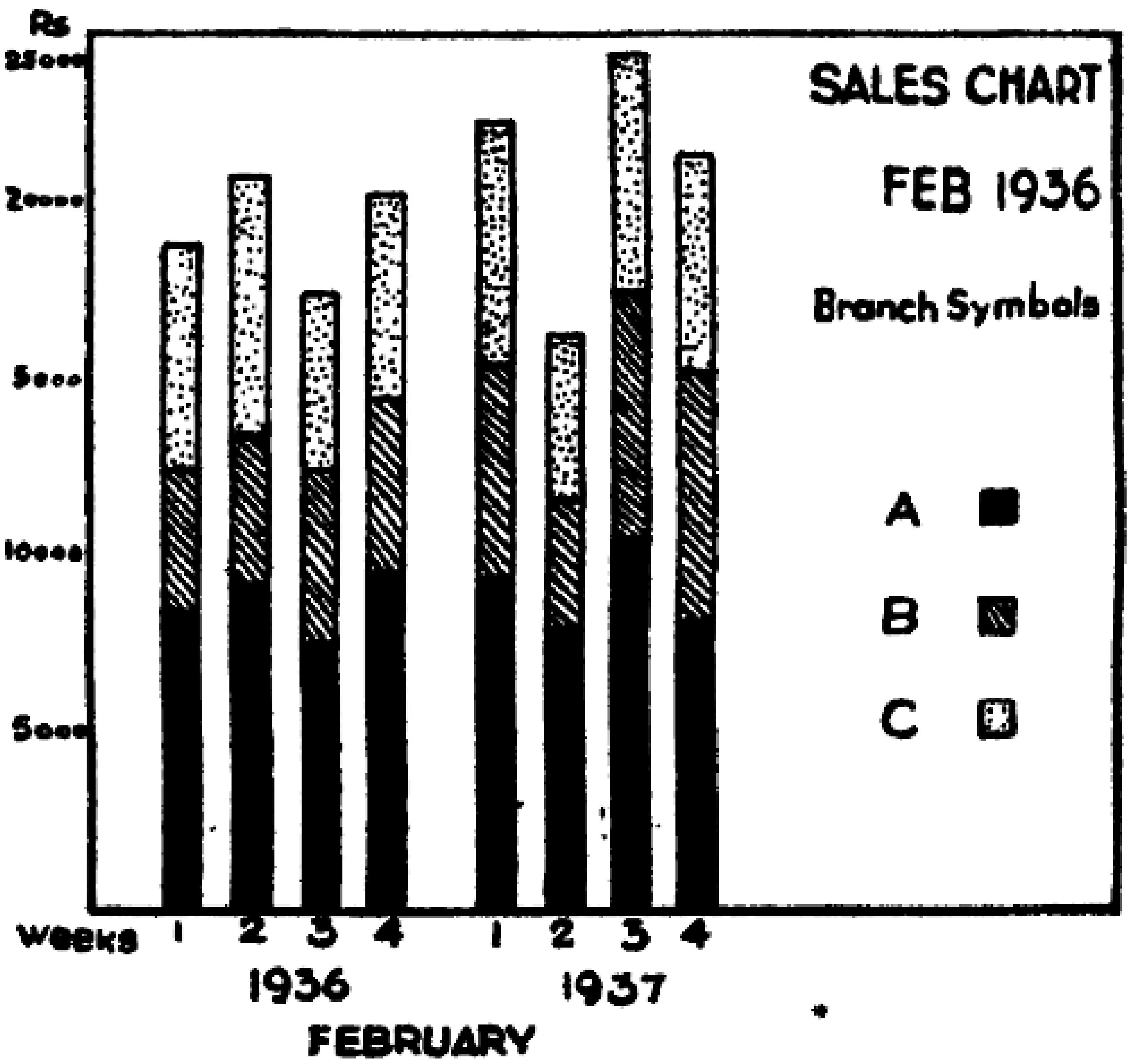
Relative portions of the bars are differently hatched, dotted or darkened out, or better, coloured differently, to give a pictorial visualisation of the behaviour of the parts and the whole. A bar diagram has this special advantage over line charts, which is not capable of being split up, in addition to being simpler to read.

The two following illustrations may be studied in detail.



Above is a simple bar-chart showing the value of sales during the year 1936. The variations from month to month from the last year's average, and the seasonal character of the demand are clearly brought out.

Following is a representation of the sales effected by different branches *A*, *B* and *C*, during February 1936 and during the same period in 1937. The length of the differently drawn bars represents the turnover of the respective branches.



It may be noted how effectively the chart visualises a heap of figures.

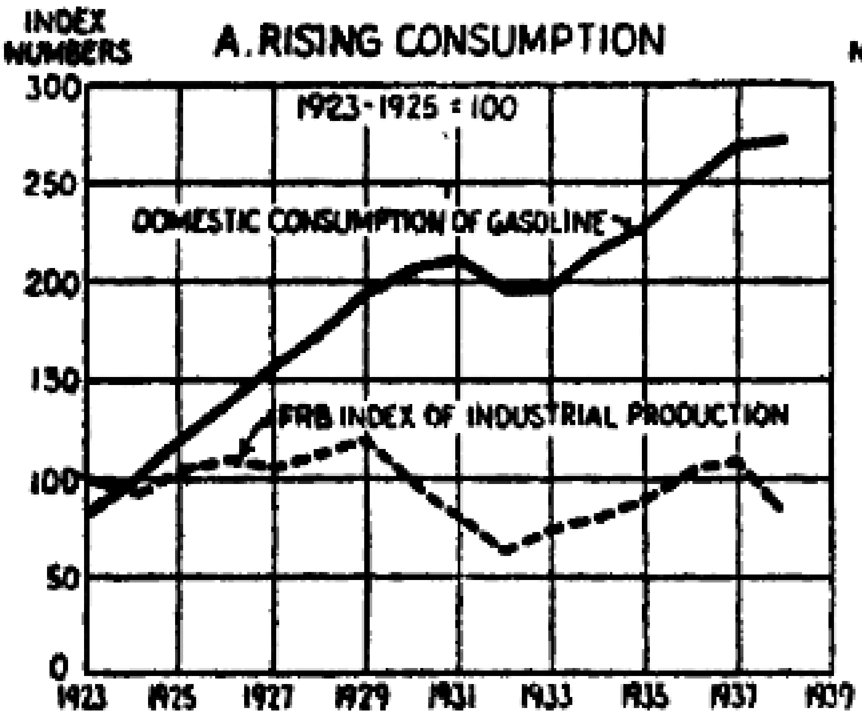
§ *Line-Charts* represent a wide field of service in the business house, the factory, or the laboratory. It has decided advantages over almost every other form of chart including that of a reliable accuracy with regard to almost all possible kind of data.

They are usually of two distinct types. In the first, the actual amounts of change are shown ; and in the second the rates of change are plotted. Each serves its own purpose in respect of data suited to it. The ratio-chart is best drawn on semi-logarithmic paper as shown below. The student is referred to advanced books on Graphics for more detailed discussions.

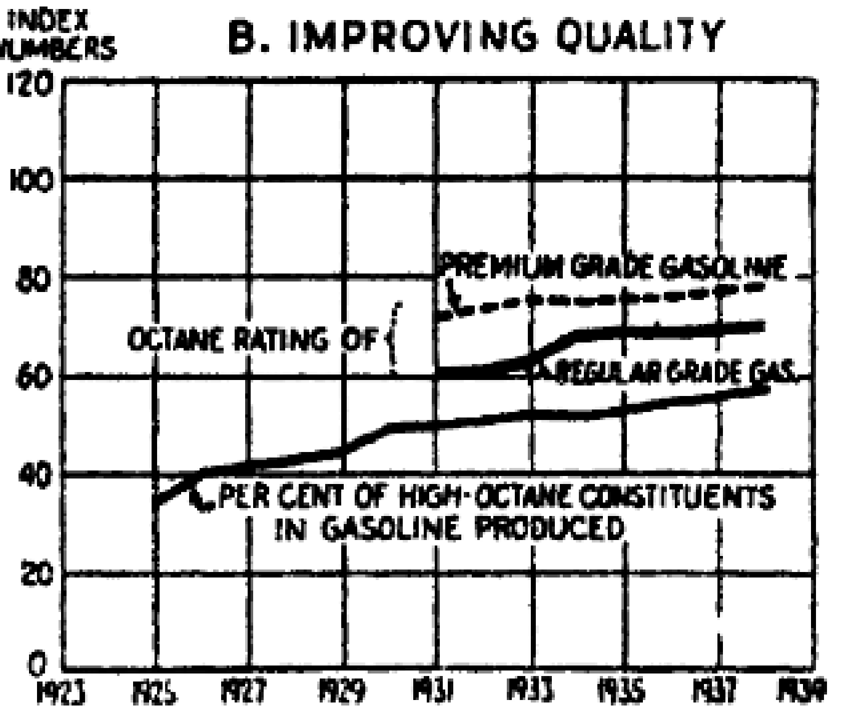
The group of charts below explain themselves.

The following general observations are offered :

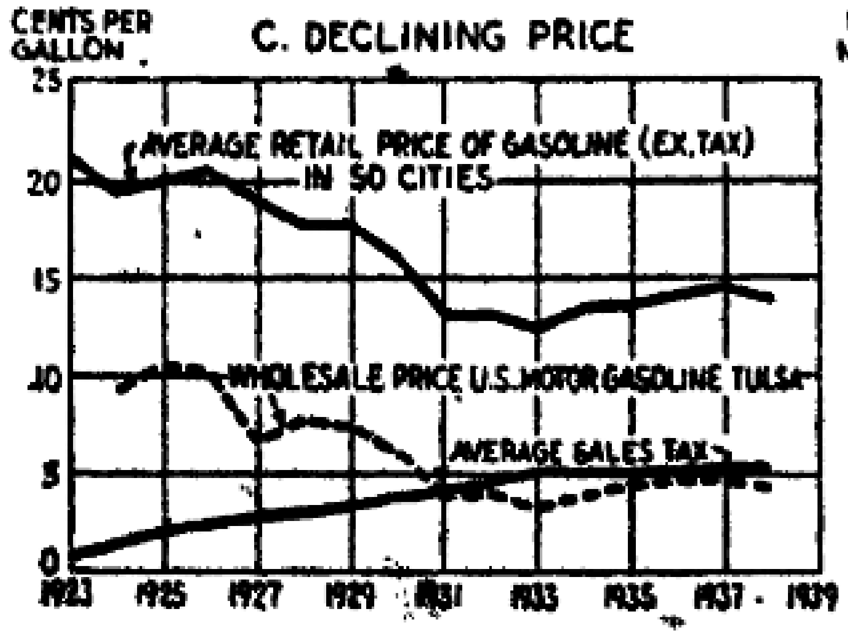
1. A full and proper heading should be given.
2. Scales should be carefully chosen, and clearly indicated.
3. While the scales need not be the same on both axes, the object of the chart should be kept in view in choosing them.
4. It may sometimes be convenient to start at the origin not from zero, but from a value about the minimum, extending to the maximum.



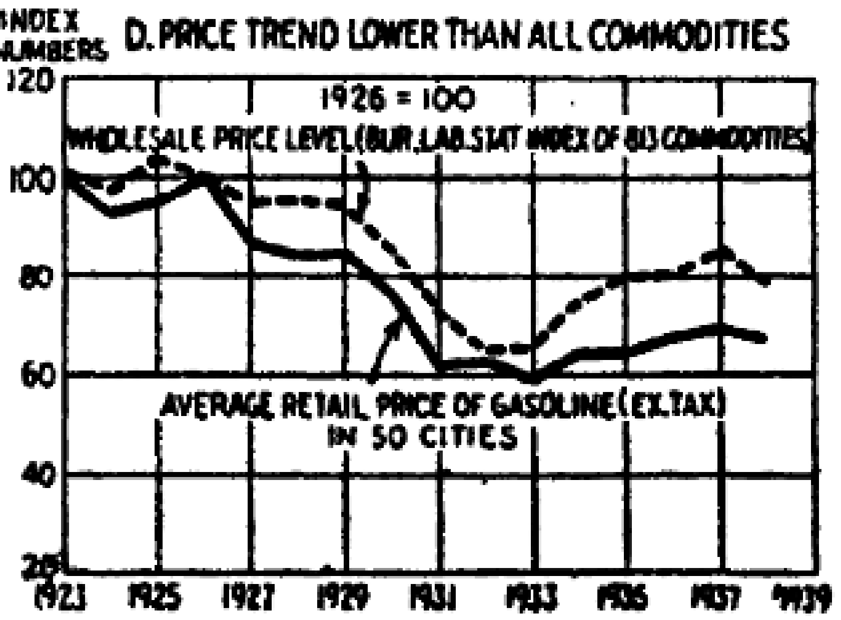
A. RISING CONSUMPTION



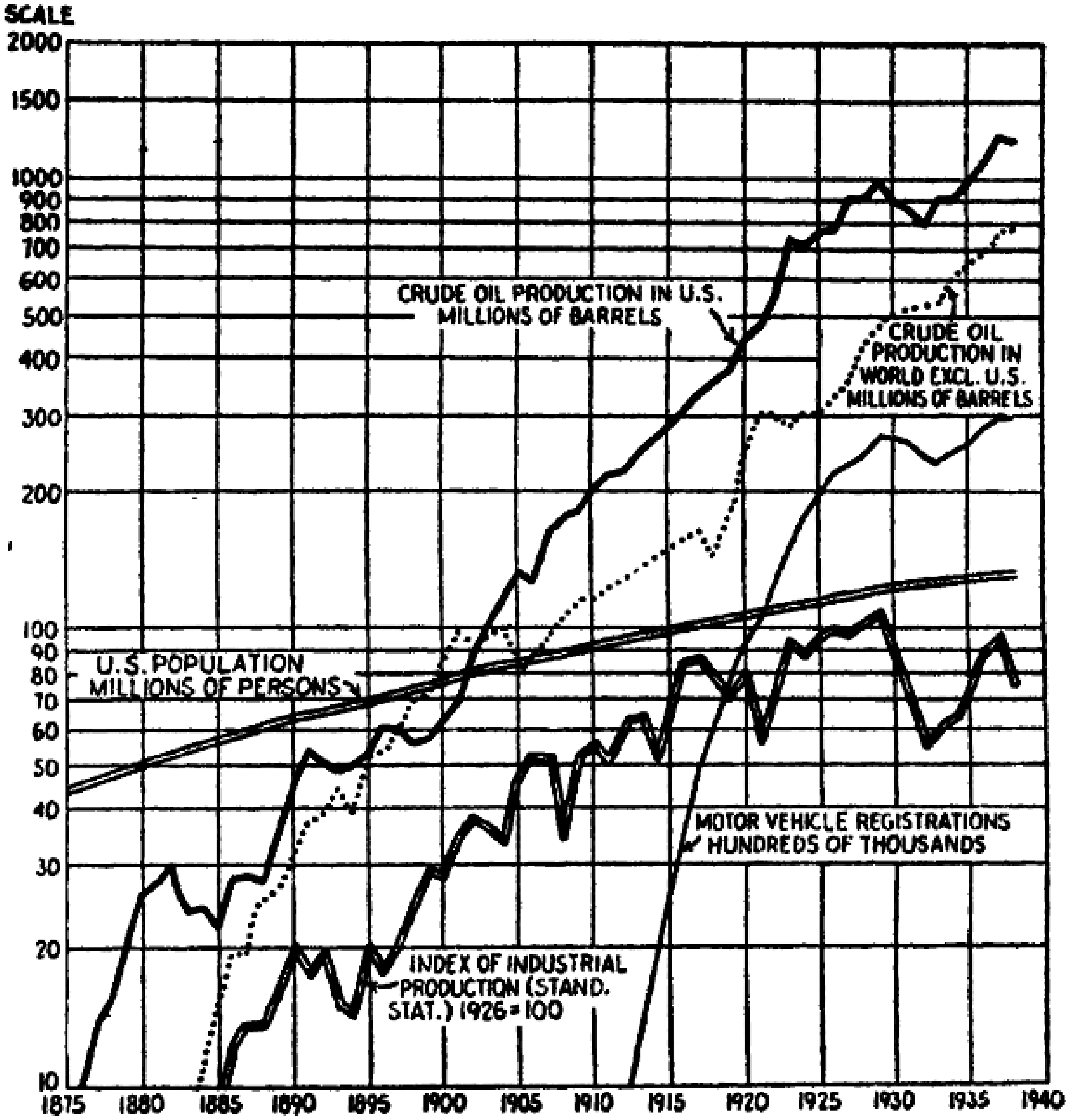
B. IMPROVING QUALITY



C. DECLINING PRICE



D. PRICE TREND LOWER THAN ALL COMMODITIES



5. A scale like 1 division = 3 is usually unsuitable, not being capable of decimal transformation.

6. When periods, not dates, are indicated, the corresponding values should be plotted in the *middle* of a small square, not on its edges.

§ *Frequency Chart.* When a group is divided into a number of smaller and more homogeneous classes indicating the number of items in each such class, the table thus formed is known as a frequency table. The science of Statistics is based on the study of such data, and the elimination of likely divergences from a probable norm, made to take in the entire range of observations (known as smoothing or graduation). A simple frequency table is illustrated below.

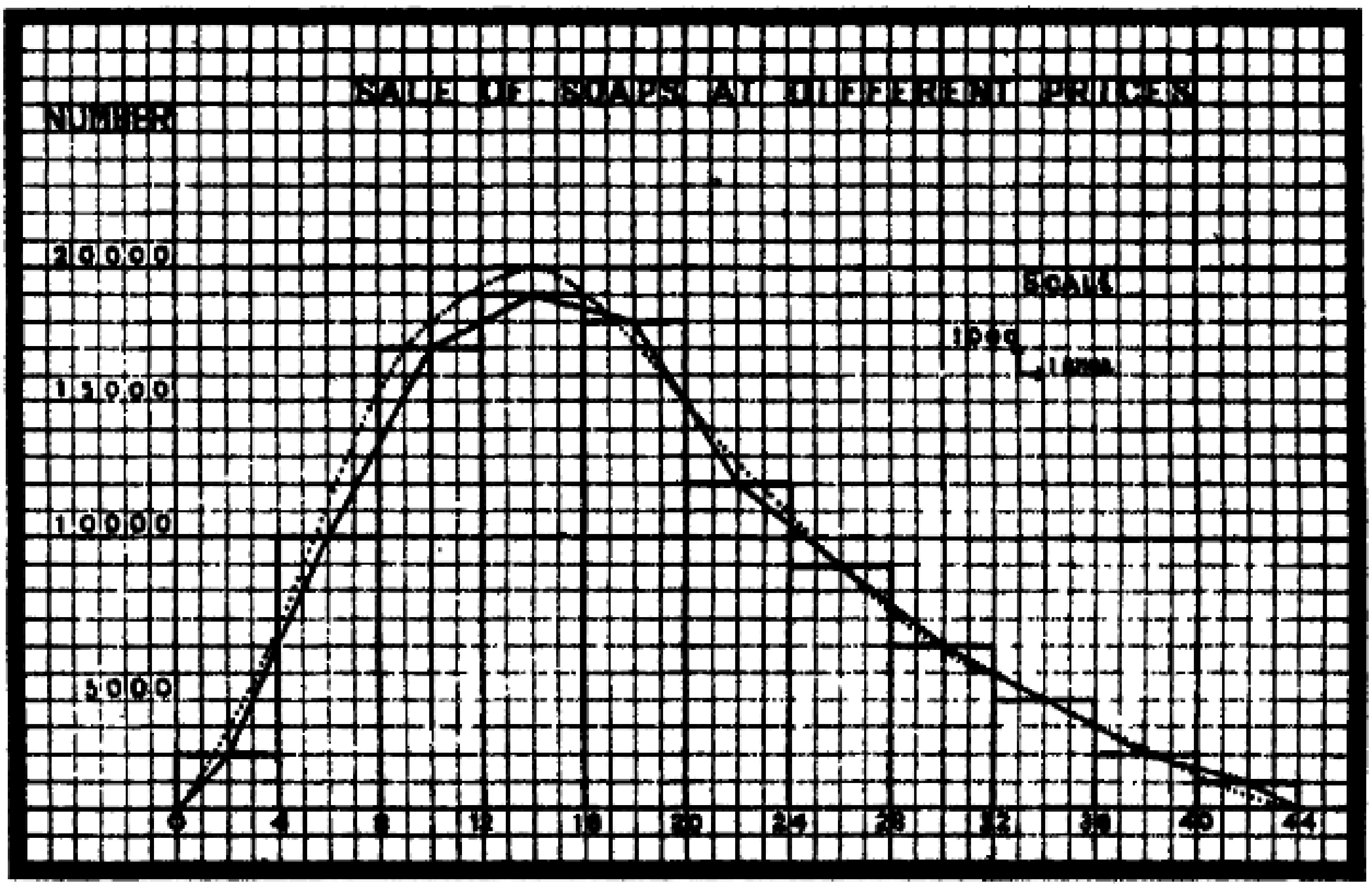
Sales Figures of a Departmental Store.
(Soap Cakes)

Price-range	No. sold in a year
Up to 4as per box	2000
4as—8as	10000
8as—12as	17000
12as—Re 1	19000
Re 1—Re 1 4as	18000
Re 1 4as—Re 1 8as	12000
Re 1 8as—Re 1 12as	9000
Re 1 12as—Rs 2	6000
Rs 2—Rs 2 4as	4000
Rs 2 4as—Rs 2 8as	2000
Rs 2 8as—Rs 2 12as	1000

Such an array of figures is known as a Frequency Table. It is evident that it is likely that the smaller the class-interval, the more homogeneous becomes the group, and the more reliable become the data for study. It does not, however, follow that the best result should be obtained by not grouping at all ; and the reverse is true. The optimum value of a class-interval is a matter of choice, in which experience plays a large part ; and very much more information can be obtained from data properly grouped than when they are not grouped at all or badly grouped.

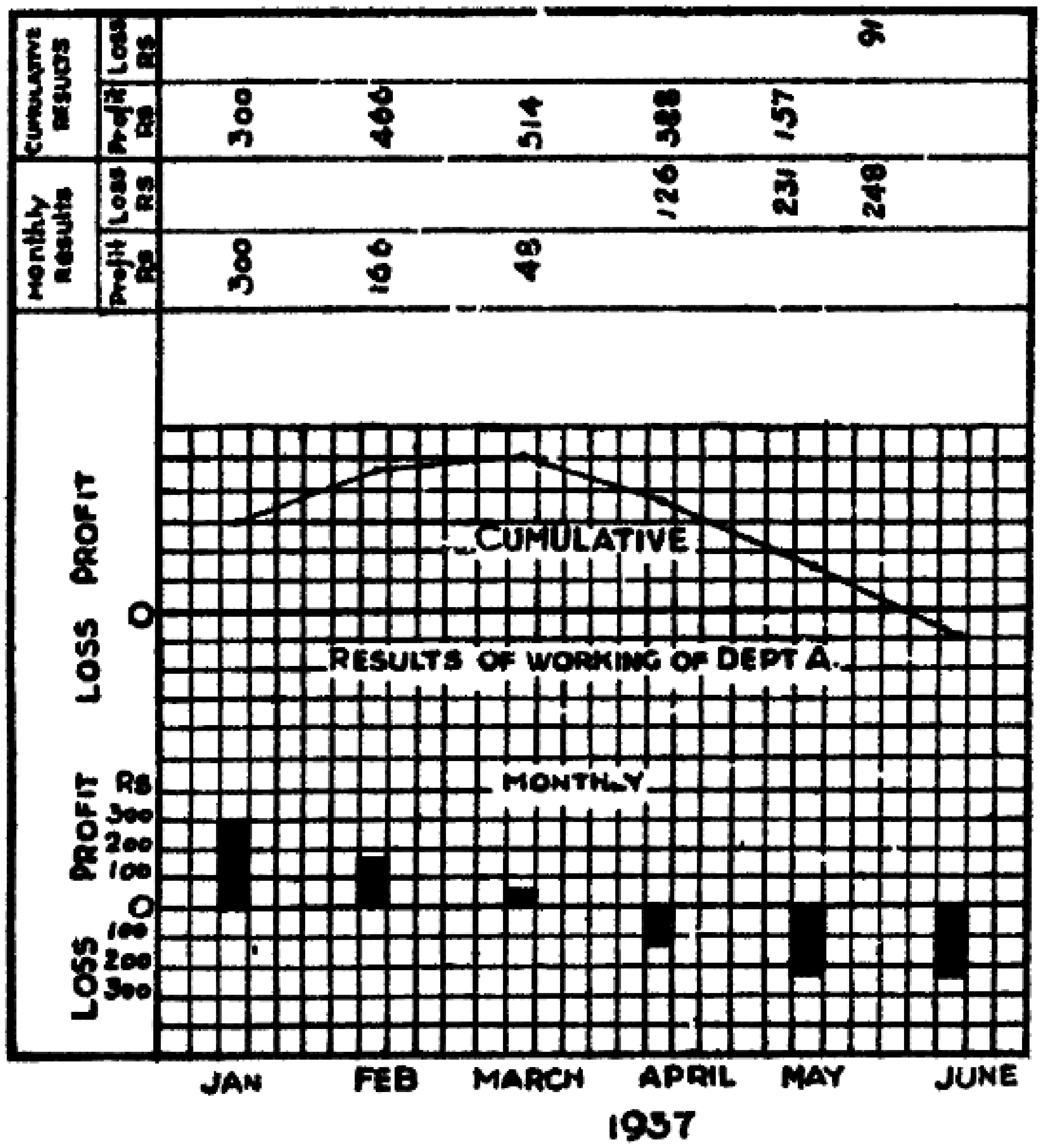
The method is to indicate class-intervals along the horizontal axis, and to plot frequencies at the mid-points of these intervals as in the figure below. The points plotted are then joined by broken lines, when the chart is known as a frequency polygon.

The polygon may be smoothed out *roughly* with the help of the eye to show the trend of the data, by drawing a *continuous line* close to the plotted points, avoiding sharp turns and eliminating irregular fluctuations, as in the figure. The total area covered by the smoothed line and the horizontal axis should be equal to that of the areas covered by the different rectangles representing the numbers sold at given prices and is also equal to that covered by the frequency polygon. The smoothing in the chart below is hardly sound statistically but shows the general trend by means of a simple line.



§ The *Divergence Chart.* It becomes frequently necessary to study the behaviour of a variable—say turnover, or prices &c. in so much as they deviate from a normal. In such cases, as the divergences may be of either sign, the base line should be drawn in the middle of the chart. In the chart below, a *cumulative value line* is also shown to scale on the top of the chart, so that the cumulative effects of the divergences may be simultaneously studied.

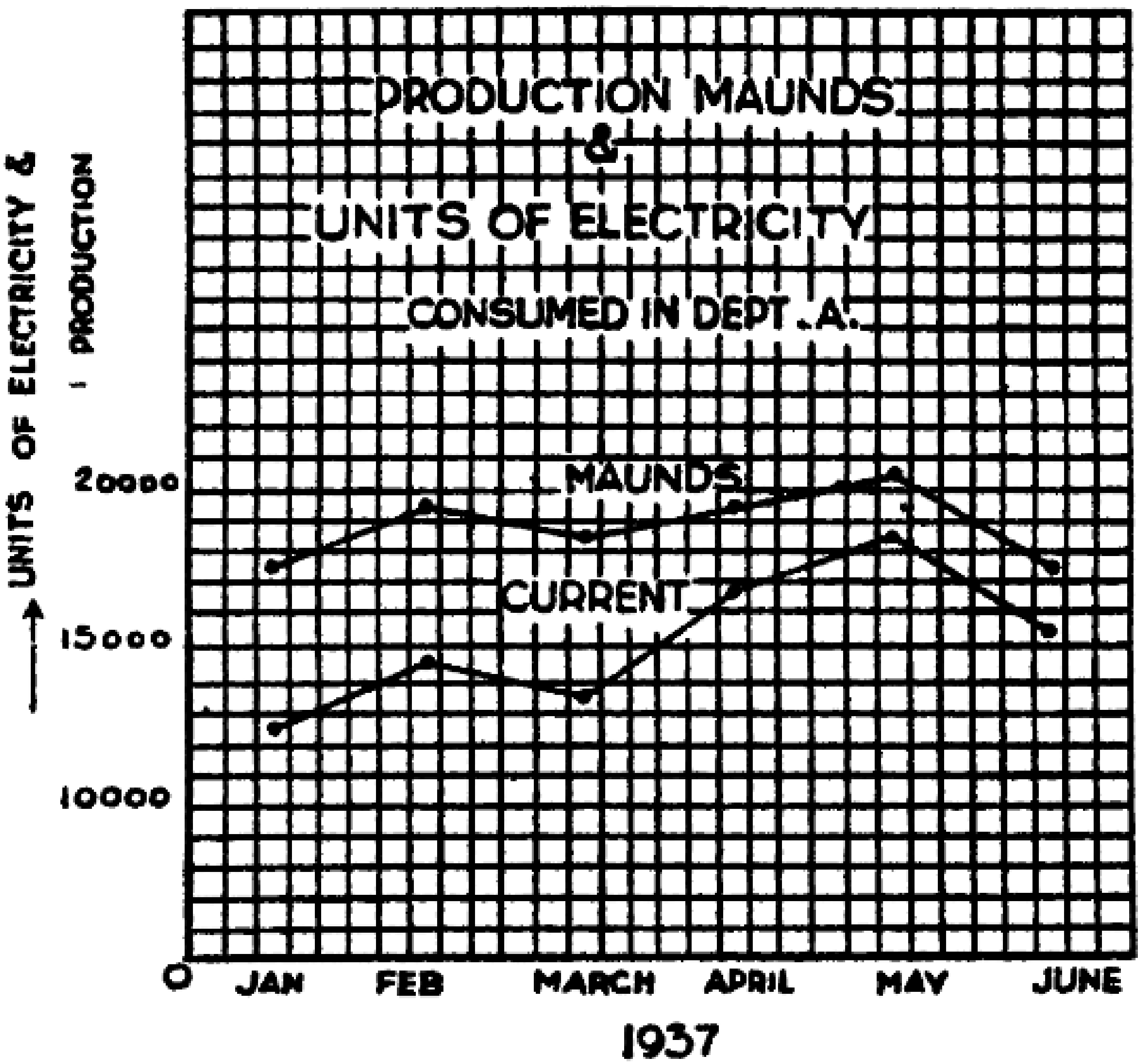
The figure following may be studied in reference to the data given therein. Such a chart is very useful to the business executive who wishes to compare the results of working with a scheduled programme or budget, or with sales record of past years' average, and in similar other instances.



§ *Historical Chart.* When time in an element, the horizontal axis is marked for periods, and the chart is called a historical chart. Its purpose is to present an easily manageable story of sales, profits, or some such single variable with the passage of time.

The chart on the next page represents the number of maunds of Flour produced studied in relation to the number of units of Electricity consumed in the production.

It should be noted that when periods are plotted, the corresponding point should be placed in *the middle* of the space allowed horizontally for that period. The points thus plotted are joined by broken lines.



§ *Moving Annual Total Chart.* The difficulty attending wide fluctuations over small periods in almost every business usually makes actual figures perplexing. Yet there lie imbedded in these figures very useful facts about tendencies and norms which can be correctly estimated and visualised by the method of moving annual totals.

The Moving Annual Total depends on the relations between the results for a certain period in one year with the corresponding period of another, and it is useful in setting short period drops or rises in the right perspective.

The M. A. T. line has been described as a perpetual inventory of turnover. The periods of seasonal depression in trades will show the M. A. T. line at a level, although the level itself can be compared with that of the preceding year.

The following figures relate to the turnover of a motor car repairs department.

Motor Car Repairs

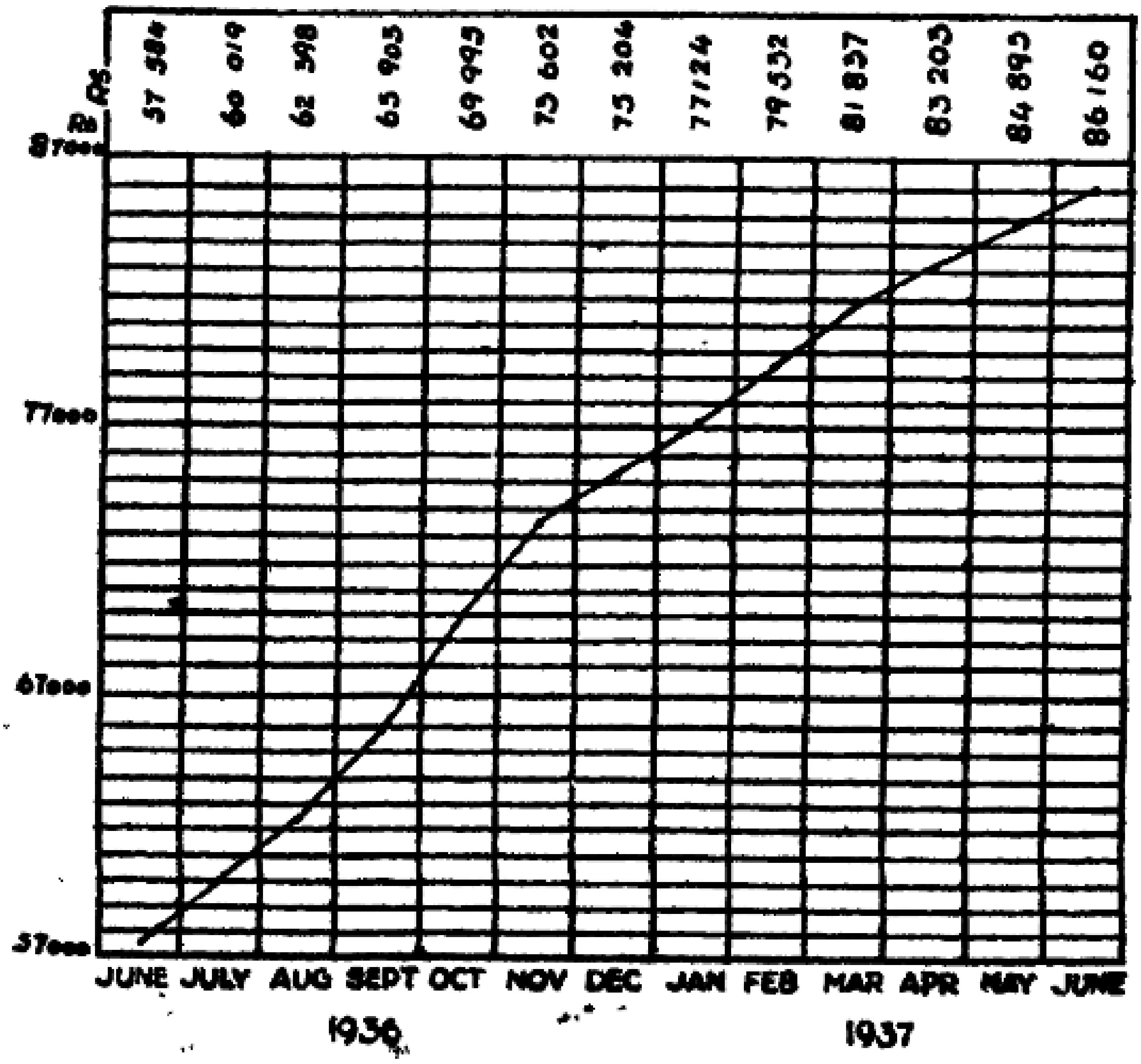
Monthly Turnover (1935-36) Rs	Months		Monthly Turnover (1936-37) Rs
	1935	*1936*	
4384	July		6819
3602	August		5981
2813	Sept.		6318
3524	Oct.		7616
5211	Nov.		8818
4623	Dec.		6225
	1936	*1937*	
4922	Jan.		6842
5018	Feb.		7426
5324	March		7629
5622	April		6988
6123	May		7813
6418	June		7685

REPAIRS DEPARTMENT

TURNOVER

MOVING ANNUAL TOTALS

JUNE 1936 – JUNE 1937



Calculations for Moving Annual Total

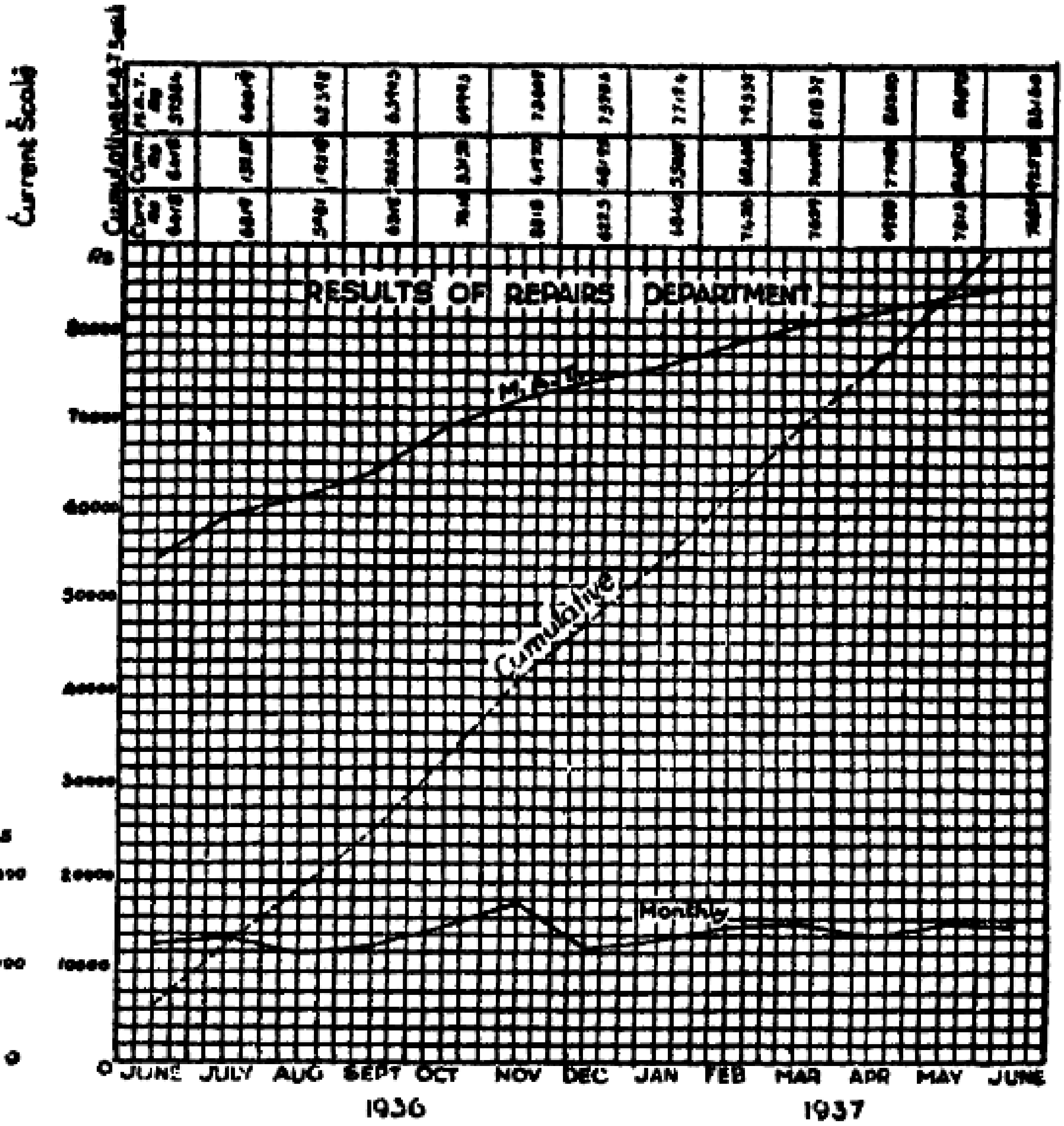
			Rs	Total for 12 mos. ending
Total for 12 mos. ending	June	1935	57584	
Less	July	1935	4384	
			53200	
Add	July	1936	6819	
			60019	July 1936
Less	Aug.	1935	3602	
			56417	
Add	Aug.	1936	5981	
			62398	Aug. 1936
Less	Sept.	1935	2813	
			59585	
Add	Sept.	1936	6318	
			65903	Sept. 1936
Less	Oct.	1935	3524	
			62379	
Add	Oct.	1936	7616	
			69995	Oct. 1936
Less	Nov.	1935	5211	
			64784	
Add	Nov.	1936	8818	
			73602	Nov. 1936
Less	Dec.	1935	4623	
			68979	
Add	Dec.	1935	6225	
			75204	Dec. 1936
Less	Jan.	1936	4922	
			70282	
Add	Jan.	1937	6842	
			77124	Jan. 1937
Less	Feb.	1936	5018	
			72106	
Add	Feb.	1937	7426	
			79532	Feb. 1937
Less	March	1936	5324	
			74208	
Add	March	1937	7629	
			81837	March 1937

			Rs	Total for 12 mos. ending
		B/f	81837	March 1937
Less	April	1936	5622	
			76215	
Add	April	1937	6988	
			83203	April 1937
Less	May	1936	6123	
			77080	
Add	May	1937	7813	
			84893	May 1937
Less	June	1936	6418	
			78475	
Add	June	1937	7685	
			86160	June 1937

If the moving annual totals are thus obtained for each of the months July 1936 to June 1937, and the points plotted and joined by a line, the trend of the turnover at once comes to the top. The erratic sales of the months are now seen in their true perspective in regard to the growth of the business as a whole.

§ *Z-Chart.* A useful record of short-period (monthly, say,) results, their cumulative effects and moving annual total is obtained by plotting them on the same chart. If the following instructions are followed, the graphs will together make up a figure of the form *Z*, from which it derives its name. The figures refer to the previous illustration.

1. The title of the chart should be clearly shown.
2. Different suitable scales will have to be selected, and shown on the graph, for the current result, and the Moving Annual Total. *The Cumulative and the M. A. T. Scale should be about 2 to 3 times the Current Scale.*
3. Besides the 12 totals for the period under review, the last month's result is also shown to provide continuity.
4. If possible, the 3 lines should be drawn in 3 different colours ; or at least, differently lined or dotted.
5. The data are usually also shown as part of the chart, if possible in colours corresponing to those of the different lines.



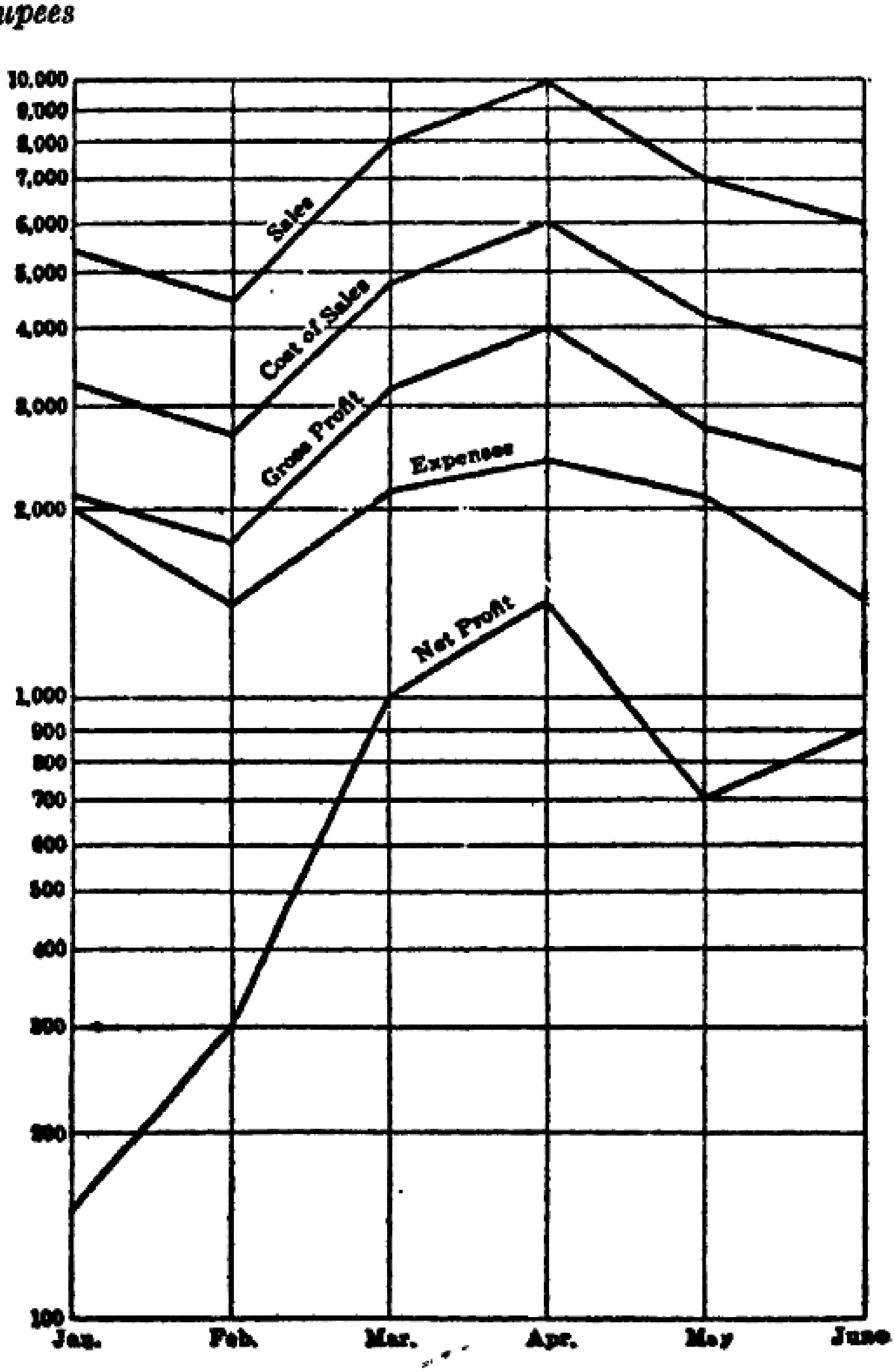
§ *Ratio Charts.* When percentage changes in the data are sought to be visualised, or when a large range of values has to be accommodated within a comparatively small space, the Ratio Chart is used.

The ruling in a Ratio Chart (the semi-logarithmic type which is commonly used in business) is made on a logarithmic scale parallel to the x-axis. Thus, on the vertical scale divisions are made not equal to the natural numbers, but equal to their logarithms.

The divisions marked on the vertical scale are :
1 is at the origin for log 1 = 0;
2 ,, at a height ·30 above the x-axis, for log 2 = ·30
3 ,, ,, ,, ,, ·47 ,, ,, ,, ,, log 3 = ·47
4 ,, ,, ,, ,, ·60 ,, ,, ,, ,, log 4 = ·60
5 ,, ,, ,, ,, ·69 ,, ,, ,, ,, log 5 = ·69
6 ,, ,, ,, ,, ·77 ,, ,, ,, ,, log 6 = ·77
7 ,, ,, ,, ,, ·84 ,, ,, ,, ,, log 7 = ·84
8 ,, ,, ,, ,, ·90 ,, ,, ,, ,, log 8 = ·90
9 ,, ,, ,, ,, ·95 ,, ,, ,, ,, log 9 = ·95
10 ,, ,, ,, ,, 1·00 ,, ,, ,, ,, log 10 = 1·00
20 ,, ,, ,, ,, 1·30 ,, ,, ,, ,, log 20 = 1·30
30 ,, ,, ,, ,, 1·47 ,, ,, ,, ,, log 30 = 1·47
40 ,, ,, ,, ,, 1·60 ,, ,, ,, ,, log 40 = 1·60
50 ,, ,, ,, ,, 1·69 ,, ,, ,, ,, log 50 = 1·69

It will be observed that the lines parallel to the horizontal axis will be unevenly spaced; but that the spacing will be similar at intervals 10, 10^2, 10^3, 10^4 &c. &c. A complete group of such lines is known as a *cycle*. In the absence of a properly ruled graph paper, one may be improvised on a millimetre graph paper.

Since $\log \frac{y_1}{y_2} = \log y_1 - \log y_2$, the distance between the lines $y = y_1$ and $y = y_2$ on such a chart will really indicate the value of the ratio $\frac{y_1}{y_2}$ (not the difference $y_1 - y_2$).



The following figures relating to a Departmental Stores may be studied on the chart above :

1941	Sales	Cost	Gross Profit	Expense	Net Profit
January	Rs 5400	Rs 3240	Rs 2160	Rs 2000	Rs 1600
February	4500	2700	1800	1500	300
March	8000	4800	3200	2200	100
April	10000	6000	4000	2500	1500
May	7000	4200	2800	2100	700
June	6000	3600	2400	1500	900
	Rs 40900	Rs 25540	Rs 16330	Rs 11800	Rs 4560

It will be observed that the chart effectively visualises the *rate* of movement of the various items. It does not, however, exhibit correct numerical magnitudes.

§ *Business Control Charts.* A wide variety of ingenious methods of keeping control over the different sections of a factory or an office is in use to meet particular needs, which cannot for obvious reasons find a place in this treatise. For example, effective controls have been devised over the work in factory in connection with the stock in the stores, the quantity in the shops under manufacture etc., and correct information may be obtained at a glance in respect of every part of every unit under production. They naturally help to remove difficulties of shortage in rough stores or in the finished parts stores, and enable the completed machines to be assembled in any combination of units that may be called for. The student desirous of further information on the subject is referred to advanced treatises on Graphics.

EXAMPLES 41

1. Solve graphically : (i) $1{\cdot}5x^2 - 37x + 8 = 0$. *Ans.* $x = {\cdot}21, 24{\cdot}4$

(ii) $6x + 5y = 18$

$9x - 3y = 5$ [*Ans.* $x = 1{\cdot}25$; $y = 2{\cdot}09$

2. A person had three deposits of Rs 3000 each in different banks. On the first he got Rs 3185 at the end of a year ; on the second he got Rs 3098 at the end of six months and on the third

he was paid Rs 3035 at the end of three months with interest. Find graphically which bank paid the highest rate of interest.

[*Ans.* 2nd highest ; 3rd lowest

3. Draw a graph to convert tons into maunds, given 1 ton $= 27\frac{2}{9}$ mds, and read off the value of $3\frac{1}{2}$ tons, $2\frac{3}{4}$ tons, $1\frac{1}{2}$ tons.

[*Ans.* 95·3 mds ; 74·9 mds ; 40·8 mds.

4. Draw a graph to represent the rate Re $1 =$ 1s $6\frac{5}{32}$d. Read off the value of Rs 28 ; Rs 16 12as ; Rs 35 6as. Find the value in Rupees of £1 6s ; 18s 7d ; 7s 9d.

[*Ans.* £2 2s 4d ; £1 5s 3d ; £2 13s 6d ; Rs 17 3as ;
Rs 12 4as 6p ; Rs 5 2as.

5. A commercial traveller was offered a fixed salary of Rs 300 per mensem, or a salary of Rs 100 per mensem together with a commission of 5% on sales, and chose the latter. If he completed Rs 4856 of business, was his choice profitable ? What should be the value of business completed to assure him of an income of Rs 350 a month on the second terms ? [*Ans.* Yes. Rs 5000

6. Under the British postal rules the largest parcel may be cylindrical in shape such that its length added to its girth is 6 ft. Tabulate corresponding values of the length and girth, consistent with this rule, and find the length at which the volume is the maximum. [*Ans.* 2 ft.

7. What is the time taken by Rs 1500 to accumulate to Rs 1587 12as at $5\frac{1}{2}$% per annum simple ? [*Ans.* 1 year $0\frac{3}{4}$ month

8. A dividend of 5as 9p in the rupee is declared on a bankrupt's liabilities. Find graphically how much would creditors for Rs 565, Rs 385 and Rs 436 receive.

[*Ans.* Rs. 203 ; Rs 138 6as ; Rs 156 11as

9. The admission of a preferential claim for Rs 2000, alters the rate of dividend from 4as to 1a 3p in the rupee. Find the assets and the liabilities.

[*Ans.* Assets : Rs 2682 ; Liabilities : Rs 10728

10. Tea is packed in packets of 2 oz. at 1a 9p ; 8 oz. at 4as 6p ; of 1 lb at 8as. What would be a fair price for packets of 6 oz and 12 oz each ? [*Ans.* 3as 7p ; 6as 3p

11. A retailer marks his goods at 10% above cost. Draw a graph from which can be read off selling price per unit, given cost price of Rs 3 14as per doz ; Rs 31 8as per gross ; Rs 3 7as each. [*Ans.* 5as 8p ; 3as 10p ; Rs 3 12as 6p

12. Draw a graph to obtain the amount of freight chargeable at 22*s* 6d per ton on 18 cwts ; $16\frac{1}{2}$ cwts ; 1 ton 16 cwts.

[*Ans.* £1 0s 3d ; 18s 7d ; £2 0s 6d

13. A discount of $6\frac{3}{4}$% is allowed on all cash purchases from a stores. Obtain graphically the discount on bills for Rs 8 12as ; Rs 15 6as ; Rs 12 14as. [*Ans.* 9as 6p ; Re 1 0a 6p ; 14as

14. Draw graphs to obtain the yield on 4% stock and $3\frac{1}{2}$% stock at prices ranging between 80 and 120. Read off yield on 4% stock at 85 ; $86\frac{1}{16}$; $98\frac{3}{16}$; $110\frac{7}{16}$; and on $3\frac{1}{2}$% stock at $81\frac{3}{8}$; $86\frac{5}{16}$; $93\frac{3}{16}$; $98\frac{9}{16}$.

15. A pint of water weighs $1\frac{1}{4}$ lbs. Find the weight of $10\frac{3}{4}$ pints, $11\frac{1}{2}$ pints, $12\frac{3}{8}$ pints of water.

[*Ans.* 13·4 lb ; 14·4 lb ; 15·5 lb

16. An importer of French silk desires to read off from a graph the length in yards of rolls of silk of which the lengths are given in metre. Given that a metre = 1·0936 yds. Draw the graph, and read off the length 16·5 m and 16·75 m in yards.

[*Ans.* 18 yds ; 18·3 yds.

17. A track rises 11 ft in 183 ft ; and another 3 ft in 54 ft ; which is the steeper track ? [*Ans.* First

18. Draw a graph to represent the weight of circular metal sheets corresponding to their diameter in inches, given that 1 sq. in, of the sheet weighs $\frac{1}{8}$ oz.

19. Find the area of a triangle of sides 2′ 3″, 3′ 6″ and 2′ 8″ in length. [*Ans.* 3 sq. ft.

20. Indian postal rates for Parcels are as below :

Not exceeding 20 tolas	... 2as
Above 20 but below 40 tolas	... 4as
For every additional 40 tolas	... 4as

Parcels exceeding 440 tolas must be registered at 3as per packet. Draw a graph from these data, and find the charge for sending parcels weighing 48 tolas, 83 tolas, 125 tolas, 340 tolas, 460 tolas, 480 tolas.

[*Ans.* 8as ; 12as ; Re 1 ; Rs 2 4as ; Rs 3 3as ; Rs 3 3as

21. Under a profit-sharing plan, labourers are benefited to the extent of one-eighth of the profits in excess of 10% on capital ; a further eighth of profits in excess of 15% ; and another quarter of profits in excess of 25% on capital.

Draw a graph from the data above, and find the amount to be distributed among labourers if profits are Rs 7895 ; Rs 18500 ; Rs 25000 ; Rs 40000 on a capital of Rs 80000.

[*Ans.* Nil ; Rs 2125 ; Rs 5000 ; Rs 12500

22. Draw the graph of the amount of Rs 100 at 5% p.a. compound interest from the Tables at the end. Read off from the graph the amount in 9¼ years, 10½ years.

[*Ans.* Rs 157 1a ; Rs 166 15as

23. A manufacturer of Duplicators sells the largest size for Rs 1350 and the smallest for Rs 240. It is observed that the offtake is largest in the smaller sizes. The prices are accordingly revised, according to a linear law, so as to sell the largest size at Rs 1000 and the smallest at Rs 300. Obtain this relationship between the new and old prices, and ascertain the reduced price of machines which were originally selling at Rs 300, Rs 350 and Rs 1000. [*Ans.* $y = \cdot 63x + 150$; Rs 340, Rs 370, Rs 780

24. A fixed royalty is paid to the holder of special patents by a manufacturer of students' telescopes. The cost of producing 25 machines is Rs 885 ; of 40 machines it is Rs 1335. What is the amount of royalty and the cost per machine ?

[*Ans.* Royalty = Rs 135 ; Cost = Rs 30

25. The total cost of production stands in the following relationship to the number of cigarettes weekly turned out from a factory.

No. of boxes of cigarettes Doz.	72000	55000	42400	20000
Cost of production Rupees	1900	1630	1430	1070

Find a possible linear relationship subsisting between the cost and output. Find the cost per dozen boxes when the total production stands at 60000 doz. [*Ans.* $y = {\cdot}016x + 748$; 5·5p.

26. Bonus is paid to the employees in a factory roughly in accordance with the law $W = w + \frac{b}{n}$, where W is the total wages, w is the normal wages, b is the total amount of the bonus available for a department and n is proportional to the number of labourers in the department. The following are the corresponding values of W and n in different departments to the nerarest 10.

W	Rs 290	Rs 330	Rs 370	Rs 410	Rs 500	Rs 690	Rs 950
n	160	120	90	70	50	30	20

What should be the most likely values for w and b ?

[*Ans.* $W = 200 + \frac{15000}{n}$.

27. The statistics of production and sales of a patent medicine are as follows :

Selling price in annas ...	6	8	10	12	14	16
Number sold (Thousands) ...	10	8	7	5	2	1·8

If a phial of the medicine has a total cost of 6as, find the probable number that may be sold at any price from 6as to Re 1 each, and find at what price it will be most profitable for the manufacturer to place it on the market. [*Ans.* 10as

28. The following table gives the quantities of a certain brand of tea sold at prices noted against each :—

Price of tea per lb.	Re as 1 4	Re as 1 8	Re as 1 12	Re as 1 14	Rs as 2 0	Rs as 2 4	Rs as 2 8
Quantity sold in thousand lb.	82·5	70·8	6·31	60·7	35·0	48·9	39·8

Estimate the probable sales when the prices are

(*a*) Re 1 10as ; (*b*) Re 1 15as ; (*c*) Rs 2 7 as. —G.C.

29. Draw a pie-chart (or other suitable chart) for the following data relating to the languages spoken in India in 1931 :—

Language	Spoken by
Western Hindi	71,547,000
Bengali	53,468,000
Telegu or Andhra	26,374,000
Bihari	27,927,000
Marathi	20,890,000
Tamil	20,412,000
Others	132,220,000
Total	352,838,000

—G.C.

30. Prepare a pie-chart for the following data relating to the revenue of Bengal for 1997-38 (in thousand rupees)

Customs	2,37,81
Income Tax	25,00
Salt	57
Land Revenue	3,53,03
Provincial Excise	1,54,56
Stamps	2,89,35
Forest	21,54
Registration	22,86
Receipts under Motor Vehicles Act	21,09
Other Taxes & Duties	39,18

What method would you suggest for representing (*a*) the comparative values, (*b*) the actual values in comparison with the total revenue ? —G. C.

PROMISCUOUS EXERCISES**

1. The radius of the earth may be taken as 4000 miles, and was at one time rather greater. Assuming that it was once 10 miles greater than at present, calculate approximately (i) the superficial area, (ii) the volume which it has lost.—B. C. S. [*Ans.* (i) 1007×10^3 sq. miles ; 2010×10^6 cu. miles.

2. Calculate compound interest on Rs 3350 for 2½ years at 4% p. a. with half-yearly rests. [*Ans.* Rs 348 10as 9p

3. A person owes Rs 6000 and desires to pay in nine months. The creditor demands a bill at 9 months for an amount that, if immediately discounted at 4%, would yield the amount due. For how much should the bill be drawn ?—G. C. [*Ans.* Rs 6185 9as

4. The prime cost of an article is 3 times the value of the raw materials used. The cost of raw materials increases in the ratio 5 : 12, and working expenses in the ratio 3 : 5. Find the prime cost of an article which used to be made for Rs 4 3as 6p.—G. C. [*Ans.* Rs 8 1a

5. Calculate interest (i) @ 3¾% p. a. for 6 months on Rs 2500 ; (ii) @ 6¼% p. a. for 2 mos. on Rs 5863 4as ; (iii) @ 4½% p. a. for 149 days on Rs 6468 8as.—G. C. [*Ans.* Rs 46 14as ; Rs 61 1a 3p ; Rs 118 13as 3p

6. On a piece of squared paper draw a circle 2″ in diameter. Count off the number of small squares inside the circle and so estimate the area of the circle in square inches to 2 places of decimals. Also calculate the area of the circle from the formula πr^2, assuming $\pi = \frac{355}{113}$.—B. C. S.

7. Compute by contracted methods, correct to 2 decimal places,

(i) $98{\cdot}561632 \times 23{\cdot}465187$, [*Ans.* 2312·77 app.

and (ii) $\cdot09764111 \div \cdot0031213$. [*Ans.* 31·28 app.

8. Find by trial the least number which when divided by 17, leaves a remainder 10, and when when divided by 13 leaves a remainder 4.—P. S. C. [*Ans.* 95

9. *A*, *B* and *C* engage in a walking contest over a distance of 1 mile. *A* defeats *B* by 30 seconds ; *A* defeats *C* by 88 yards ; and *B* defeats *C* by 20 seconds. What time is taken by each competitor to cover the mile ?—P. S. C. [*Ans.* 15⅚ min. ; 16⅓ min. ; 16⅔ min.

10. Find the cost of papering the walls of a square room 14 ft high and 18 ft long, with 2 doors, each 8 ft by 4 ft, and 3 windows each 2 ft by 3 ft, the amount saved by each window being 6s 9d. What additional height would increase the cost by 9s ?—I. P. S. [*Ans.* £52 1s 9d ; 1½ in.

11. Find the sum of all the products of a multiplication table running up to 10×10.—C. C. [*Ans.* 3025

**These examples have been selected from examination questions. The abbreviations indicate the examinations at which they have been set ; I. C. S. (Indian Civil Service) ; B. C. S. (Bengal Civil Service) ; C. C. (Calcutta Corporation Clerkship) ; P. S. C. (Public Service Commission Recruitment Examinations) ; S. C. C. (Bengal Secretariat Clerkship Examination) ; A. G. (Accountants-General, Bengal, and Posts and Telegraphs Office Recruitment Examinations) ; G. C. (Government Commercial Course Examinations); I. Com. (Inter. Commerce. Cal. Univ.) ; I. B. (Dacca Intermediate Board's Examination) ; I. P. S. (Imperial Police Service).

12. A discount of 33⅓% is allowed on the list price of a certain make of guns. The importer has to pay duty at 20% on the net invoice price ; and his expenses are estimated at 10% of the same. If the guns are sold at the list price plus duty calculated on the list price, what is the profit made ?—G. C. [*Ans.* 38$\frac{6}{13}$%

13. A merchant fails for Rs 47800. Besides his stock-in-trade, his assets are : Cash...Rs 6251 10as 8p ; Good Debts...Rs 5794. Winding up Expenses were Rs 2491 1a 4p. An average of 4as in the rupee is realised on other Debts, valuing Rs 6653 ; and he is enabled to pay a dividend of 6as 3p in the rupee. Find the value of his stock-in-trade.—G. C. [*Ans.* Rs 7454 0a 8p

14. A sum of money is deposited at interest. At the end of the first year Rs 292 is withdrawn ; at the end of the second year Rs 384 is withdrawn ; and the total sum then remaining at credit is Rs 448. If no withdrawals had been made the annual simple interest would have been Rs 42. Find the original sum deposited and the rate of interest.—P. S. C.
[*Ans* Rs 1050 ; 4% p. a.

15. Find the discount on a bill for £16000 drawn on 1st April at 6 months and discounted on 20th July @ 2½% p. a. How much more could have been obtained on the bill if it were discounted on 21st July ?
[*Ans.* £83 5s 9d ; £1 1s 11¼d

16. An importer examines two *pro forma* invoices for two similar brands of toothpaste. The American rate works out at $1250 for 25 gross, and the English quotation is £11 15s per gross *c.i.f.* The import duty on the American article is 60% and that on the English is 40% *ad valorem.* Where should the order be placed ? Taking Re 1 = 1s 6d and $ 4·75 = £1, determine the selling price, allowing a margin of 20% to the importer.
[*Ans.* Last : Rs 21 15as per doz.

17. A man sold his goods at a profit of 15%. The money thus realised was invested in goods which had to be sold at a loss of 16%. How much does he gain or lose by the two transactions ?—G. C. [*Ans.* Loss 3·4%

18. A man purchases an estate for £4000 and sells it on the same day for £4500 to be paid in 2 equal instalments in 3 and 6 months, respectively. How much per cent does he make by the bargain, the rate of interest being 6% p. a. ?—G. C. [*Ans.* 10%

19. The following are ten measured values of X :—

20·6 ; 19·8 ; 21·1 ; 21·8 ; 21·7 ; 24·7 ; 21·4 ; 21·5 ; 21·2 ; 21·0.

(i) Calculate the average value of X, and the differences of the measured values from the average. [*Ans.* 21·48

(ii) If S denotes the sum of the squares of the differences, compute $\cdot 6745\sqrt{S/10}$ correct to 2 places of decimals.—P. S. C. [*Ans.* ·82

20. The 3½ per cents are quoted at 68. On this income tax is to be paid at 26 pies in the rupee. The 5 per cents, free of income tax, are however quoted at 101. Which is the better investment, and what difference does it make to a man who invests Rs 100000 ?—G. C. [*Ans.* Latter ; Rs 500 8as. p.a.

21. A and B are partners in a trading venure, A contributing Rs 20000 and B Rs 30000. A acts as manager and is to receive 25% of the profits as remuneration for his services ; it being agreed that the balance is to be shared between the partners in proportion to their contributions. A's share of profit and remuneration in a year amounts to Rs 5500. What is B's shares ?—G. C. [*Ans.* Rs 4500

22. A rectangular grass plot, the lengths of whose sides are in the ratio 5 : 6, costs £16 11s 6⅜d to turf at the rate of 5d per sq. yd. Find the length of the sides.—I. Com. [*Ans.* 25·75 yd ; 30·9 yd.

23. What should be the marked price of a motor car which cost £250, if it is desired to make a profit of 30% on net returns, after allowing the agent a commission of 20% and a cash discount of 20% ?—G. C.
[*Ans.* £558 0s 9d

24. *A* buys goods from *B* for Rs 5000. *B* offers him a discount of 4% for cash or payment in full after 6 months. If *A* accepts the latter terms and gives a bill for the amount, which *B* discounts immediately, what will *B* gain or lose, the bank rate of discount being 5% p. a. ?—G. C. [*Ans.* Gain Rs 75

25. A railway engine running at 45 miles an hour picks up water from a trough ¼ mile long, at the rate of 169 gallons per minute. Taking a gallon to be 0·16057 cubic foot, find the number of cubic feet of water it picks up during the quarter mile.—I. Com. [*Ans.* 9·05 c. ft.

26. A milkman adulterates a quantity of milk as follows. He first takes out 9 seers of milk and replaces it by water. After thorough mixture, he again takes out 9 seers and replaces it by water. The proportion of milk to water in the mixture is now 9 : 16. If the adulterated milk is sold at 5as per seer, what is the amount of illicit profit made by the milkman ?—P. S. C.
[*Ans.* Rs 4 8as

27. The longitude of Mukden is 123°30′E, and that of Moscow 37°30′E. Find what time it is at Moscow when it is 11 A.M. at Mukden. Also find the times at these two places when it is noon at Calcutta, whose longitude is 88°E.—I. Com. [*Ans.* 5 hr. 16 min. ; 2.22 P.M. ; 8 hr. 38 min. 40 sec. A.M.

28. Is it profitable for a Calcutta merchant to buy a Paris draft in order to pay a debt of 10000 francs, when 164·30 francs = Rs 100 ; or to remit the amount through London, the course of exchange being 1s 4d to Re 1, and 25 francs = £1 ? What difference does it make to him in rupees ?—G. C.
[*Ans.* Rs 86 7as

29. The price of petrol is increased by 15%. Find by how much per cent a man must reduce his consumption so as not to increase his expenditure.
—G. C. [*Ans.* 13%

30. Find $\dfrac{638{\cdot}3461 \times {\cdot}008193}{{\cdot}1645 \times 43{\cdot}618}$ correct to the nearest integer. [*Ans.* 1

31. What sum will amount to Rs 1000 in 12 years' time at 4½% p. a. compound ? Given log 1·045 = 0·0191163 and log 589·6636 = 2·7706044.
—I. Com. [*Ans.* Rs 589 10as 6p

32. What is the amount and the present value of an annuity-certain of Rs 150 for 12 years, reckoning interest at 3½% p. a. ?
Given $1{\cdot}035^{12} = 1{\cdot}511066$.—I. Com. [*Ans.* Rs 2190 4as 6p ; Rs 1449 8as

33. The three-half per cents are quoted at 75. On this income tax is deducted at 32½ pies in the rupee. What difference in annual income is obtained by transferring Rs 50000 stock carrying 3½% to 5% tax-free stock at 108 ?—G. C. [*Ans.* Rs 281 3as 6p +

34. The book debts of a bankrupt are equal in amount to his liabilities. Part he recovers in full, but of Rs 4800 he recovers only 6 annas in the rupee, and of Rs 3600 only 5as 4p in the rupee, and he pays his creditors 10as 8p in the rupee. What was the total amount of his debts, if the liquidation expenses amount to Rs 515 5as 4p. ?—P. S. C. [*Ans.* Rs 17746

35. A merchant in England orders goods from Germany worth 960 Marks. The customs duty, insurance, and the carriage of goods to England amount to £2 10s. He then sends the goods to India, thereby incurring an additional expense of Rs 101 4as. Find his profit if he sells the goods in India for Rs 1026. [1 Mark = 11¾d and £1 = Rs 13 8as]—I. P. S. [*Ans.* Rs 256 8as

36. The price of gold is Rs 38 15as per ounce. An alloy of gold and silver weighing 18 lb is worth Rs 7246 8as ; but if the weights of gold and silver in the alloy were interchanged, it would be worth only Rs 1717 8as. Find the proportion of gold and silver in the alloy and the price of silver per ounce.—S. C. C. [*Ans.* 23 : 4 ; Rs 2 9as

37. A merchant marks on an article a price which was 50% above cost price, but to purchasers he gave 12% discount on the marked price, thereby gaining 15s. What was the cost price ?—G. C. [*Ans.* £2 6s 10½d

38. Four persons enter into partnership ; the second has twice as large capital as the first, and the third as much as half the sum of the capital of the first two, and the fourth has a sum equal to the capital of all others. Distribute total profits of Rs 20000 among the partners in proportion to capital.—G.C.

[*Ans.* Rs 2222 3as 6p ; Rs 4444 7as ; Rs 3333 5as 6p ; Rs 10000

39. A merchant has a 3 months' bill for Rs 1000, which his broker discounts at 2½%. Find what rate per cent per annum he must earn on the discounted value so that he may not lose by selling the bill.—G. C.

[*Ans.* 2·516%

40. Find the rate per cent per annum at which a certain sum of money, invested at compound interest, payable half yearly, will double itself in 12 years. —S. C. [*Ans.* 5·95% p.a.

41. A person directs his broker to purchase eight Rs 1000 shares in a certain mine, quoted at Rs 2725 per share. To accomplish this he authorises his broker to sell out Rs 8500 stock in the 4 per cents at 96⅛, and a certain amount of 4½ per cent stock at 105⅛, just so much as to realise the amount required for the purchase. The broker's charge for each of the three transactions is ⅛th per cent. (i) What amount of 4½ per cent stock is to be sold ? (ii) What had the broker to receive on the whole ?—S. C.

[*Ans.* Rs 13000 ; Rs 36 14as

42. The salary of a clerk is increased by a fixed sum every year. After 5 years, it is Rs 85 ; while after 13 years it becomes Rs 125. Draw a graph from which his salary may be read off for any year and determine from it (i) his initial salary, and (ii) the annual increment.—S. C.

[*Ans.* Rs 60 p.m. ; Rs 5

43. A well, supplied by a steady stream of water, can irrigate 3 acres of land for just 15 days, or 6 acres for 7 days. For how long can it be used for irrigating 5 acres ? [*Ans.* 8⅓ days app.

44. An investor purchased 400 ten-rupee shares (Rs 6 paid up) in a company when they were selling at 11 and paying a dividend of 7½%, brokerage being charged at 4½ as per share. He invests another Rs 49000 in the 4 per cents at 97¼, paying brokerage at 2as%. After receiving one dividend on each he sells out, the shares at 10⅜ and the stock at 104, paying brokerage as before. What is the average return on his investment ?

—G. C. [*Ans.* 8¾%

45. At One O'clock on Monday I find that my watch is 17 seconds slow : at One O'clock on Wednesday it is 31 seconds slow. What is the correct time when my watch shows 8 O'clock on Saturday morning ?—B. C. S.

[*Ans.* 8 hr 0 m 54 sec.

46. A cotton mill has a capital of Rs 400000. The profit made during a year is Rs 37480. Rs 7480 is placed in the Reserve Fund, and the rest is paid out as dividend. A person holds shares of the nominal value of Rs 15000 bought at a discount of 25%. What rate of interest per cent per annum does he derive on his investment ?—G. C. [*Ans.* 10%

47. The following table gives the height of the spring tides at points on the River Thames, at specified distances from the mouth of the river :—

Place	Distance in miles	Height ft.	Height in.
London Docks	60	18	10
Putney	67·5	10	2
Kew	73	7	1
Richmond	76	3	4
Teddington	79	1	4½

Draw a graph based upon the table, and use it to determine :—

(i) Height of the spring tides at Barnes Bridge, 70·5 miles from the mouth.

(ii) The places where the height is 10 ft and 5 ft respectively.

(iii) The probable height of the spring tides at Greenwich, 50 miles from the mouth.—B. C, S.

48. Two brothers sold shares and both obtained the same sum, although their selling prices were different. If the elder brother had sold his shares at the younger brother's price, he would have obtained Rs 245 ; and if the younger sold his at the other's price he would have obtained Rs 125. The total number of shares was 60 ; how many had each brother and at what prices were they actually sold ?—S. C.

[*Ans.* *A*—35 shares @ Rs 5 each ; *B*—25 shares @ Rs 7 each

49. An officer can form the men of his regiment into a hollow square 5 deep and also into a hollow square 6 deep ; but the front of the latter formation contains 4 men fewer than in the former. Find the number of men in the regiment. —I. P. S. [*Ans.* 600 men

50. A rectangular tank contains 350 gallons of water. Its length is 6 ft and breadth 3 ft. Find its depth correct to within one-tenth of an inch, being given 1 gallon = 277·174 cu. ins.—C. C. [*Ans.* 37·4 in.

51. A rectangular block of stone is to be polished on all faces but that resting on the ground. By putting 3 different faces down, the total surfaces to be treated are found to be 412, 394 and 404 square feet, respectively. Find the lengths of the edges of the block.—C. C. [*Ans.* 8 ft : 9 ft ; 10 ft.

52. (i) A labourer in charge of a horse-roller traverses 99 miles whilst rolling a field of 40 acres. Find the width of the roller. 1 acre = 4840 sq. yd.

[*Ans.* 40 in.

(ii) 30 loads of gravel are laid evenly on a path 400 yds long and 4½ ft wide. If each load contains a cubic yard, find the thickness of the layer. —C. C. [*Ans.* 1·8 in.

53. When a boy was asked to divide 6739546 by 94321, he made a mistake in writing the divisor, and consequently got 1559 as the quotient and 3107 as the remainder. What was the amount of the error ?—S. C.

[*Ans.* 90000

54. Show that the number represented by *abcde*, where *a*, *b*, *c*, *d* and *e* are each a significant digit, is exactly divisible by 11, whenever $(a+c+e)-(b+d)$ is divisible by 11. —S. C.

55. Find the total cost of carpeting a room 18 ft × 14 ft × 10 ft with carpet 2 ft 3 in wide at 8s 4½d per yard, and of papering the walls, if the cost per piece of paper of length 12 yards and width 21 inches be 1s 8d.—G. C.

[*Ans.* £7 4s 4d

56. If the manufacturer makes a profit of 20%, the wholesaler a profit of 25%, and the retailer a profit of 40%, what was the cost of manufacture of an article bought at the shop for 17s 6d ?—G. C. [*Ans.* 8s 4d

57. A person loses in the first year 10% of his capital. But in the next he gains 20% of what he had at the end of the first year, and his capital is now Rs 7200 more than it was at first. Find his original capital.—G. C. [*Ans.* Rs 90000

58. In a firm *A*, *B* and *C* contribute capitals Rs 25000, Rs 35000 and Rs 40000 respectively. They agree to divide the profit in proportion to their capital, after paying 5% interest on capital, and Rs 2000 and Rs 3000 as salaries to *A* and *B* respectively for their services. What would each receive out of profits amounting to Rs 20000 ?—G. C. [*Ans.* Rs 5750 ; Rs 8250 ; Rs 6000

59. A dealer altered the trade discount from 10% to 15%. By what percentage should he increase the selling price so as to receive the same net amount for his goods ?—G. C. [*Ans.* $5\frac{15}{17}$%

60. Two men run a 3 mile race round a course measuring $\frac{3}{10}$ths of a mile. If their rates are as 4 : 3, how often and where does the winner pass the other ?—G. C. [*Ans.* Twice, at starting point

61. A merchant buys 4000 maunds of wheat, one-fifth of which he sells at a gain of 5%, one-fourth at a gain of 10%, one-half at a gain of 12% and the remainder at a gain of 16%. Had he sold the whole at a gain of 11% he could have made a profit of Rs 728 more. What was the cost of the wheat per maund ?—S. C. [*Ans.* Rs 26

62. A cargo is valued at £5270 6s ; the premium on the insurance is @ 5 guineas%, policy duty at 4s% and commission @ $\frac{7}{16}$%. What sum must be insured to cover the cargo and the expenses of the insurance, and what premium must be paid ?—S. C. [*Ans.* £5600 ; £294

63. A Government officer received an increment on April 1st 1927, and drew Rs 400 p.m. from that date in the grade 150-25-200-50/2—750. On October 1st 1927 he was deputed to a foreign service for 5 years on his grade pay. Contributions were payable to Government for leave and pension charges. Contribution for leave salary was payable at the rate of 12½% of the pay drawn in foreign service, and contributions for pension were payable at the following rates :—

Length of service in grade	Rate
1—4 years	5% of maximum monthly pay of grade
5—7 years	6% ,, ,, ,, ,,
8—10 years	7% ,, ,, ,, ,,
11—13 years	8% ,, ,, ,, ,,
14—16 years	9% ,, ,, ,, ,,
17 and above	10% ,, ,, ,, ,,

Calculate the total amount to be recovered by Government.—S. C. [*Ans.* Rs 7092 8as

64. A merchant sells a blend of tea at Rs 2 8as per lb and thus obtains 25% on his cost price. The blend is made up from teas costing him respectively Re 1 4as, Re 1 5as, Re 1 13as and Re 1 14as per lb. In an order for 18 lb of blended tea, he takes as his first quantity 7 lbs of tea of the Re 1 14as per lb. How many lbs of each of the other qualities must he add ? —R. A. [*Ans.* 1 lb ; 2 lb ; 8 lb.

65. In the following division sums find the dividends :—

(a) 8*ab*) 9*cde*4 (*fgh*
klm
2*npq*
*rs*3*t*

(b) 417) 154*abc* (*def*
ghkl
mnpq
*r*9*st*
uvw
xyz

—S. C. [*Ans.* (a) 90434 (b) 154707

66. If 60 cubic inches of lead, together with 54 cubic inches of cork, are equal in weight to 1538⅔ cubic inches of fir, and the weight of equal quantities of lead and fir are represented by the numbers 11·324 and ·45, what number represents the weight of an equal quantity of cork ?—G. C. [*Ans.* ·24

67. A candidate in an examination, wherein he has to gain at least half full marks to pass, has two papers of questions placed before him, of which the first carries half as many marks again as the second. On the first paper he gains marks which are ·58 of the maximum ; on the second, marks which are ·43 of its maximum. Does he pass or fail ? —G. C. [*Ans.* Pass

68. (a) If the tax on tea be reduced by 10%, by how much must the consumption increase so that there may be an increase of 8% on the revenue from this head ? [*Ans.* 20%

(b) A parliamentary grant is made at the rate of 5s per head for all children at elementary schools. If the grant be distributed at the rate of 5s 9d per child in town and 3s 3d per child in country schools, what percentage of the total number of children is in each class of school ?
—S. C. [*Ans.* 70% in town ; 30% in country

69. A money-lender borrows money at 4% p. a., paying interest at the end of the year. He lends it at 6% p. a., payable half-yearly, and receives the interest at the end of the year. By this means he gains Rs 104 8as in a year ; how much money does he borrow ?—S. C. [*Ans.* Rs 5000

70. A man on 1st January borrows Rs 10000 from a friend. This sum may be repaid together with simple interest at 5% either (i) on the 31st December by a single payment, or (ii) by equal monthly instalment on the last day of each month.

Show in detail how much he pays by each method—G. C.

71. To estimate the cubic content of a log of timber the following formula may be used $C = L \times \left(\frac{G+g+g'}{3}\right)^2$, where C is the cubic content of the log, L its length, and G, g, g' are equal to ¼-girth of the log at the middle, and at the two ends respectively. Estimate the cubic content of a log 21·5 ft long with girths 15·2 ft, 12·4 ft and 17·4 ft at the middle and the two ends respectively. Give your answer to the nearest cubic foot.
—P. S. C. [*Ans.* 302 cu. ft.

72. A man drove his motor car for 20 miles at an average speed of 25 m. p. h. At what average speed must he drive for the next 20 miles, if his average speed for the whole distance is to be 30 m. p. h. ?
—P. S. C. [*Ans.* 37½ m. p. h.

73. A person sold out £6450 India 3½% Stock at 64½, and with the proceeds bought as many Dunlop Rubber Company 5% Preference Shares of £1 each as he could @ 16s 3d each. What was the gain in income derived by the change in investment ? Brokerage ⅛% for India Stock and 3d for £1 share.—I. Com. [*Ans.* £25 8s

74. A multiplication sum having been worked is partially rubbed out, the figures remaining being :—the multiplicand 7699, the first two (of four) figures in the multiplier 42, and the last two figures 47 in the product. Restore the complete work.—P. S. C. [*Ans.* Multiplier—4253

75. A rectangular sheet of tin, the lengths of whose sides are 45 in. and 33 in., has four equal square portions removed from the corners, each side of the removed corner being 9 in., and the sides are then turned up to form an open rectangular box 9 in. deep. If the box be filled with water, find the weight of water when a cubic foot weighs $62\frac{1}{2}$ lb. How long will it take to discharge at the rate of 3 gallons a minute, assuming that a gallon of water weighs 8 lb ? —P. S. C. [*Ans.* $131\frac{5}{6}$ lb ; $5\frac{1}{2}$ min. nearly

76. The distance between Santahar and Poradaha is $70\frac{1}{2}$ miles. No. 1 Up Train leaves Poradaha at 6-57 and travels uniformly at a speed of 31 miles per hour. No. 2 Down Train leaves Santahar at 7-3 and travels at a uniform speed of 33 miles per hour. Where do they meet, and at what time ?—G. C.

[*Ans.* $36\frac{41}{128}$ miles from Santahar at 7 hr. 59 m.

77. Find the respective times between 4 and 5 o'clock when the hour and minute hands of a clock are (1) exactly opposite each other, (2) at right angles to each other.

—G. C. [*Ans.* (i) 4 hr $54\frac{6}{11}$m ; (ii) 4 hr $5\frac{5}{11}$m or 4 hr $38\frac{2}{11}$ m

78. A tradesman imported a quantity of foreign goods for which he had to pay a duty of 15%, but the demand for them having fallen he is obliged to sell them at a loss of $5\frac{5}{7}$% ; a month earlier he could have sold them for Rs 1242 more, and then he would have cleared $2\frac{6}{7}$% on the transaction. What price did he pay for the goods ?—P. S. C. [*Ans.* Rs 12600

79. Two monkeys, having stolen a pile of walnuts and filberts from a garden, are on the point of beginning their feast, when they see the injured owner of the nuts approaching with a stick. At once they see that he will take $2\frac{1}{2}$ minutes to reach them. There are twice as many filberts as walnuts, and one monkey finishes the latter at the rate of 15 per minute in $\frac{4}{5}$-time and runs away ; the other manages to eat the filberts just in time. If the first monkey had stopped to help the other till all were finished, find when they would have got away (i) if they eat filberts at equal rates, (ii) if the first monkey eats filberts at the same rate as he eats walnuts.

—P. S. C. [*Ans.* $2\frac{1}{4}$ min ; 2 min $18\frac{6}{11}$ sec.

80. A person owns $\frac{1}{5}$·th of a ship, worth £3484, which is insured for $91\frac{2}{3}$% of its real value. What would he lose in the event of the ship being lost ?

—G. C. [*Ans.* £58 1s 4d

81. Deduce the horse-power developed by a boiler from the formula H. P. $=\frac{10G\times F}{33000}$, where G is the number of gallons lifted per minute, and F is the lift in feet. The boiler works from 8 A.M. to 9 A.M. Delivery 100000 gallons. Net lift 150 ft. Add 70% to cover frictional losses.

—G. C. [*Ans.* 129 H. P.

82. A golfer has two clubs mended in Calcutta, a new head put on one, a leather face on the other. The former costs four times as much as the latter, the corresponding ratio in Lahore being 5. The leather face costs him twice the Lahore price. Including Re 1 for a ball, he pays twice the Lahore charge for these repairs. What is the Calcutta charge for each ?

—P. S. C. [*Ans.* Face Re 1 ; Head Rs 4

83. Barbier's formula for resistance to engine and tender in lb per ton is $R=8\cdot51+3\cdot24V\times\left(\frac{1\cdot609V+30}{1000}\right)$, where V is the speed in miles per hour. Calculate R when $V=52$, and answer to 3 significant figures.
—P. S. C. [*Ans.* 27·7 lb per ton.

84. A train travels from Colston Basset to Cropwell-Bishop at the rate of 20 miles an hour including stoppages. If there were no stoppages, it would travel at the rate of 25 miles per hour. After it has lost half an hour by stopping, an accident occurs owing to which it decreases its speed to $\frac{2}{5}$-ths of its former rate, and arrives at Cropwell-Bishop half an hour late. Where does the accident happen, and what is the distance from Colston Basset to Cropwell-Bishop ?—G. C. [*Ans.* 50 miles from start ; 60 miles

85. Two clocks, of which one gains and the other loses one minute in an hour, strike 1 O'clock together. What will be the interval by a correct clock between their respectively striking 2 O'clock ? —G. C. [*Ans.* $2\frac{2}{3599}$ min.

86. B is a place on a river between A and C, and is twice as far from A as from C. A man who rows at a uniform speed can row against the current from A to C in 2 hr 20 minutes ; and can row from A to B and back again in 2 hr 35 minutes. How long will he take to row from C to A ?
—P. S. C. [*Ans.* 1 hr 32½ min.

87. A weir is 1200 yards wide. If the water rises an inch while the stream is flowing at the rate of 3 miles an hour, how many additional tons of water will pass over the weir per minute ? A cubic yard of water weighs ¾ths of a ton.—G. C. [*Ans.* 2200 tons

88. A bill for 10 lb of butter, 7 fowls and 52 quarts of milk amount to 42s 7d, when a bill for 1 lb butter, 2 fowls and 4 quarts of milk is 6s 10d. The next week, butter rises 1d per lb, and fowls 3d each, and the bill for 8 lb butter, 5 fowls and 46 quarts of milk is 35s 11d. Find the prices for the first week.—S. C.

[*Ans.* Butter—14d p. lb ; Fowl—2s 3d each ; Milk—3½d p. qt.

89. A formula for ascertaining the average pay of a time-scale of pay is

$$\frac{A+B}{2}+\frac{B-A}{2}\left[1-(R+1)\left\{\cdot021+\frac{1-\cdot015R}{F-E}\right\}\right]$$

where A is the minimum pay of the scale, B the maximum pay, R the period of rise in years, E the average age at entry into the scale, and F the average age of retirement on super-annuation pension.

What is the average annual cost (to the nearest Rs 100) of the creation of 8 posts on the time-scale of Rs 80-5-200 per month (average age at entry 35 years) ? The average age of retirement is 55 years in both cases.—S. C.
[*Ans.* Rs 11600

90. A man after spending 5% of his income in insuring his life, and 5% of the remainder in income tax, has Rs 20 more than 90% of his whole income left. Find his whole income.—S. C. [*Ans.* Rs 8000

91. A tradesman sells two articles together for Rs 46 making 10% profit on one and 20% on the other. If he had sold each article at 15% profit, the result would have been the same. At what price does he sell each article ?
—S. C. [*Ans.* Rs 22 ; Rs 24

92. One tap will fill a cistern in 4 hours and a second will fill it in 6 hours. Find with the aid of a graph the time they will take to fill the cistern together. —S. C. [*Ans.* 2 hours 24 min.

93. In a long division sum the quotient consists of 2 figures; if the dividend is 34256 and the two remainders are 305 and 560, find the divisor and the quotient.—P. S. C. [*Ans.* 624 ; 54

94. Rupees 25600 was invested for 3 years at compound interest, the rate being 5% for the first year, 4% for the second and 3% for the third. Interest was added half-yearly. Find the amount earned.—G. C.
[*Ans.* Rs 8228 6as

95. A bill for £750 was drawn on 6th March payable 6 months after date the rate being 4½% p. a. It was discounted 28th June. What did the banker pay the holder of the bill ?—G. C. [*Ans.* £743 5s

96. A contractor undertook to construct a railway 400 miles long. He was able to shorten the line by 10 miles, but the cost of construction exceeded his estimate by Rs 150 per mile. If the total cost was unaltered, what was his estimated expenditure per mile ? [*Ans.* Rs 5850

97. During a sale a trader reduced the price of his goods 25% below marked prices, which had originally been fixed to allow 25% profit on cost price after deducting 10% discount for cash. What per cent does he gain or lose ?—G. C. [*Ans.* 4⅙% gain

98. An excess of 25% was charged by mistake on an invoice for 24 tons 15 cwt 3 qr @ £7 13s 6½d per ton. Calculate the amount of the Credit Note. —G. C. [*Ans.* £47 8s 2d

99. An Accomodation Bill was accepted on the 8th January 1935 at 4 months. It was discounted on the 11th February following to meet a bill for Rs 9000 due on that date. If the rate of discount be 4 per cent p. a. what was the amount of the first bill ?—G. C. [*Ans.* Rs 9997 8as

100. In terms of a partnership agreement between *A*, *B* and *C*, *B* and *C* are allowed to draw at the rate of Rs 250 and Rs 200 p.m. respectively in anticipation on profits, no interest being charged thereon. It is agreed that the Capital Accounts will be debited with these amounts in the event of loss or insufficient profits. Profits are to be divided in proportion to capital and left in the business, less drawings. During the first year there was a loss of Rs 2400 ; and during the second year there was a profit of Rs 20880. If *A* brought to the business Rs 30000, *B* Rs 20000 and *C* Rs 10000, show the balances of the capital accounts at the end of the second year.—G. C.

[*Ans.* *A*—Rs 40320 ; *B*—Rs 19680 ; *C*—Rs 7680

101. It was estimated that a new waterworks scheme would cost a municipality Rs 365585 10as. If the rateable value of the town is Rs 8 crores, find to the nearest ¼-pie the additional rate that will have to be charged to raise this amount. What excess, if any, would result if 98% only of the total dues are collected ?—G. C. [*Ans.* 1p in Re 1 ; Rs 42747 11as

102. An analysis showed that the cost of production of an article is made up as follows : Materials 40% ; wages 30% ; overhead charges 30%. If wages rise by 10% and the cost of materials fall by 5%, find the reduction necessary in the overhead charges in order that the cost of production may remain unaltered.—G. C. [*Ans.* 3⅓%

103. A tourist buys a bill in London for RM 10260·25 at a cost of £455. He encashes it in Paris when the rate is 100 francs = RM 16·50 and spends a sum of 37076·33 francs. When he returns to London he finds that he has still left with him the equivalent of £200. What are the rates between London—Paris, and London—Berlin ?—G. C.

[*Ans.* £1 = 125·585 francs = RM 22·55

104. A railway entered into a contract with a colliery to clear a certain quantity of stock at its siding in 35 weeks. 50 trucks of capacity 19¼ tons each were employed for the purpose for two days in the week. At the end of 13 weeks it was found that only $\frac{2}{11}$ths of the stock had been cleared. If the railway then allocated 65 trucks for 3 days in the week to finish the work in the stipulated time, what must have been the capacity of each new truck ? What will be the total cost of clearance, if the expenses up to the end of the first 13 weeks amounted to Rs 300 ?—G. C.

[*Ans.* 26¼ tons ; Rs 1850

105. A broadcast receiving set was purchased on the following terms :—Rs 50 down on 1st January 1938 and 12 instalments of Rs 15 each payable on the first day of every succeeding month. After the third monthly payment was duly made, the purchaser desired to pay the balance in a lump sum. What is the last date on which the dealer would be prepared to receive this lump sum payment without charging interest ? What was the average interest charged on the instalment terms if the cash price was Rs 200 ? —G. C. [*Ans.* 1st Sept. 1938 ; 37% p. a.

106. A Bank charges the customer Rs 5198 12as for a remittance to London when the rate of exchange stands at 1s 6$\frac{15}{16}$d to the rupee. What is the value of the remittance in £ sterling ?—G. C. [£411 13s 10d

107. Two grades of motor oil *A* and *B* are mixed in the proportion 3 : 1 to make 96 gallons of grade *C*. When half of *C* has been sold, a further quantity of *A* is added to increase the proportion of *A* to *B* in the resulting mixture to 7 : 2. Find the quantity of *A* last added.—G. C. [*Ans.* 6 gallons

108. A speculator invested in Jute shares at 285 and sold out at 289¾ ; he re-invested in Cotton shares at 81⅞ and sold out at 82½ ; then, in coals at 55 and sold out at 54½ ; then in Government Paper at 45½ and sold out at 42¾ ; then he bought War Loan at 100¼, and sold at 105. Assuming that every time all the amount realised was fully re-invested, does he gain or lose by the transactions ?—G. C. [*Ans.* Neither

109. A man sold a horse at a profit of 8%. If the price had been Rs 27 lower the result would have been a loss of 10%. Find the cost price of the horse.—A. G. [*Ans.* Rs 150

110. Galvanized sheets were despatched to a factory as follows :—

235 pcs.	weighing	1 ton	15 cwt 3 qr
115 pcs.	,,		18 cwt 3 qr 6 lb
460 pcs.	,,	2 ton	14 cwt 2 qr 8 lb
90 pcs.	,,		8 cwt 1 qr

Make out an invoice for these at Rs 4 8as per cwt. Find also the average weight and price of a sheet.—G. C. [*Ans.* Rs 528 3as ; 14·6 lb ; 9as 5p

111. 11 B. W. G. copper wire (⅛-in. thick) is wound closely round a cylindrical drum 2 ft. high. The drum has a diameter of 3 ft. 6 in. and a base of diameter 7 ft. If 100 ft. of the wire weigh 4·75 lb, find the length of the wire fully covering the drum, and its weight.

[*Ans.* 132 miles 128 yd, 3·591 tons

112. A scheme of street electrification is estimated to cost a City Municipality Rs 12½ lakhs per annum. If the rateable value of the city be Rs 2½ crores, find to the nearest pie the additional rate that will have to be levied to meet this expenditure. If 15% of the additional rate cannot be collected, find whether it will still be possible to meet the entire cost of the scheme from the levy.—G. C. [*Ans.* 10p in the rupee ; Yes

113. *A*, *B* and *C* enter into partnership in which *C* has a ⅔rds share, the others holding equal shares. It is agreed that 6¼% of the profits will be set aside as a Reserve; and that *A* and *B* are to receive 10% and 15% respectively of the profits as salary. Find what the partners receive from the business if the profits during a year are Rs 4654 8as.—G. C.

[*Ans.* *A*—Rs 998 12as 6p ; *B*—Rs 1231 8as ; *C*—Rs 2133 5as

114. A train runs as follows :—

A to *B*...5 miles in 8 minutes...stoppage 3 mins.
B to *C*...4 ... 6 ... ... 3 ...
C to *D*...6 ... 10 ... ... 4 ...
D to *E*...8 ... 11 ... ... 4 ...
E to *F*...7 ... 10 ... ... 3 ...
F to *G*...4 ... 6 ... ... 3 ...

Find the average speed of the train (i) with stoppage, and (ii) without stoppage—G. C. [*Ans.* 30 m.p.h. ; 40 m.p.h.

115. The ratio of expenses to income of an insurance company is 48 per cent. If expenses be curtailed by 25% all round, how will the expense ratio be altered, if at the same time the income falls by 10% ?—G. C. [*Ans.* 40%

116. A man received in $ 146 and exchanged it @ £1=$5. He also received £38 17s 6d and exchanged it at £1=$4·75. Find his gain per cent in each case and calculate his total gain if exchange stood at £1=$ 4·867. —I. Com. [*Ans.* 2·73% ; 2·46% ; $8·42

117. A man has £2100 stock 3 per cent which he transferred to the 3½ per cents at £87½, thus increasing his income by £7. What was the price of the 3 per cents ?—I. Com. [*Ans.* 83⅓

118. A loan is to be repaid as follows :—20 per cent at the end of a month, and the balance in equal monthly instalments thereafter @5% of the outstanding. After the 10th monthly instalment has been paid, the borrower desires to pay off the balance in a lump sum. For how long may he postpone this payment ?—G. C. [*Ans.* 5½ mos. after last payment

119. A Transport Company has an average monthly income of Rs 80,000 from 40 motor lorries doing an average of 229½ miles a day for 6 days in the week. What should be his estimated monthly income with 85 similar lorries doing 202½ miles a day with a rest of two days in the week ?

—G. C. [*Ans.* Rs 1,25,000

120. An author received a sum of money for a book, and in addition, a fixed amount for every copy sold. When 500 copies had been sold, he had received £85 16s 8d, and when 1350 copies had been sold he had received altogether £142 10s. How much will he have received when 10000 copies have been sold ?—I. Com. [*Ans.* £719 3s 8d

121. It was noted that two dockyard cranes took 7 hours 45 minutes to clear 310 packages from the ship's hold. How long will three cranes take to clear 3444 similar packages, if the third crane can work 1½ times as fast as the others ?—G. C. [*Ans.* 49½ hours

122. A firm makes a profit of 10% in excess of its previous year's profit for 4 consecutive years. The difference between the profit made in the second and last years amounts to Rs 12,705. What was the first year's profit ?

—G. C. [*Ans.* Rs 55000

123. An article was marked 20% above cost. If the trader allowed the usual trade discount he could get Rs 24 more than what he got by selling it at a discount of 20% to clear shop-soiled stock. What was the rate of trade discount, which if calculated on the cost price would come to Rs 20 ?—G. C. [*Ans.* 10%

124. A grocer buys flour at 8as 6p a seer to adulterate barley at 15as 6p per seer. How would he mix them in order to sell the mixture at Re 1 2as 9p per seer to clear 25% ? How much flour will thus be used with 5¾ seers of barley ?—G. C. [*Ans.* 1 : 23 ; ¼ seer

125. A difference of 15as in the cost price of an article would mean a loss of 5% to the dealer instead of a gain of 7½%. What is the cost price of the article ?—G. C. [*Ans.* Rs 7 8as

126. The list price of an article is 25% above the selling price ; and the cost price is 40% below the list price. Find the rates of discount and profit. [*Ans.* 20% ; 33⅓%

127. A bankrupt's assets are as follows : Cash and realised assets, Rs 55825 ; good debts Rs 18695 ; debts 60% good Rs 24300 ; debts 40% good, Rs 38500. His liabilities amount to Rs 209000. How much can the receiver pay as dividend ? [*Ans.* 8as in Re 1

After some litigation a claim for Rs 52250 was ordered by the court to be treated as preferential. How would a creditor for Rs 42000 be affected ? —G. C. [*Ans.* Further loss Rs 7000

128. A cubical box, of external dimensions 17 inches each way, would contain crushed ore of the value of £421 17s 6d, if it were made of material 1 inch thick ; but by mistake it has been made of thicker material and the difference in the value of the ore which it will hold is consequently £78 17s 6d. What is the real thickness of the material ?—I. Com. [*Ans.* 1·5 in.

129. An increase of 50% in the import duty resulted in an increase of 20% in the price of a motor car. What percentage of the old price represented duty ?—G. C. [*Ans.* 40%

130. A discount of 33⅓% is allowed on the list price of a certain make of guns. The importer has to pay duty at 20% on the net cost price ; and his expenses are estimated at 10% on the same. If the guns are sold at the list price plus duty calculated thereon, what is the profit made ?—G. C. [*Ans.* 38$\frac{6}{13}$%

131. 1 maund 38 seers of vegetable ghee was mixed with pure ghee to form a mixture 61% pure. A dealer tries to improve the quality by putting a further maund of pure ghee into the mixture. What is the percentage of ghee in the new mixture ?—G. C. [*Ans.* 67·5%

132. A city municipality decides to meet half of a new recurring expenditure of Rs 20 lakhs out of its ordinary revenue, and to raise the other half by increasing the rates. If the rateable value of the city be 12½ crores, calculate the additional rate that will be levied to raise the necessary funds. Answer correct to the tenth of a pie.—G. C. [*Ans.* 1·6 pies in Re 1

133. 200 2-in. hexagonal nuts weigh 52¾lb. If a cubic foot of the metal weighs 480 lb, and the nuts have been made out of a plate 1¾ in. thick find the inside diamater of a nut. [*Ans.* 1 inch

134. *A* and *B*, the principal ingredients in the manufacture of a commodity, are used in the proportion 5 : 6. The respective costs are as 4 : 3. The finished product is equal in quantity to that of *A*, and is sold at a profit of 10% at Rs 22 per cwt. If the other expenses of production amount to Rs 5 per cwt produced, find the altered selling price when the cost of the ingredients go up by 10%. [*Ans.* Rs. 23 10as 6p per cwt.

135. A merchant in New York buys goods in Geneva to the value of 4004 francs. Find the value of the goods in dollars, when the rate of exchange between London and Geneva in 25·48 francs to the £.—I. Com. [*Ans.* $764·50

136. A man left Rs 18 000 with the direction that it should be divided in such a way that his 3 sons, aged 9, 12 and 15 years, should each receive the same amount when they reached the age of 25. If the rate of interest is $3\frac{1}{3}$% p.a., what should each son receive when he is 25 years old ?—I. Com. [*Ans.* Rs 9350 8as

137. Of 3 pipes *A*, *B*, *C*, *A* and *B* together can fill a cistern in 20 minutes ; *B* and *C* together can fill it in 18 minutes ; *A* and *C* together can fill it in 15 minutes. How long will *B* alone take to fill the cistern ? —P. S. C. [*Ans.* $51\frac{3}{7}$ min.

138. The rents on a certain estate are 15% higher now than they were 10 years ago ; the landlord allows 20% off the total rental for repairs and pays his agent 4% of the remainder for collection ; after paying income tax at 8d in £1 on his gross income, his net income is £6670. What was the rental of the estate 10 years ago ?—P. S. C. [*Ans.* £7812 10s

139. *A* can beat *B* by 1 foot if he gives *B* 3 yards start in a race of 100 yards ; if *B* gives *C* a start of 8 yards in a race of 120 yd, *C* wins by 28 in. Who will win and by how much if *A* gives *C* a start of 10 yd in a race of 100 yd ?—S.C. [*Ans.* *C* beats *A* by $\frac{25}{28}$yd.

140. A river which flows at a uniform rate passes through two towns which are $13\frac{1}{2}$ miles apart. A steamer travelling down stream from one town to the other takes 45 minutes for the journey, but a man rowing a boat takes twice as long. On a journey upstream the row boat takes 3 times as long as the steamer. What is the rate of flow of the river and what would be the speed of each of the boats in still water ?—S. C.

[*Ans.* $2\frac{1}{4}$ m.p.h. ; $6\frac{3}{4}$ m.p.h. ; $15\frac{3}{4}$ m.p.h.

141. At three positions *A*, *B*, *C* in town, which are in the same straight line and of which each is $\frac{3}{4}$ mile apart from the next, the distant boom as of a gun is heard at different moments. At *B* and *A* the boom is heard $\frac{3}{4}$ sec. and $1\frac{7}{8}$ sec. respectively later than at *C*. Calculate the distance of the source from *B* to the nearest mile, the velocity of sound being 1120 ft per second.—B. C. S. [*Ans.* 12 miles

142. An ordinary wooden match-box contains 70 matches. The section of each match is a square ; 5 matches are placed side by side, forming a rectangle $\frac{1}{2}$ in. wide and 2 in. long. Find the internal cubical contents of the box, supposing $\frac{1}{4}$th of the whole space is wasted—P. S. C. [*Ans.* $1\frac{7}{8}$ cu. in.

143. The gradients on a railway between two stations *A* and *B* are as follows :—

1 in 500	rise	for	720 yards
level	—	,,	160 ,,
1 in 2500	fall	,,	2120 ,,
1 in 570	fall	,,	324 ,,
level	—	,,	150 ,,
1 in 2400	rise	,,	1520 ,,
1 in 200	rise	,,	520 ,,

Find the difference in level between *A* and *B*. —B. C. S. [*Ans.* 3·3 yd.

144. A model of a machine is made from the same material as the machine itself, the linear dimensions being reduced on the scale 1·5 inches to the foot. If the model weighs 74 lbs, what will be the weight of the machine to the nearest ton ?—B. C. S. [*Ans.* 17 tons

145. In order to calculate the diameter of a piece of wire, a length of it is dropped into a vessel completely full of water. If 35 cubic centimetres of water spill over, and the length of the wire is one metre, what is its diameter ?—B. C. S. [*Ans.* 2·23 mm.

146. Newsprint, ·003 in. thick is rolled tightly into the shape of a cylinder. If a roll contains 20 miles of paper, find its diameter to the nearest foot.—P. S. C. [*Ans.* 6 ft

147. In 1940, in a market of rising money rates, an investor has the choice of the two following investments : 4% stock (1950-70) at 85 and 4½% stock (1955-75) at 95. Which investment should be preferred ?

Which should give the better average yield, assuming that there will be a steady fall in money rates ? [*Ans.* The former in both cases.

148. A bucket 12 inches high is placed out in the rain ; and in an hour it collects rain water to a height of 5 inches. If the top has a diameter of 8 inches, and the bottom 4 inches, find the rate of rainfall per hour. [*Ans.* 1·843 inches.

149. The expenses of a hotel are partly constant, and partly vary directly with the number of boarders. Each boarder pays Rs 300 a month. The profits are Rs 54 a head per month when there are 50 boarders, and Rs 64 when there are 60. What is the profit on each boarder when there are 80 ?—G. C. [*Ans.* Rs 76 8as

150. A vessel trading between two ports left one of them under steam, but 36 hours after starting, a fair wind enables her to go 3 miles per hour faster during the rest of the journey, and to reach distination 21 hours sooner. Had she been able to utilize the wind during the whole journey she would have arrived 6 hours before she actually did. Find the rate under steam and the time for the journey under steam only.—G. C. [*Ans.* 15 m. p. h. ; 162 hours

151. A stone sphere, 18 inches in diameter, on a College bridge at Cambridge was rolled off into the Cam. If it had been placed in a rectangular tank containing water, how much would the water have risen if the tank had been 4½ ft long and 2½ ft broad ?—P. S. C. [*Ans.* 1·9 in. app.

152. A person invests Rs 43,700 in the purchase of 4% Port Trust Loan at 115 and sells out when the price goes up to 125, investing the proceeds in a 3½% issue at 95. What is the change in his net annual income, assuming that an income tax of 2as 8p in the rupee was payable by him throughout ? —G. C. [*Ans.* Rs 191 10as 8p

153. If 1·143 and 2·571 are both correct to 3 places of decimal, find the limits of error in their product. —G. C. [*Ans.* ±·001470

154. A father wishes to provide a dowry for his daughter's marriage. He puts in a small sum every month in Savings Bank and expects an interest of 3% per annum on monthly balances. Find how much he should lay by every month to obtain at least Rs 5000 at the end of 15 years. [*Ans.* Rs 22 1a

155. Find the cost of making a road half-a-mile long, 36ft wide ; the soil being first excavated to a depth of 1 ft at a cost of 1s per cu. yd., rubble being then laid 9in. deep at a cost of 1s 6d per cu. yd., and then 3 in. of gravel at 3s 3d per cu. yd. being laid on the top, and the whole consolidated by a steam roller at a cost of 2d per sq. yd.—I. Com. [*Ans.* £605

156. What price should be marked on articles which cost £2 6s 8d so that a profit of 10% may be made after allowing a discount of 12½% for cash discount ? —I. Com. [*Ans.* £2 18s 8d

157. A man embarks his whole capital in 4 successive ventures ; in the first he obtains 100% and in each of the others he loses 20%. Show what percentage he has gained on his original capital. —I. Com. [*Ans.* 2·4%

158. A workman is able to save 12½% of his wages ; but if his wages were raised 2s a week and his expenses were increased by 10%, his annual savings would be diminished by 17s 4d. What are the man's weekly wages, a year being taken as 52 weeks exactly ? —I. Com. [*Ans.* 26s 8d

159. *A* and *B* are partners in a trading venture *A* contributing £2000 and *B* £3000. *A*, however, acts as manager, the understanding being that, of the profits *A* shall get 25% for his services as manager, the remainder to be divided in the ratio of their contributions. *B* gets £120 in his share. What would *A* get ? —I. Com. [*Ans.* £146 13s 4d

160. An estate consists of house property of gross rental of £150, but land tax £8 10s, sewers tax £1 15s and insurance £3 15s are paid. The losses on account of bad tenants amount to 12½% of the rental, ordinary repairs to 17½% and collection charges to 2½%. Find the value of the property at 4% simple interest. —I. Com. [*Ans.* £2306 5s

161. A yard measure is too long by ·18 inch. It is used to measure the sides of a rectangle ; find how much per cent the true area exceeds the calculated area. —I. Com. [*Ans.* 1% *less.*

162. In mixing tea 1 lb. in every 100 lb is wasted. In what proportion must a dealer mix teas which cost him 1s 9d and 1s 4d per lb respectively, so as to gain 10% by selling the mixture at 1s 6d per lb ?—B. C. S, [*Ans.* 4 : 1

163. The capital of a certain Railway is £1,000,000 in 20,000 shares of £50 each fully paid up. The gross annual receipts are £105,000, of which 48% is absorbed in working expenses, £4,600 goes to the reserve fund, and the remainder to pay dividend. Find what annual income a person will obtain from the investment of £4,500 in the undertaking, the shares being at £62 10s. —B. C. S. [*Ans.* £180 p.a.

164. A dealer bought a horse for £110, and sold it the same day for £121 15s allowing the buyer 5 months' credit. Money being worth 3½% per annum, what was his gain per cent ? —B. C. G. [*Ans.* 9$\frac{1}{11}$%

USE OF TABLES

The appended Tables are meant to be used to simplify calculations. The values obtained from them may be depended upon for most practical purposes, with the limitation that the Logarithm Tables can give only four dependable significant figures in the answer. Interest Tables will be found to be more accurate, while the decimalised values of annas and pies will give exact values. The Conversion Tables are necessarily approximate, but will be found of extensive use in checking results otherwise obtained. The Calendar for the current century may be found to be useful for different purposes. Foreign Money Tables have been omitted in the present uncertain international conditions.

The illustrations following indicate the manner in which the Tables may be used with advantage.

ILLUSTRATIONS

TABLE I G. *Weights and Measures* :

(i) Reduce $53\frac{1}{2}$ standard Maunds into cwt.

[1 md = $\frac{1}{2}$ cwt + $\underline{2}$ − 2 lb − $\underline{7}$)]

$\frac{1}{2}$ of $53\frac{1}{2}$ cwt = 26 cwt 3 qr
$\underline{2}$ = 13 ” 1 ” 14 lb
40 cwt 0 qr 14 lb

less $2 \times 53\frac{1}{2}$ lb = 3 qr 23 lb
$\underline{7}$ = 15 ” 3 qr 8 lb
39 cwt 1 qr 6 lb

Note. A somewhat rougher value is obtained by taking 1 md. = $\frac{1}{2}$ cwt + $\underline{2}$ − $\underline{16}$.

(ii) Reduce 3 ton 15 cwt to maunds.

[1 ton = 25 md + $\frac{1}{9}$th of 20 mds]

$3\frac{3}{4} \times 25$ = 93 md 30 seers
$3\frac{3}{4} \times 20 \times \frac{1}{9}$ = 8 ” 13 ”
102 md 3 seers

(iii) Find the weight of 6 Madras candys (500 lb.)

[1 candy = 4 cwt + $\underline{8}$ − $\underline{14}$]

6 × 4 cwt	= 1 ton 4 cwt
$\underline{8}$	3 cwt
	1 ton 7 cwt
less $\underline{14}$ i.e. $\frac{1}{14}$ of 3 cwt	0 qr 24 lb
	1 ton 6 cwt 3 qr 4 lb

(iv) Find the length of a railway 36½ miles long in Kilometers. [1 mile = 1 Km + $\underline{2}$ + $\underline{5}$ + $\underline{9}$% + $\underline{3}$%

36½ × 1 Km =	36·5 Km
$\underline{2}$ =	18·25
$\underline{5}$ =	3·65
$\underline{9}$% =	·33
	58·73 Km

Note. If the work were continued to the next stage, a slightly bett result would be obtained 50·74 Km app.

(v) Convert 368·50 Kg into lb. [1 Kg = 2 lb + $\underline{10}$ + $\underline{2}$% + $\underline{10}$ +

		lb
368·50 × 2	=	737
$\underline{10}$	=	73·7
$\underline{2}$%	=	1·474
$\underline{10}$	=	·147
$\underline{2}$	=	·073
		812·394 lb
	or	812¾ lb nearly.

TABLE IV. *Customs*: Find duty on the following :

(i) 1½ cwt cardamon.

Tariff value @ Rs 55 per cwt	= Rs 82 8as
Duty @ 50%...Rs 42 4as	
less @ 5%...Rs 4 4as	= Rs 38

(ii) 1000 pairs of Continental boots and shoes, valued at Rs 3580.

Duty @ 25% *ad. val.* = Rs 895

Duty at 4as per pair ... Rs 250
at 1a " $\frac{4}{}$ ... Rs 62 8as
Rs 312 8as

∴ Duty payable is Rs 895

TABLES VI-VII. *Money and Exchange* :

(i) Convert Rs 8500 into Yens at Rs 100 = 77'50 Yens.

Rupee 1 = '7¾ Yen
850 × 8 = Yen 6800
× ¼ = 212'50
Yen 6587'50

(ii) Reduce $ 385'60 into Milreis at $ 100 = 8'70 Milreis.

3'856
8'7
30'848
2'9992
33'8472

∴ $ 385'60 = 33'85 Milreis.

(iii) Find the value of 15½ lbs of Tea @ 10as 7p per lb.

10as 7p = Re '6614$\frac{7}{12}$ from the Table.

10 lbs cost Rs 6'6146
5 lbs cost 2 Rs 3'3073
'5 lb costs 10 Rs '3307
Rs 10'2526

i.e. Rs 10 4as from the Table.

(iv) Convert £334 5s 9d into Rupees @ Re 1 = 1s 5$\frac{17}{32}$d

£300 = 3 × Rs 1368 15as 8'9p = Rs 4106 15as 2'7p
30 = 3 × Rs 136 14as 4'5p = 410 11as 1'5p
4 = 4 × Rs 13 11as 0'4p = 54 12as 1'6p
5s = 3 13as 6'2p
9d = 9 × 11'0p = 8as 3'0p
Rs 4576 12as 3p

(v) Convert £65 8as 10½d into Rupees @ Re 1 = 1s 6$\frac{31}{32}$d..

£50 =		Rs 632 9as 10'9p
£10 =		126 8as 4'6p
£5 = $\underline{2}$		63 4as 2'3p
5s =		3 2as 7'3p
3s = 3 ×	10as 1'5p	1 14as 4'5p
10d = 10 ×	10'1p	8as 5'0p
½d =		5'0p
		Rs 828 0a 3'6p
	or,	Rs 828 0a 3p

(vi) Convert Rs 5834 into £'s @ 1s 6$\frac{15}{32}$d.

Reqd. value = £5834 × '076953⅛

= £448'9445 = £448 18s 10⅝d

(vii) Convert Rs 863 10as into £ sterling at Re 1 = 1s 6$\frac{19}{32}$d.

Reqd. value = £863'625 × '0774739583 = £66'9084

= £66 18s 2d

(viii) Convert $ 453'50 into £ sterling at £1 = $ 4'90$\frac{3}{32}$.

At this rate $1 = £'2040426. Multiply correct to 4 decimal places, 453'50 × '2040426.

```
20'4~~0426~~
 4'535
81'6170        ∴ Required value = £92'5332
10'2021
  '6121                         = £92 10s 8d
  '1020
92'5332
```

TABLE VIII. *Brokerage, Commission and Discount.*

(i) Calculate brokerage on Rs 8620 12as @ 3¾%.

@ 3¾% on Rs 8000	...	Rs 300
600	...	22'5
20	...	'75
'7	...	'026¼
'05	...	'001⅞
		Rs 323'278

∴ Required brokerage = Rs 323' 4as 6p

(ii) Find discount on £83 15s @ 4¼%

@ 4¼% on £80 ... £3·4

3 ... ·1275

10s ... ·0212

5s ... ·0106

£3·5593

∴ Required discount = £3 11s 2¼d

TABLE IX. *Simple Interest, Banker's Discount.*

(i) Find simple interest on Rs 5325 for 68 days @ 2¾% p. a.

Reqd. interest = Interest on Rs 5325 × 68 @ 2¾% p. a. for 1 day

= Interest on Rs 362100 @ 2¾% p. a. for 1 day

Interest @ 2¾% p. a. for 1 day on Rs 300000 ... Rs 22·6027

60000 ... 4·5205

2000 ... ·1507

100 ... ·0075

Rs 27·2814

∴ Required interest = Rs 27 4as 6p

(ii) Calculate Banker's Discount @ 3½% on a bill for Rs 6385 to mature after 39 days.

Discount = Interest on Rs 6385 × 39 for 1 day @ 3½% p. a.

= Interest on Rs 249015 for 1 day @ 3½% p. a.

Interest @ 3½% for 1 day on Rs 200000 ... Rs 19·1781

40000 ... 3·8356

9000 ... ·8630

10 ... ·0010

5 ... ·0005

Rs 23·8782

∴ Required discount = Rs 23 14as

TABLE X. *Table of Days*

(i) Calculate the number of days between 18th March 1937 to 27th July 1937.

27th July 1937 is 208 days from 1st Jan 1937

and, 18th March 1937 is 77 days from 1st Jan 1937

∴ Intervening Period = 131 days.

(ii) Find the number of days to maturity of a bill due on 13th Feb. 1936 as at 22nd November 1935 on which it is discounted.

Bill due on 13. 2.36
Days of Grace 3 From 22.11.35 to 31.12.35 = 39 days
Bill matures on 16. 2.36
Discounted on 22.10.35 From 31.12.35 to 16.2.36 = 47 days
86 days

∴ Intervening Period = 86 days

TABLE XI. *Depreciated Book Value.*

Find the depreciated book value of an asset costing Rs 105000 at $7\frac{1}{2}$% p. a. on reduced values at the end of 11 years.

From the Table Rs 1000 thus depreciated becomes Rs 424

∴ Required value = Rs 424 × 105
= Rs 44520

TABLE XII. *Years' Purchase represented by prices of a $3\frac{1}{2}$% perpetuity.*

If $3\frac{1}{2}$% Irredeemable Government stock is quoted at $94\frac{3}{16}$, find the number of years' purchase represented by the price. Also, find the relative yield.

A price of 90 represents 25·7 years' purchase
" " 4 " 1·14 " "
" " 3as " ·05 " "
26·89 years' purchase

∴ Required answer = 26·9 years' purchase
⇒ 27 years' purchase

$$\text{Also, yield per annum} = \frac{100}{26{\cdot}89}\%$$
$$= \underline{3{\cdot}72\%} \qquad \text{(See Reciprocals Table)}$$

TABLE XVII. *Yield on Investments.*

To find yield on investment in the $3\frac{1}{2}$ per cent at $89\frac{13}{16}$

@ $89\frac{8}{16}$ yield = 3·91%   @ 90 yield = 3·89%
@ 90 " " 3·89   Diff. 3as " ·01 *Add*
Diff. 8as " " ·02   ∴ @ $89\frac{13}{16}$ " 3·90%
3as " " ·01

N. B. The method used is linear Interpolation.

TABLES I

WEIGHTS AND MEASURES

A. Imperial Standard Units (F. P. S.)*

Linear Measure

12 inches	1 foot	6 feet	1 fathom
3 feet	1 yard	10 chains	1 furlong
5½ yards	1 rod	8 furlongs	1 mile
22 yards	1 chain	1760 yards	1 mile
100 links	1 chain	6082 feet	1 Br. Nautical mile

Square Measure

144 sq. ins.	1 sq. ft.	10 sq. chains	1 acre
9 sq. ft.	1 sq. yd.	4840 sq. yds.	1 acre
484 sq. yds.	1 sq. chain	8 roods	1 acre
10^5 sq. links	1 acre	1 sq. mile	640 acres

Cubic Measure

1728 cubic inches	1 cu. ft.	277·274 cubic ins.	1 gallon (Imp.)
27 cubic ft.	1 cu. yd.	231 cubic ins.	1 U. S. gallon
128 cubic ft.	1 cord of wood	40 cubic ft.	1 shipping ton (standard)
2150·42 cubic ins.	1 standard bushel	100 cubic ft.	1 Register ton (shipping)

Paper Measure

24 sheets	1 quire	20 quires	1 ream
10 reams	1 bale		

Liquid Measure

16 fluid ounces	1 pint	4 gills	1 pint
2 pints	1 quart	4 quarts	1 gallon
31½ gallons	1 barrel	2 barrels	1 hogshead

---

*The Imperial Standard Units are used in Great Britain, British Colonies and Dominions and also in the U. S. A. (with some differences noted below.)

** 1 knot is a speed of 1 Nautical mile per hour.

60 Nautical miles = 1° of earth's surface.

Avoirdupois Measure

27$\frac{11}{32}$ grains	... 1 dram (dr.)	20 cwts.	... 1 ton
16 drams	... 1 ounce (oz.)	2000 lbs.	... 1 Short Ton (American)
16 ounces	... 1 pound (lb.)		
28 pounds	... 1 quarter (qr.)	2240 lbs.	... 1 Long Ton
4 quarters	... 1 hundred-weight (cwt.)	437½ grs.	... 1 oz.
		7000 grs.	... 1 lb.

Troy Measure

24 grains	... 1 penny-weight (dwt.)	12 ozs.	... 1 lb.
		5760 grains	... 1 lb.
20 dwts.	... 1 ounce (oz.)	1 carat	... 3·2 grains
480 grains	... 1 oz.		

Apothecaries Measure

Weight		*Fluid*	
20 grains	... 1 scruple (℈)	60 minims	... 1 drachm
30 scruple	... 1 dram (ʒ)	8 drachms	... 1 ounce
8 drams	... 1 ounce (℥)	20 ozs.	... 1 pint
12 ozs.	... 1 pound (Troy)	8 pints	... 1 gallon (Imp.)

N. B. A grain has the same wieght in all Measures.

B. Indian Standard and Provincial Measures.

WEIGHTS

Indian Standard & Bengal

180 grains	... 1 tola	40 seers	... 1 maund
5 tolas	... 1 chhittack	1 Maund	... 82$\frac{2}{7}$ lbs.:
16 chhittacks	... 1 seer	1 Bazar Md.	... 82 lbs.
Factory seer	... 72$\frac{4}{7}$ tolas.		

Bombay

36 tanks	... 1 tipari	4 seers	... 1 payli
2 tiparis	... 1 seer	16 paylis	... 1 phara
25 pharas	... 1 muda	8 pharas	... 1 candy
		Maund (40 seers)	... 1 quarter (*Br.*)
1 seer	... ·84 stand. seer	Candy (cotton)	... 784 lbs.
		Candy (standard)	... 560 lbs.
Candy (wheat)	... 656 lbs.		

Madras

3 tolas	...	1 palam	8 palams	...	1 seer
5 seers	...	1 viss	8 viss	...	1 maund
20 maunds	...	1 Candy	1 Candy (bazar)	...	500 lbs.
1 Maund	...	$24\frac{24}{35}$ lbs.	1 Seer	...	·3 stand. seer

LAND MEASURES

Bengal

2 hath (cubits)	...	1 yard	20 sq. cubits	...	1 chhatak
16 chhataks	...	1 cottah	20 cottah	...	1 bigha
1 cottah	...	720 sq. ft.	1 bigha	...	1600 sq. yds.

U. P.

20 kuchvansi	...	1 bishvansi	20 bishvansis	...	1 bigha
		1 bigha	...	3025 sq. yd.	

Punjab

9 sarsi	...	1 maria	20 marias	...	1 canal
4 canals	...	1 bigha	1 bigha	...	1620 sq. yd.

Madras

1 bigha	...	6400 sq. yd.

Bombay

39¼ sq. cubits	...	1 kathis	20 kathis	...	1 pand
20 pands	...	1 bigha	6 bighas	...	1 rukeh
20 rukehs	...	1 chahur	1 bigha	...	3927 sq. yd.

C. The Metric System (C. G. S.)

The following prefixes are used in connection with standard units of weights and measures, given below.

(d) deci —$\frac{1}{10}$th	Deca—	10	times	(D)
(c) centi—$\frac{1}{100}$th	Hecto—	100	,,	(H)
(m) milli—$\frac{1}{1000}$th	Kilo—	1000	,,	(K)
	Myria—	10000	,,	(M)

Unit of *length*	... 1 meter
Unit of *area*	... 1 sq. Decameter or 1 Are.
Standard Linear Unit	... 1 Kilometer (Km.)
Unit of *volume*	... 1 Stere or cubic meter
Unit of *weight*	... 1 gramme
Standard cubic unit	... 1 Litre or 1000 c. cms.
Standard unit of weight	... 1 Kilogramme (weight of 1 litre of water)

The Metric System is in general use in the Continent of Europe, with local differences in the names of units. It is also steadily replacing the Imperial System in many advanced countries of the East and West – Persia, Thailand, Japan etc. are adopting the Metric System for Home and Foreign Trade purposes.

D. Monetary Units

Country	Money
Great-Britain	£1 = 20s = 240d
India	Re 1 = 16as = 192 pies
U. S. A.	$1 = 100 cents
France	1 franc = 100 cents
Germany	1 R. Mark = 100 pfenniges
Italy	1 Lira = 100 cents
Japan	1 Yen = 100 Sens
Shanghai	$1 (standard) = 100 cents
Ceylon	1 Rupee = 100 cents
Holland	1 Florin = 100 cents
Strs. Settlements	$1 = 100 cents
Dutch East Indies	1 Florin = 100 cents
Thailand	1 Baht = 100 satangs
Belgium	1 Belga = 5fr. = 500 cents
Denmark	1 Krone = 100 öre
Norway	Ditto
Sweden	Ditto
Czechoslovakia	1 Krone = 100 heller
U. S. S. R.	1 Tch. = 10 roubles = 100 Kps.
Canada	$1 = 100 cents
Brazil	1 Milreis = 100 cents
Argentine	1 Peso (paper) = 100 cents
Austria	1 Schilling = 100 groschen
Greece	1 Drachme = 100 lepta
Portugal	1 Escude = 100 centavos

E. Some Foreign Weights and Measures with their F. P. S. equivalents

France

Metric Ton	= 1000 Kgs.	= 2205 lbs. (Av.)
Quintal	= 100 Kgs.	= 220·5 lbs. (Av.)
Kilogramme	= 1000 grammes	= 2·2046212 lbs. (Av.) = 32·151 oz. (Tr.)
Gramme	= ·03527 oz.	= 15·4323 grains
Kilometer	= 1000 meters	= ·62138½ miles
Meter	= 1·093633 yds.	= 39·37079 ins.
Litre	= 1000 c. cm.	= ·8803 quarts = 1¾ pint (app.)
Cubic meter	= 35·3148 c. ft.	= 1⅓ cubic yard (app.)
Hectare	= 100 Ares	= 2·47114 acres.
Square meter	= 1·1960 sq.yd.	Sq. centimeter = ·155 sq. in.

U. S. A.

Bushel	= ·9694 Imp. bushels.	
Gallon (dry)	= ·1212 Imp. galls.	
Gallon (liq.)	= 231 c. ins. = ⅚ Imp. gall.	
Pint	= ⅛ gallon	= 20 fluid oz.
Cental	= 100 lbs.	

U. S. S. R.

Vershok	= 1¾ in.	Vedro	= 2·704 Imp. gall.
Stopa	= 14 ins.	Cheivart	= 46·2 gallons
Sachsine	= 7 ft.	Funt	= ·9026 lb.
Vert	= 1166⅔ yds.	Pond	= 36·114 lbs.
Dessiatine	= 2·7 acres	Berkowitz	= 361·273 lbs.

Japan

Shaker	= 11·930 in. = 30 cms.	Ri	= 5·9553 sq. miles
Ken	= 1·9884 yard	Sq. cho	= 2·4507 acres
Ki	= 2·4403 miles	Tsabo	= 3·9538 sq. yds.
Cho	= 5·423 chains	(Liq.) Koku	= 39·6804 Imp. galls.

(Dry) Koku	= 4·9629 bushels
(Capacity) Koku	= ·1 ton
Momme	= 3¾ gms.
Kwan	= 1000 momme
	= 8·267 lbs. (Avoir)
Kin	= 1·323 lb (Avoir)

Straits Settlements

Catty	= 1⅓ lb (Av.)	Gantang	= 1 gall.
Picul	= 5333⅓ lbs. (Av.)	Chupak	= 1 quart

Dutch East Indies

Picul	= 133⅓ lb (Av.)	Tjengkal	= 4 yards
Catty	= 1⅓ lb (Av.)	Poal (Java)	= 1507 meters

Italy …	Grammo	= 1 gramme.	*Greece* …	2 Mnas	= 1 Kg.
Greece …	Drachme		*Holland* …	1 Pond	
Spain …	Grams		*Austria* …	2 Centners	

S. America—	C. Libra	…	1·014 lb.
	Quintal	…	101·43 lb.

F

Some Rough Equivalent Measures and Useful Constants

Measure	Rough Equivalent	Measure	Rough Equivalent
8 Km.	5 miles	1 wineglassful	59·2 c. cm.
11 meters	12 yds	Ditto	2 fluid oz.
5 meters	1 pole	1 teacupful	118·4 c. cm.
5 cm.	2 in.	Ditto	4 fluid oz.
25 mm.	1 in.	Ditto	⅕ pint
		1 fluid ounce	1·805 cub. in.
2 Hectares	5 acres	Ditto	29·59 c. cm.
5 Sq. meter	6 sq. yd.	1 gallon (Imp.)	·1604 c. ft.
1 acre	3 bighas (stand.)	1 gallon (Imp.)	8 pints
1 Cottah	720 sq. ft. (,,)	Ditto	4·546 litres
1 Chittack	45 sq. ft. (,,)	1 U. S. gallon	3·785 litres
5 Kg.	11 lb.	1 pint	5·68 c. cm.
1 Metric Ton	1 ton *minus* ⅓ cwt.	Ditto	16 fluid oz.
2 gm.	31 grains	1 cubic ft.	6·228 gall.
200 gm.	7 oz.	1 litre	1¾ pints
1 Quintal	2 cwt.	1 Hectolitre	22 gallons
15 Kg.	14 seer (stand.)	40 c.ft. rough timber	1 load or ton
		50 c.ft. heavy timber	Ditto
Weight of		100 c.ft.	1 Register ton of Shipping
1 c. cm. water	1 gm.		
1 litre water	1 Kg.	1 cubic yard	1 load of earth
	2·205 lb.	128 cubic ft.	1 cord of wood
1 litre gasoline	1·59 lb.	108 cubic ft.	1 stack of wood
Ditto	·72 Kg.		
1 Imp. gallon water	10 lb.	**Specific gravity**	
1 gallon gasoline	7·2 lb.	Water	1
1 cubic ft. water	1000 oz	Gold	19·36
1 teaspoonful	3½ c. cm.	Tin	7·3
1 dessertpoonful	7·5 c. cm.	Steel	7·8
1 tablespoonful	15 c. cm.	Aluminium	2·67
Ditto	½ fluid oz.	Coal (Anthracite)	1·50
		Cork	·24

G

Conversion of Weights and Measures and Aliquotised Parts

	Equivalents	Aliquotised Parts (*app.*)
Indian-Imperial		
Stand. maund	$82\frac{2}{7}$ lb.	$\frac{3}{4}$ cwt $+\underline{2}-\underline{16}$
Factory maund	...	$\frac{1}{30}$ ton
Bombay maund	28 lb.	1 qr.
Madras maund	$24\frac{24}{35}$ lb.	25 lb $-(\underline{1}\% + \underline{4})$
Madras candy	500 lb.	4 cwt $+\underline{8}-\underline{14}$
Bale (Jute & Catton)	400 lb.	3 cwt $+\underline{6}+\underline{7}$
Candy (wheat)	656 lb.	5 cwt $+\underline{7}+\underline{5}$
Candy (cotton)	784 lb.	7 cwt.
Candy (Bombay Stand.)	560 lb.	5 cwt.
Stand. seer (Bengal)	$2\frac{2}{35}$ tb.	2 lb + 1 oz. $-\underline{12}$
Madras seer	·$6\frac{6}{35}$ lb.	$\frac{1}{2}$ lb $+\underline{5}+\underline{7}+\underline{5}$ or ·6lb $+\underline{35}$
Bombay seer	$\frac{7}{10}$ lb	·7 lb.
Cubit	$1\frac{1}{2}$ ft.	1 ft. $+\underline{2}$
Bigha (Stand.)	·33058 acres	$(\underline{3}-$·$002\frac{3}{4})$ acre
Imperial-Indian		
Ton	$27\frac{2}{9}$ md. (stand.)	25 md + $\frac{1}{9}$th of 20 mds.
Cwt.	$54\frac{4}{9}$ sr. ,,	1 md $+\underline{4}+\underline{3}+\underline{3}$
Stone	$6\frac{29}{36}$ sr. ,,	7 sr. $-\underline{36}$
Lb. (Av.)	$\frac{35}{72}$ sr. ,,	$\frac{1}{2}$ sr. $-\frac{2}{9}$ ch.
Lb. (Tr.)	32 tolas ,,	$\frac{1}{2}$ sr. $-\underline{5}$
Oz. (Av.)	$2\frac{3}{7}$ tola ,,	2 tolas $+\underline{3}$
Acre	3 bighas $0\frac{1}{2}$ cott.	3 bigh. + $\frac{1}{2}$ cott.
Imperial-Metric		
Ton	1016·0475 Kg.	1 Met. ton $+\underline{1}\%+\underline{2}+\underline{5}+\underline{16}$
Cwt.	·5083 Quintal	$\frac{1}{2}$ Quintal $+\underline{60}$
Pound (Av.)	453·593 gms.	$\frac{1}{2}$ Kg $-(\underline{11}+\underline{2}\%)$
Pound (Tr.)	373·242 gms.	·3 Kg $+\underline{4}-(\underline{2}\%+\underline{6})$
Ounce (Av.)	28·35 gms.	30 gm $-(\underline{20}+\underline{10})$
Grain	·0648 gms.	6% gm. $+\underline{8}\%$

G (*Contd.*)

	Equivalents	Aliquotised Parts
Imperial-Metric		
Mile	1·60315 Km.	1 Km + $\underline{2}$ + $\underline{5}$ + $\underline{9}$% + $\underline{3}$%
Yard	·9143835 m.	1 m − ($\underline{12}$ + $\underline{9}$%)
Foot	·3048 m.	·3 m + $\underline{16}$%.
Inch	2·54 cm.	2 cm. + $\underline{4}$ + $\underline{12}$
Acre	·40467 Hectares	·4 Hect. + $\underline{1}$% + $\underline{17}$%
Sq. yd.	·8361 Sq. m.	1 Sq. m − ($\underline{6}$ − $\underline{60}$)
Sq. in.	6·4516 Sq. cm.	6 Sq. cm. + $\underline{7}$% + $\underline{8}$%
Cubic yard	·7645 cubic metre	½ c. metre + $\underline{2}$ + $\underline{6}$%
Cubic foot	·02832 cubic metre	$\frac{1}{40}$ cubic m. + $\frac{1}{300}$ cubic m.
Pint	5682 c. cm.	½ litre + $\underline{8}$ + $\underline{11}$
Gallon	4·54346 litres	4 litres + ½ litre + $\underline{9}$%
1 lb water	·4543 litre	½ litre − $\underline{9}$%
Metric-Imperial		
Metric Ton	·984206 Ton	1 Ton − [$\underline{1}$% + $\underline{2}$ + $\underline{6}$]
Kg	2·20462125 lb.	2 lb. + $\underline{10}$ + $\underline{2}$% + $\underline{10}$ + $\underline{2}$
Gramme	15·43235 grs.	15 grs. + $\underline{3}$%
Gramme	·035274 miles	$\underline{3}$% + $\underline{17}$%
Km	·6213824 miles	½ m + [$\underline{4}$ − ($\underline{8}$% − $\underline{30}$)]
Metre	1·093633 yd.	1 yd + $\underline{12}$ + $\underline{8}$
Centi-metre	·39371 in.	⅓.in. + $\underline{5}$ − $\underline{10}$
Milli-metre	$\frac{1}{25}$ in.	4% in.
Sq. Km	·3861 Sq. mile	⅓ Sq. mile + $\underline{17}$%
Hectare	2·47114 acres	2 acres + $\underline{5}$ + $\underline{6}$
Sq. meter	1·1960 Sq. yd.	1 Sq. yd + ($\underline{5}$ − $\underline{2}$%)
Sq. cm.	·155 Sq. in	·1 Sq. in. + $\underline{2}$ + $\underline{10}$
Cubic cm.	·061024 c. in	6% c. in. + $\underline{60}$
Litre	1·7607734 pints	1 Pint + $\underline{2}$ + $\underline{2}$ + $\underline{4}$%
Litre	·2201 gall.	½ gall. + $\underline{10}$
1 Litre water	2·205 lb.	2 lb + $\underline{10}$ + $\underline{40}$
Cub. Meter	35·3148 cub. ft. 1·3080215 cu. yd.	30 c. ft. + $\underline{6}$ + $\underline{6}$% + $\underline{5}$%

4

Weights and Volumes of Common Materials

Material	Wt. in lb. of one cu. ft.	cu in.	No. of cu. in in 1 lb.	Specific Gravity
Aluminium (cast)	160	0·093	10·8	2·57
Brass, cast (average)	505	0·292	3·42	8·11
Brass, Sheet	527	0·305	3·28	8·46
Copper, wire	554	0·321	3·12	8·89
Copper, cast	537	0·311	3·22	8·62
Gun-metal (av.)	544	0·315	3·18	8·74
Iron, cast (av.)	449	0·260	3·85	7·21
Iron, wrought (av.)	480	0·278	3·60	7·71
Steel (average)	490	0·284	3·53	7·87
Tin	462	0·267	3·74	7·42
Lead	710	0·410	–	11·3
Ash	43-53	0·025-31	40-32	0·6–0·9
Ebony	70-83	0·041-48	24-21	1·1–1·3
Oak, English	48-58	0·028-34	36-29	0·6–0·9
Yellow Pine	29-41	0·017-24	59-42	0·4–0·8
Teak	41-50	–	–	–
Brick, common	100-125	0·058-72	17-14	1·6–9·0
Clay, common	119-125	–	–	–
Cement, Portland	80-90	0·046-52	22-19	1·3–1·4
Concrete	120-140	0·069-81	15-12	1·9–2·3
Earth (dry)	80-120	0·046-69	22-15	1·3–1·9
Mortar (hardened)	90-120	0·052-69	19-15	1·4–1·9
Coal (anthracite)	50-56	...	40-45 cu. ft. per ton	1·3–1·8
Coal (bituminous)	47-52	...	43-58 cu. ft. per ton	1·2–1·5
Sand, dry	100	–	–	–
Sand, wet	117	–	–	–
Asphalt	156	–	–	–

TABLE II

Some Common Units of Trade

CALCUTTA	*per*	**BOMBAY**	*per*
Jute Firsts	Bale (400 lbs.)	Oilseeds	Cwt.
Jute Bags	100 bags	Sugar	Cwt.
Hessian	100 yards	Gold	Tola
Rice	Md. ($82\frac{2}{7}$ lbs.)	Silver	100 tolas
Linseed	Ditto		
Shellac	Ditto	**RANGOON**	
Raw Hides	20 lbs.	Rice	100 baskets (75 lbs each)
Goat Skins	100 pieces		
Pig Iron	Ton	**CHICAGO**	
Sugar	Md. ($82\frac{2}{7}$ lbs.)	Cotton	Bale (500 lbs.)
Bone-meal	Ton	Wheat	Bushel
Vegetable Product	Case (60 lbs.)		
		LONDON	
MADRAS		Jute Firsts	Ton
Castor oil	Candy(500 lbs.)	Wheat	Parcel (64 lb.)
Tanned Hides, Skins	lb.	Rice	Cwt.
Groundnut	Candy	Rubber	lb.
(Decorticated)	(500 lbs.)	Oilseed	Ton
		Tea	lb.
COCHIN		Groundnut	Ton
Cocoanut Oil	Candy (654 lbs.)	Sugar	Cwt.
		Hides & Skins	lb.
KARACHI		Wool	Pack(240 lbs.)
		,,	Sack (364 lbs.)
Kandahar Wool	Md. (82 lbs.)	Pig Iron	Ton
Wheat (Delhi)	Cwt.	Hessian	Yard
Do (Punjab White)	Candy (656 lbs.)	Shellac	Cwt.
Do (Lyallpur)	Md.	Soap	Barrel (256 lb.)
Rape-seed	Candy		
		JAPAN	
BOMBAY		Round Steel Bar	100 Kg.
Cotton	Candy (784 lbs.)	Coal	Ton
Do	Bale (400 lbs.)	Electric Copper	100 Kg.
Cotton Yarn	lb.	Crude Rubber	lb.
Linseed	Cwt.	Cement (bag)	50 Kg.
Groundnut	Cwt.	Gasoline (case)	10 galls.

TABLE III

An Extract from the Chambers of Commerce

TONNAGE SCHEDULE

Articles	Bombay &/ or Karachi	Calcutta	Madras	Colombo	Singapore
Bullion	……………………………ad valorem……………………………				
Coal	20 cwts.	20 cwts.	20 cwts		
Rice	18 ,,	20 ,,	20 ,,	20 cwts.	20 cwts.
Jute		50 c.ft.			
Cotton	50 c.ft. 40 ,,		50 c.ft.		
Pig Iron		20 cwts.			
Raw Silk		10 ,,			
Gunnies		{20 ,, or, 50 c.ft.			
Soap (in bags)		15 cwts.			
,, (in cases)	40 c.ft.	50 c.ft.			
Tea		50 c.ft.			
Shellac	16 cwts.	20 cwts.	20 cwts.		
Rubber (in bags)				50 c.ft.	
Castor seeds	14 cwts.				
Rape &c.	16 ,,				
Coir (in bundles)	5 ,,				6 cwts.
Wheat	18 ,,	20 cwts.			
Sugar	19 ,,	20 ,,		20 cwts.	
Hides	40 c.ft.	50 c.ft.		14 ,,	
Groundnuts	14 cwts. 18 ,,		20 cwts.	16 ,,	16 cwts.

Some Japanese Freightage Units

Coal—Metric Ton	Wheat—Metric Ton
Log—1000 c.ft.	Lumber—1000 c.ft.

Cement—Metric Ton

TABLE IV

An Extract from Indian Customs Tariffs

Articles	Tariff Unit	Tariff Value	Standard Rate	U. K. Rate
Import				
Spirits	Imp. Gall.		Rs. 37 8as	
Milk		*ad. val.*	30%	20%
Butter	lb.	Re 1 1a	25%	
Cardamon	cwt.	Rs 55	45%	
Pepper White	,,	Rs 50	45%	
Sugar	,,	*ad. val.*	Rs 9 1a	
Tea	lb.	,,	5 annas	
Tobacco (cigarettes)	1000	,,	Rs 10 10as to Rs 15	
Coal &c.	Ton	,,	10 annas	
Motor Spirit	Imp. gall.	,,	3as 9p	
Peppermint Oil	lb.	Rs 8 6as	25%	
Petroleum Jelly	,,	2as 11p	25%	
Cotton raw	,,		6 pies	
Raw Silk (Bokhara)	,,	Rs 12	25%	
Boots & Shoes		*ad. val.*	25% or 5as per pair	
Cinematograph Film (exposed)	foot	4as 6p	37½%	
Motor Cars		*ad. val.*	37½%	30%
Machinery		,,	10%	
Pig Iron	Ton	Rs 75	20%	10%
Cotton Greys			31¼% or 4⅜as per lb	25%
Silk Japan	lb	Rs 10 12as	50%	
Twill ,,	lb	Rs 15	50%	
Mixed Hosiery (Japan)	lb	Rs 25	50%	
Aeroplanes		*ad. val.*	2½%	
Export				
Raw Jute	Bale-400 lbs.		Rs 40 8as	
Jute Bags	Ton		Rs 20	
Hessians	,,		Rs 32	
Raw Hides			5%	
Goat Skins	Piece		Re 1	
Sheep Skins	,,		Annas 8	
Rice	Maund		2as 3p	

TABLE V

Decimals of a Rupee

Annas	0p	1p	2p	3p	4p	5p	6p	7p	8p	9p	10p	11p
0	——	·0052 1/12	·0104 1/6	·0156 1/4	·0208 1/3	·0260 5/12	·0312 5	·0364 7/12	·0416 2/3	·0468 3/4	·0520 5/6	·0572 11/12
1	·0625	·0677 1/12	·0729 1/6	·0781 1/4	·0833 1/3	·0885 5/12	·0937 5	·0989 7/12	·1041 2/3	·1093 3/4	·1145 5/6	·1197 11/12
2	·1250	·1302 1/12	·1354 1/6	·1406 1/4	·1458 1/3	·1510 5/12	·1562 5	·1614 7/12	·1666 2/3	·1718 3/4	·1770 5/6	·1822 11/12
3	·1875	·1927 1/12	·1979 1/6	·2031 1/4	·2083 1/3	·2135 5/12	·2187 5	·2239 7/12	·2291 2/3	·2343 3/4	·2395 5/6	·2447 11/12
4	·2500	·2552 1/12	·2604 1/6	·2656 1/4	·2708 1/3	·2760 5/12	·2812 5	·2864 7/12	·2916 2/3	·2968 3/4	·3020 5/6	·3072 11/12
5	·3125	·3177 1/12	·3229 1/6	·3281 1/4	·3333 1/3	·3385 5/12	·3437 5	·3489 7/12	·3541 2/3	·3593 3/4	·3645 5/6	·3697 11/12
6	·3750	·3802 1/12	·3854 1/6	·3906 1/4	·3958 1/3	·4010 5/12	·4062 5	·4114 7/12	·4166 2/3	·4218 3/4	·4270 5/6	·4322 11/12
7	·4375	·4427 1/12	·4479 1/6	·4531 1/4	·4583 1/3	·4635 5/12	·4687 5	·4739 7/12	·4791 2/3	·4843 3/4	·4895 5/6	·4947 11/12
8	·5000	·5052 1/12	·5104 1/6	·5156 1/4	·5208 1/3	·5260 5/12	·5312 5	·5364 7/12	·5416 2/3	·5468 3/4	·5520 5/6	·5572 11/12
9	·5625	·5677 1/12	·5729 1/6	·5781 1/4	·5833 1/3	·5885 5/12	·5937 5	·5989 7/12	·6041 2/3	·6093 3/4	·6145 5/6	·6197 11/12
10	·6250	·6302 1/12	·6354 1/6	·6406 1/4	·6458 1/3	·6510 5/12	·6562 5	·6614 7/12	·6666 2/3	·6718 3/4	·6770 5/6	·6822 11/12
11	·6875	·6927 1/12	·6979 1/6	·7031 1/4	·7083 1/3	·7135 5/12	·7187 5	·7239 7/12	·7291 2/3	·7343 3/4	·7395 5/6	·7447 11/12
12	·7500	·7552 1/12	·7604 1/6	·7656 1/4	·7708 1/3	·7760 5/12	·7812 5	·7864 7/12	·7916 2/3	·7968 3/4	·8020 5/6	·8072 11/12
13	·8125	·8177 1/12	·8229 1/6	·8281 1/4	·8333 1/3	·8385 5/12	·8437 5	·8489 7/12	·8541 2/3	·8593 3/4	·8645 5/6	·8698 11/12
14	·8750	·8802 1/12	·8854 1/6	·8906 1/4	·8958 1/3	·9010 5/12	·9062 5	·9114 7/12	·9166 2/3	·9218 3/4	·9270 5/6	·9322 11/12
15	·9375	·9427 1/12	·9479 1/6	·9531 1/4	·9583 1/3	·9635 5/12	·9687 5	·9739 7/12	·9791 2/3	·9843 3/4	·9895 5/6	·9947 11/12

TABLE VI

Rupee-Stérling Exchange

Equivalents of Re 1.

@ 1s 6d			*d*	@ 1s 5d		
£·076	000	000	—	£·070	833	3333̇
	130	2083̇	$\frac{1}{32}$		963	5416̇
	260	4166̇	$\frac{2}{32}$	·071	093	7500
	390	6250	$\frac{3}{32}$		223	9583̇
	520	8333̇	$\frac{4}{32}$		354	1666̇
	651	0416̇	$\frac{5}{32}$		484	3750
	781	2500	$\frac{6}{32}$		614	5833̇
	911	4583̇	$\frac{7}{32}$		744	7916̇
·076	041	6666̇	$\frac{8}{32}$		875	0000
	171	8750	$\frac{9}{32}$	·072	005	2083̇
	302	0833̇	$\frac{10}{32}$		135	4166̇
	432	2916̇	$\frac{11}{32}$		265	6250
	562	5000	$\frac{12}{32}$		395	8333̇
	692	7083̇	$\frac{13}{32}$		526	0416̇
	822	9166̇	$\frac{14}{32}$		656	2500
	953	1250	$\frac{15}{32}$		786	4583̇
·077	083	3333̇	$\frac{16}{32}$		916	6666̇
	213	5416̇	$\frac{17}{32}$	·073	046	8750
	343	7500	$\frac{18}{32}$		177	0833̇
	473	9583̇	$\frac{19}{32}$		307	2916̇
	604	1666̇	$\frac{20}{32}$		437	5000
	734	3750	$\frac{21}{32}$		567	7083̇
	864	5833̇	$\frac{22}{32}$		697	9166̇
	994	7916̇	$\frac{23}{32}$		828	1250
·078	125	0000	$\frac{24}{32}$		958	3333̇
	255	2083̇	$\frac{25}{32}$	·074	088	5416̇
	385	4166̇	$\frac{26}{32}$		218	7500
	515	6250	$\frac{27}{32}$		348	9583̇
	645	8333̇	$\frac{28}{32}$		479	1666̇
	776	0416̇	$\frac{29}{32}$		609	3750
	906	2500	$\frac{30}{32}$		739	5833̇
079	036	4583̇	$\frac{31}{32}$		869	7916̇

TABLE VII

Sterling-Rupee Exchange

1s 5d

32-nds. of 1d.	£1000 = Rs 13000 +			£100 Rs 1300 +			£50 = Rs 650 +			£10 = Rs 130 +			£1 = Rs 13 +			10s Rs 6 +			5s = Rs 3 +		1s = 10 as +		1d = 10p +	32-nds. of 1d.
	Rs	as	p	Rs	as	p	Rs	as	p	Rs	as	p	Rs	as	p	Rs	as	p	as	p	as	p	p	
—	1117	10	4·2	111	12	2·8	55	14	1·4	11	2	9·9	1	1	10·6	1	0	11·3	8	5·6	1	3·5	1.3	—
1	1091	11	10·7	109	2	9·5	54	9	4·8	10	14	8·1	1	1	5·6	1	0	8·8	8	4·4	1	3·3	1·3	1
2	1065	14	11·3	106	9	5·9	53	4	9·0	10	10	6·6	1	1	0·7	1	0	6·3	8	3·2	1	3·0	1·3	2
3	1040	3	6·1	104	0	4·2	52	0	2·1	10	6	5·2	1	0	7·7	1	0	3·9	8	1·9	1	2·8	1·2	3
4	1014	9	6·9	101	7	4·3	50	11	8·1	10	2	4·0	1	0	2·8	1	0	1·4	8	0·7	1	2·5	1·2	4
5	989	1	1·6	98	14	6·2	49	7	3·1	9	14	3·0		15	9·9		15	11·0	7	11·5	1	2·3	1·2	5
6	963	10	2·2	96	5	9·8	48	2	10·9	9	10	2·2		15	5·0		15	8·5	7	10·3	1	2·1	1·2	6
7	938	4	8·5	93	13	3·2	46	14	7·6	9	6	1·5		15	0·2		15	6·1	7	9·0	1	1·8	1·2	7
8	913	0	8·3	91	4	10·4	45	10	5·2	9	2	1·0		14	7·3		15	3·7	7	7·8	1	1·6	1·1	8
9	887	14	1·8	88	12	7·4	44	6	3·7	8	14	0·7		14	2·5		15	1·2	7	6·6	1	1·3	1·1	9
10	862	13	0·6	86	4	6·1	43	2	3·0	8	10	0·6		13	9·7		14	10·8	7	5·4	1	1·1	1·1	10
11	837	13	4·9	83	12	6·5	41	14	3·2	8	6	0·6		13	4·9		14	8·4	7	4·2	1	0·8	1·1	11
12	812	15	2·3	81	4	8·6	40	10	4·3	8	2	0·9		13	0·1		14	6·0	7	3·0	1	0·6	1·1	12

1s 5d

13	788	2	5·0	78	13	0·5	39	6	6·2	7	14	1·3	12	7·3	14	3·7	7	1·8	1	0·4	1·0	13
14	763	7	0·6	76	5	6·1	38	2	9·0	7	10	1·8	12	2·6	14	1·3	7	0·6	1	0·1	1·0	14
15	738	13	1·3	73	14	1·3	36	15	0·7	7	6	2·5	11	9·8	13	10·9	6	11·5		11·9	1·0	15
16	714	4	6·9	71	6	10·3	35	11	5·1	7	2	3·4	11	5·1	13	8·6	6	10·3		11·7	1·0	16
17	689	13	5·2	68	15	8·9	34	7	10·5	6	14	4·5	11	0·4	13	6·2	6	9·1		11·4	1·0	17
18	665	7	8·2	66	8	9·2	33	4	4·6	6	10	5·7	10	7·8	13	3·9	6	7·9		11·2	·9	18
19	641	3	8·9	64	1	11·2	32	0	11·6	6	6	7·1	10	3·1	13	1·6	6	6·8		11·0	·9	19
20	617	0	4·1	61	11	2·8	30	13	7·4	6	2	8·7	9	10·5	12	11·2	6	5·6		10·7	·9	20
21	592	14	8·7	59	4	6·1	29	10	4·0	5	14	10·4	9	5·8	12	8·9	6	4·5		10·5	·9	21
22	568	14	5·7	56	14	3·0	28	7	1·5	5	11	0·3	9	1·2	12	6·6	6	3·3		10·3	·9	22
23	544	15	6·9	54	7	11·5	27	3	11·7	5	7	2·3	8	8·6	12	4·3	6	2·2		10·0	·8	23
24	521	2	0·3	52	1	9·6	26	0	10·8	5	3	4·6	8	4·1	12	2·0	6	1·0		9·8	·8	24
25	497	5	9·8	49	11	9·4	24	13	10·7	4	15	6·9	7	11·5	11	11·7	5	11·9		9·6	·8	25
26	473	10	11·4	47	5	10·7	23	10	11·4	4	11	9·5	7	6·9	11	9·5	5	10·7		9·3	·8	26
27	450	1	4·8	45	0	1·7	22	8	0·8	4	8	0·2	7	2·4	11	7·2	5	9·6		9·1	·8	27
28	426	9	2·1	42	10	6·2	21	5	3·1	4	4	3·0	6	9·9	11	5·0	5	8·5		8·9	·7	28
29	403	2	3·1	40	5	0·3	20	2	6·2	4	0	6·0	6	5·4	11	2·7	5	7·4		8·7	·7	29
30	379	12	7·9	37	15	8·0	18	15	10·0	3	12	9·2	6	0·9	11	0·5	5	6·2		8·4	·7	30
31	356	8	4·2	35	10	5·2	17	13	2·6	3	9	0·5	5	8·5	10	10·2	5	5·1		8·2	·7	31

1 farthing = 2¼ pies.

TABLES VII (*Contd.*)

Sterling-Rupee Exchange

1s 6d

1s 6d

32-nds of 1d	£1000= Rs 12000+			£100= Rs 1200+			£50= Rs 600+			£10= Rs 120+			£1= Rs 12+			10s= Rs 6+		5s= Rs 3+		1s= 10as+	1d= 10p+	32-nds of 1d.
	Rs	as	p	Rs	as	p	Rs	as	p	Rs	as	p	Rs	as	p	as	p	as	p	p	p	
—	1333	5	4·0	133	5	4·0	66	10	8·0	13	5	4·0	1	5	4·0	10	6·0	5	4·0	8·0	·7	—
1	1310	3	7·3	131	0	4·3	65	8	2·2	13	1	4·6	1	4	11·6	10	5·8	5	2·9	7·8	·6	1
2	1287	3	1·9	128	11	6·2	64	5	9·1	12	13	11·4	1	4	7·1	10	3·6	5	1·8	7·6	·6	2
3	1264	3	11·8	126	6	9·6	63	3	4·8	12	10	3·4	1	4	2·7	10	1·4	5	0·7	7·3	·6	3
4	1241	6	0·8	124	2	2·5	62	1	1·2	12	6	7·4	1	3	10·3	9	11·2	4	11·6	7·1	·6	4
5	1218	9	5·0	121	13	8·9	60	14	10·5	12	2	11·7	1	3	6·0	9	9·0	4	10·5	6·9	·6	5
6	1195	14	0·2	119	9	4·8	59	12	8·4	11	15	4·1	1	3	1·6	9	6·8	4	9·4	6·7	·6	6
7	1173	3	10·4	117	5	2·2	58	10	7·1	11	11	8·6	1	2	9·3	9	4·6	4	8·3	6·5	·5	7
8	1150	10	11·5	115	1	1·2	57	8	6·6	11	8	1·3	1	2	4·9	9	2·5	4	7·2	6·2	·5	8
9	1128	3	3·4	112	13	1·5	56	6	6·8	11	4	6·2	1	2	0·6	9	0·3	4	6·2	6·0	·5	9
10	1105	12	10·0	110	9	3·4	55	4	7·7	11	0	11·1	1	1	8·3	8	10·2	4	5·1	5·8	·5	10
11	1083	7	7·3	108	5	6·7	54	2	9·4	10	13	4·3	1	1	4·0	8	8·0	4	4·0	5 6	·5	11
12	1061	3	7·1	106	1	11·5	53	0	11·8	10	9	9·6	1	0	11·8	8	5·9	4	2·9	5·4	·5	12

13	1089 0 9·5	103 14 5·7	51 15 2·9	10 6 3·0	1 0 7·5	8 3·7	4 1·9	5·2	·4	13
14	1016 15 2·2	101 11 1·4	50 13 6·7	10 2 6·5	1 0 3·3	8 1·6	4 0·8	5·0	·4	14
15	994 14 9·4	99 7 10·5	49 11 11·3	9 15 2·3	15 11·0	7 11·5	3 11·8	4·8	·4	15
16	972 15 6·8	97 4 9·1	48 10 4·5	9 11 6·1	15 6·8	7 0·4	3 10·7	4·5	·4	16
17	951 1 6·5	95 1 9·0	47 8 10·5	9 8 2·1	15 2·6	7 7·3	3 9·7	4·3	·4	17
18	929 4 8·2	92 14 10·4	46 7 5·2	9 4 8·2	14 10·4	7 5·2	3 8·6	4·1	·3	18
19	907 9 0·1	90 12 1·2	45 6 0·6	9 1 2·5	14 6·3	7 3·1	3 7·6	3·9	·3	19
20	885 14 6·0	88 9 5·4	44 4 8·7	8 13 8·9	14 2·1	7 1·0	3 6·5	3·7	·3	20
21	864 5 1·7	86 6 11·0	43 3 5·5	8 10 3·5	13 10·0	6 11·0	3 5·5	3·5	·3	21
22	842 12 11·4	84 4 5·9	42 2 3·0	8 6 10·2	13 5·8	6 8·9	3 4·5	3·3	·3	22
23	821 5 10·8	82 2 2·3	41 1 1·1	8 3 5·0	13 1·7	6 6·9	3 3·4	3·1	·3	23
24	800 0 0	80 0 0	40 0 0	8 0 0	12 9·6	6 4·8	3 2·4	2·9	·2	24
25	778 11 2·8	77 13 11·1	38 14 11·5	7 12 7·1	12 5·5	6 2·8	3 1·4	2·7	·2	25
26	757 7 7·2	75 11 11·5	37 13 11·8	7 9 2·4	12 1·4	6 0·7	3 0·4	2·5	·2	26
27	736 5 1·1	73 10 1·3	36 13 0·7	7 5 9·7	11 9·4	5 10·7	2 11·3	2·3	·2	27
28	715 3 8·5	71 8 4·5	35 12 2·2	7 2 5·2	11 5·3	5 8·7	2 10·3	2·1	·2	28
29	694 3 5·3	69 6 8·9	34 11 4·5	6 15 0·9	11 1·3	5 6·6	2 9·3	1·9	·2	29
30	673 4 3·3	67 5 2·7	33 10 7·4	6 11 8·7	10 9·3	5 4·6	2 8·3	1·7	·1	30
31	652 6 2·6	65 3 9·9	32 9 10·9	6 8 4·6	10 5·3	5 2·6	2 7·3	1·5	·1	31

1 farthing = 2¼ pies

TABLES VII (*Contd.*)

1s 7d Sterling-Rupee Exchange 1s 7d

32-nds of 1d	£1000= Rs 12000+			£100= Rs 1200+			£50= Rs 600+			£10= Rs 120+			£1= Rs 12+		10s= Rs 6+		5s= Rs 3+		1s= 9as+		1d=9p +	32-nds of 1d
	Rs	as	p	Rs	as	p	Rs	as	p	Rs	as	p	as	p	as	p	as	p	as	p	p	
—	631	9	3·2	63	2	6·3	31	9	3·2	6	5	0·6	10	1·3	5	0·6	2	6·3	1	1·3	1·1	—
1	610	13	4·8	61	1	4·1	30	8	8·0	6	1	8·8	9	9·3	4	10·6	2	5·3	1	1·1	1·1	1
2	590	2	7·5	59	0	3·1	29	8	1·6	5	14	5·1	9	5·3	4	8·7	2	4·3	1	0·9	1·1	2
3	569	8	11·2	56	15	3·5	28	7	7·8	5	11	1·6	9	1·4	4	6·7	2	3·3	1	0·7	1·1	3
4	549	0	3·8	54	14	5·2	27	7	2·6	5	7	10·1	8	9·4	4	4·7	2	2·4	1	0·5	1·0	4
5	528	8	9·2	52	13	8·1	26	6	10·1	5	4	6·8	8	5·5	4	2·7	2	1·4	1	0·3	1·0	5
6	508	2	3·5	50	13	0·4	25	6	6·2	5	1	3·6	8	1·6	4	0·8	2	0·4	1	0·1	1·0	6
7	487	12	10·5	48	12	5·9	24	6	2·9	4	14	0·6	7	9·7	3	10·8	1	11·4		11·9	1·0	7
8	467	8	6·2	46	12	0·6	23	6	0·3	4	10	9·7	7	5·8	3	8·9	1	10·4		11·7	1·0	8
9	447	5	2·5	44	11	8·7	22	5	10·3	4	7	6·9	7	1·9	3	6·9	1	9·5		11·5	1·0	9
10	427	2	11·4	42	11	5·9	21	5	9·0	4	4	4·2	6	10·0	3	5·0	1	8·5		11·3	0·9	10
11	407	1	8·8	40	11	4·5	20	5	8·2	4	1	1·6	6	6·2	3	3·1	1	7·5		11·1	0·9	11
12	387	1	6·6	38	11	4·3	19	5	8·1	3	13	11·2	6	2·3	3	1·2	1	6·6		10·9	0·9	12

13	367	2	4·8	36	11	5·8	18	5	8·6	3	10	8·9	5	10·5	2	11·2	1	5·6	10·7	0·9	13
14	347	4	3·2	34	11	7·5	17	5	9·8	3	7	6·8	5	6·7	2	9·3	1	4·7	10·5	0·9	14
15	327	7	2·0	32	11	11·0	16	5	11·5	3	4	4·7	5	2·9	2	7·4	1	3·7	10·3	0·9	15
16	307	11	0·9	30	12	3·7	15	9	1·8	3	1	2·8	4	11·1	2	5·5	1	2·8	10·2	0·8	16
17	288	0	0	28	12	9·6	14	6	4·8	2	14	1·0	4	7·3	2	3·6	1	1·8	10·0	0·8	17
18	268	5	11·2	26	13	4·7	13	6	8·4	2	10	11·3	4	3·5	2	1·8	1	0·9	9·8	0·8	18
19	249	12	10·3	24	14	1·0	12	7	0·5	2	7	9·7	3	11·3	1	11·9		11·9	9·6	0·8	19
20	229	4	9·5	22	14	10·5	11	7	5·3	2	4	5·3	3	8·0	1	10·0		11·0	9·4	0·8	20
21	209	13	8·5	20	15	9·3	10	7	10·6	2	1	6·9	3	4·3	1	8·1		10·1	9·2	0·8	21
22	190	7	7·4	19	0	9·1	9	8	4·6	1	14	5·7	3	0·6	1	6·3		9·1	9·0	0·8	22
23	171	2	6·1	17	1	10·2	8	8	11·1	1	11	4·6	2	8·9	1	4·4		8·2	8·8	0·7	23
24	151	14	4·6	15	3	0·5	7	9	6·2	1	8	3·6	2	5·2	1	2·6		7·3	8·7	0·7	24
25	132	11	2·7	13	4	8·9	6	10	1·9	1	5	2·8	2	1·5	1	0·7		6·4	8·5	0·7	25
26	113	9	0·4	11	5	8·4	5	10	10·2	1	2	2·0	1	9·8		10·9		5·5	8·3	0·7	26
27	94	7	9·7	9	7	2·2	4	11	7·1		15	1·4	1	6·1		9·1		4·5	8·1	0·7	27
28	75	7	6·6	7	8	9·1	3	12	4·5		12	0·9	1	2·5		7·2		3·6	7·9	0·7	28
29	56	8	2·9	5	10	5·1	2	13	2·5		9	0·5	0	10·9		5·4		2·7	7·7	0·6	29
30	37	9	10·6	3	12	2·3	1	14	1·1		6	0·2	0	7·2		3·6		1·8	7·6	0·6	30
31	18	12	5·6	1	14	0·6		15	0·3		3	0·1	0	3·6		1·8		0·9	7·4	0·6	31

1 farthing = 2½ pies

TABLE VIII

Brokerage, Commission or Discount on 1

Table of Nine Multiples

Rate %	1	2	3	4	5	6	7	8	9	¼	Rate %
¼	·0025	·0050	·0075	·01	·0125	·0150	·0175	·02	·0225	·0006	¼
½	·0050	·0100	·0150	·02	·0250	·0300	·0350	·04	·0450	·0013	½
¾	·0075	·0150	·0225	·03	·0375	·0450	·0525	·06	·0675	·0019	¾
1	·0100	·0200	·0300	·04	·0500	·0600	·0700	·08	·0900	·0025	1
1¼	·0125	·0250	·0375	·05	·0625	·0750	·0875	·10	·1125	·0031	1¼
1½	·0150	·0300	·0450	·06	·0750	·0900	·1050	·12	·1350	·0038	1½
1¾	·0175	·0350	·0525	·07	·0875	·1050	·1225	·14	·1575	·0044	1¾
2	·0200	·0400	·0600	·08	·1000	·1200	·1400	·16	·1800	·0050	2
2¼	·0225	·0450	·0675	·09	·1125	·1850	·1575	·18	·2025	·0056	2¼
2½	·0250	·0500	·0750	·10	·1250	·1500	·1750	·20	·2250	·0063	2½
2¾	·0275	·0550	·0825	·11	·1375	·1650	·1925	·22	·2475	·0069	2¾
3	·0300	·0600	·0900	·12	·1500	·1800	·2100	·24	·2700	·0075	3
3¼	·0325	·0650	·0975	·13	·1625	·1950	·2275	·26	·2925	·0081	3¼
3½	·0350	·0700	·1050	·14	·1750	·2100	·2450	·28	·3150	·0088	3½

3¾	·0375	·0750	·1125	·15	·1875	·2250	·2625	·30	·3375	·0094	3¾
4	·0400	·0800	·1200	·16	·2000	·2400	·2800	·32	·3600	·0100	4
4¼	·0425	·0850	·1275	·17	·2125	·2550	·2975	·34	·3825	·0106	4¼
4½	·0450	·0900	·1350	·18	·2250	·2700	·3150	·36	·4050	·0113	4½
4¾	·0475	·0950	·1425	·19	·2375	·2850	·3325	·38	·4275	·0119	4¾
5	·0500	·1000	·1500	·20	·2500	·3000	·3500	·40	·4500	·0125	5
5¼	·0525	·1050	·1575	·21	·2625	·3150	·3675	·42	·4725	·0131	5¼
5½	·0550	·1100	·1650	·22	·2750	·3300	·3850	·44	·4950	·0138	5½
5¾	·0575	·1150	·1725	·23	·2875	·3450	·4025	·46	·5175	·0144	5¾
6	·0600	·1200	·1800	·24	·3000	·3600	·4200	·48	·5400	·0150	6
6½	·0650	·1300	·1950	·26	·3250	·3900	·4550	·52	·5850	·0163	6½
7	·0700	·1400	·2100	·28	·3500	·4200	·4900	·56	·6300	·0175	7
7½	·0750	·1500	·2250	·30	·3750	·4500	·5250	·60	·6750	·0188	7½
8	·0800	·1600	·2400	·32	·4000	·4800	·5600	·64	·7200	·0200	8
9	·0900	·1800	·2700	·36	·4500	·5400	·6300	·72	·8100	·0225	9
15	·1500	·3000	·4500	·60	·7500	·9000	1·0500	1·20	1·3500	·0375	15
16	·1600	·3200	·4800	·64	·8000	·9600	1·1200	1·28	1·4400	·0400	16

TABLE IX

Simple Interest on Multiples of 10⁶ for 1 day.

Rate % p. a.	1	2	3	4	5	6	7	8	9	Rate % p. a.
¼	6·849315	13·698630	20·547945	27·397260	34·246575	41·095890	47·945205	54·794521	61·643836	¼
½	13·698630	27·397260	41·095890	54·794521	68·493151	82·191781	95·890411	109·589041	123·287671	½
¾	20·547945	41·095890	61·643835	82·191781	102·739726	123·287672	143·833117	164·383561	184·931506	¾
1	27·397260	54·794521	82·191781	109·589041	136·986301	164·383562	191·780822	219·178082	246·575343	1
1¼	34·246575	68·493151	102·739726	136·986301	171·232876	205·479453	239·723528	273·972602	308·219178	1¼
1½	41·095890	82·191781	123·287671	164·383562	205·479452	246·575342	287·671232	328·767122	369·863013	1½
1¾	47·945205	95·890411	140·835616	191·780822	239·726027	307·671433	335·613938	383·561642	431·506848	1¾
2	54·794521	109·589041	164·383562	219·178082	273·972602	328·767124	383·561644	438·356165	493·150685	2
2¼	61·643836	123·287671	184·931461	246·575342	308·219177	369·863015	431·504350	493·150685	554·794520	2¼
2½	68·493151	136·986301	205·479452	273·972603	342·465754	410·958904	479·452055	547·945206	616·438357	2½
2¾	75·342466	150·684931	226·027397	301·369863	376·712329	452·054795	527·394761	602·739726	678·082192	2¾
3	82·191781	164·383562	246·575343	328·767124	410·958904	493·150685	575·342466	657·534247	739·726027	3

3¼	89·041096	176·282192	267·123288	356·164383	445·205479	534·246575	623·285172	712·328767	801·369862	3¼
3½	95·890411	191·780822	287·671233	383·561644	479·452055	575·342466	671·232877	767·123288	863·013699	3½
3¾	102·739726	205·479452	308·219178	410·958504	513·698630	616·438357	719·175583	821·917808	924·657534	3¾
4	109·589041	219·178082	328·767124	438·356165	547·945206	657·534247	767·123288	876·712330	986·301370	4
4¼	116·438356	232·876712	349·315069	465·753425	582·191781	698·630138	815·065994	931·506850	1047·945205	4¼
4½	123·287671	246·575343	369·863014	493·150685	616·438356	739·726028	863·013699	986·301370	1109·589042	4½
4¾	130·136986	260·273973	390·410959	520·547945	650·684931	730·821919	910·956405	1041·095890	1171·232877	4¾
5	136·986301	273·972602	410·958904	547·945206	684·931507	821·917808	958·904110	1095·890411	1232·876712	5
5¼	143·835616	287·671233	431·506849	575·342465	719·178082	863·013699	1006·846816	1150·684931	1294·520547	5¼
5½	150·684931	301·369863	452·054794	602·739726	753·424657	904·109588	1054·794520	1205·479452	1356·164383	5½
5¾	157·534246	315·068493	472·602739	630·136986	787·671232	945·205479	1102·737226	1260·273972	1417·808218	5¾
6	164·383562	328·767124	493·150685	657·534247	821·917808	986·301370	1150·684931	1315·068493	1479·452055	6
6½	178·082192	356·164383	534·246575	712·328767	891·410959	1068·493151	1246·575342	1424·657534	1602·739726	6½
7	191·780822	383·561644	575·342466	767·123288	958·904110	1150·684931	1342·465753	1534·246575	1726·027397	7
7½	205·479452	410·958904	616·486356	821·917809	1027·397261	1233·876712	1438·356164	1643·835616	1849·315068	7½
8	219·178082	438·356165	657·534247	876·712330	1095·890411	1315·068493	1534·246575	1753·424657	1972·602740	8
9	246·575343	493·150685	739·726027	986·301370	1232·876712	1479·452055	1726·027367	1972·602740	2219·178082	9

TABLE X

Table of Days

Number of Days from December 31

Jan.	Feb.	March	April	May	June	**Date**	July	Aug.	Sep.	Oct.	Nov.	Dec.
1	32	60	91	121	152	**1**	182	213	244	274	305	335
2	33	61	92	122	153	**2**	183	214	245	275	306	336
3	34	62	93	123	154	**3**	184	215	246	276	307	337
4	35	63	94	124	155	**4**	185	216	247	277	308	338
5	36	64	95	125	156	**5**	186	217	248	278	309	339
6	37	65	96	126	157	**6**	187	218	249	279	310	340
7	38	66	97	127	158	**7**	188	219	250	280	311	341
8	39	67	98	128	159	**8**	189	220	251	281	312	342
9	40	68	99	129	160	**9**	190	221	252	282	313	343
10	41	69	100	130	161	**10**	191	222	253	283	314	344
11	42	70	101	131	162	**11**	192	223	254	284	315	345
12	43	71	102	132	163	**12**	193	224	255	285	316	346
13	44	72	103	133	164	**13**	194	225	256	286	317	347
14	45	73	104	134	165	**14**	195	226	257	287	318	348
15	46	74	105	135	166	**15**	196	227	258	288	319	349
16	47	75	106	136	167	**16**	197	228	259	289	320	350
17	48	76	107	137	168	**17**	198	229	260	290	321	351
18	49	77	108	138	169	**18**	199	230	261	291	322	352
19	50	78	109	139	170	**19**	200	231	262	292	323	353
20	51	79	110	140	171	**20**	201	232	263	293	324	354
21	52	80	111	141	172	**21**	202	233	264	294	325	355
22	53	81	112	142	173	**22**	203	234	265	295	326	356
23	54	82	113	143	174	**23**	204	235	266	296	327	357
24	55	83	114	144	175	**24**	205	236	267	297	328	358
25	56	84	115	145	176	**25**	206	237	268	298	329	359
26	57	85	116	146	177	**26**	207	238	269	299	330	360
27	58	86	117	147	178	**27**	208	239	270	300	331	361
28	59	87	118	148	179	**28**	209	240	271	301	332	362
29	—	88	119	149	180	**29**	210	241	272	302	333	363
30	—	89	120	150	181	**30**	211	242	273	303	334	364
31	—	90	—	151	—	**31**	212	243	—	304	—	365

N. B. Add 1 to the number after February 28 in a leap year.

TABLE XI—Depreciation Table

Value of asset worth 1000 providing for depreciation on reduced values at different rates

Years.	2%	2½%	5%	6¼%	7½%	10%
1	980	975	950	937	925	900
2	960	951	902	878	855	810
3	941	927	857	823	791	729
4	922	908	814	773	732	656
5	903	880	773	725	677	590
6	885	858	735	680	626	531
7	868	837	698	637	579	478
8	850	816	663	597	535	430
9	833	796	630	550	495	387
10	817	776	598	516	458	348
11	800	757	568	484	424	313
12	784	738	540	454	392	282
13	769	720	513	426	362	254
14	753	702	487	399	335	228
15	738	684	463	374	310	205
16	723	667	440	351	287	185
17	709	650	418	329	265	166
18	695	634	397	308	245	150
19	681	618	377	289	227	135
20	667	603	358	271	210	121
21	654	588	340	254	194	109
22	641	573	323	238	179	98
23	628	559	307	223	166	88
24	615	545	291	209	153	79
25	603	531	277	196	142	71

TABLE XII

Number of years' purchase represented by the price of a 3½% perpetuity $\left[=\frac{price\%}{3\frac{1}{2}}\right]$

Price Rs.	Years' Purchase	*Price* As.	Year's Purchase
1	·2857143	1	·0178571
2	·5714286	2	·0357143
3	·8571429	3	·0535614
4	1·1428571	4	·0714286
5	1·4285714	5	·0892857
6	1·7142857	6	·1071229
7	2·0000000	7	·1250000
8	2·2857143	8	·1428571
9	2·5714286	9	·1607543

Note. The corresponding yield is obtained by dividing 100 by the number of years obtained.

TABLE XIII

Compound Interest

Amount of 1 at the end of n periods : $(1+i)^n$

n	1%	1½%	2%	2½%	3%	3½%	4%	4½%	5%
1	1·01 000	1·01 500	1·02 000	1·02 500	1·03 000	1·03 500	1·04 000	1·04 500	1·05 000
2	1·02 010	1·03 023	1·04 040	1·05 063	1·06 090	1·07 123	1·08 160	1·09 203	1·10 250
3	1·03 030	1·04 568	1·06 121	1·07 689	1·09 273	1·10 872	1·12 486	1·14 117	1·15 763
4	1·04 060	1·06 136	1·08 243	1·10 381	1·12 551	1·14 752	1·16 986	1·19 252	1·21 551
5	1·05 101	1·07 728	1·10 408	1·13 141	1·15 927	1·18 769	1·21 665	1·24 618	1·27 628
6	1·06 152	1·09 344	1·12 616	1·15 969	1·19 405	1·22 926	1·26 532	1·30 226	1·34 010
7	1·07 214	1·10 984	1·14 869	1·18 869	1·22 987	1·27 228	1·31 593	1·36 086	1·40 710
8	1·08 286	1·12 649	1·17 166	1·21 840	1·26 677	1·31 681	1·36 857	1·42 210	1·47 746
9	1·09 369	1·14 339	1·19 509	1·24 886	1·30 477	1·36 290	1·42 331	1·48 610	1·55 133
10	1·10 462	1·16 054	1·21 899	1·28 008	1·34 392	1·41 060	1·48 024	1·55 297	1·62 889
11	1·11 567	1·17 795	1·24 337	1·31 209	1·38 423	1·45 997	1·53 945	1·62 285	1·71 034
12	1·12 683	1·19 562	1·26 824	1·34 489	1·42 576	1·51 107	1·60 103	1·69 588	1·79 586

13	1·13 809	1·21 355	1·29 361	1·37 851	1·46 853	1·56 396	1·66 507	1·77 220	1·88 565
14	1·14 947	1·23 176	1·31 948	1·41 297	1·51 259	1·61 869	1·73 168	1·85 194	1·97 993
15	1·16 097	1·25 023	1·34 587	1·44 830	1·55 797	1·67 535	1·80 094	1·93 528	2·07 893
16	1·17 258	1·26 899	1·37 279	1·48 451	1·60 471	1·73 399	1·87 298	2·02 237	2·18 287
17	1·18 430	1·28 802	1·40 024	1·52 162	1·65 285	1·79 468	1·94 790	2·11 338	2·29 202
18	1·19 615	1·30 734	1·42 825	1·55 966	1·70 243	1·85 749	2·02 582	2·20 848	2·40 662
19	1·20 811	1·32 695	1·45 681	1·59 865	1·75 351	1·92 250	2·10 685	2·30 786	2·52 695
20	1·22 019	1·34 686	1·48 595	1·63 862	1·80 611	1·98 979	2·19 112	2·41 171	2·65 330
21	1·23 239	1·36 706	1·51 567	1·67 958	1·86 029	2·05 943	2·27 877	2·52 024	2·78 596
22	1·24 472	1·38 756	1·54 598	1·72 157	1·91 610	2·13 151	2·36 992	2·63 365	2·92 526
23	1·25 716	1·40 838	1·57 690	1·76 461	1·97 359	2·20 611	2·46 472	2·75 217	3·07 152
24	1·26 973	1·42 950	1·60 844	1·80 873	2·03 279	2·28 333	3·56 330	2·87 601	3·22 510
25	1·28 243	1·45 095	1·64 061	1·85 394	2·09 378	2·36 324	2·66 584	3·00 543	3·38 635
26	1·29 526	1·47 271	1·67 342	1·90 029	2·15 659	2·44 596	2·77 247	3·14 068	3·55 567
27	1·30 821	1·49 480	1·70 689	1·94 780	2·22 129	2·53 157	2·88 337	3·28 201	3·73 346
28	1·32 129	1·51 722	1·74 102	1·99 650	2·28 793	2·62 017	2·99 870	3·42 970	3·92 018
29	1·33 450	1·53 998	1·77 584	2·04 641	2·35 657	2·71 188	3·11 865	3·58 404	4·11 614
30	1·34 785	1·56 308	1·81 136	2·09 757	2·42 726	2·80 679	3·24 340	3·74 532	4·32 194

TABLE XIV

Present Value of 1 due at end of *n* periods : v^n

n	1%	1½%	2%	2¼%	2½%	2¾%	3%	3½%	4%	4½%	5%	n
1	·990099	·985222	·980392	·977995	·975610	·973236	·970874	·966184	·961538	·956938	·952381	1
2	·980296	·970662	·961169	·956474	·951814	·947188	·942596	·933511	·924556	·915730	·907029	2
3	·970590	·956317	·942322	·935427	·928599	·921838	·915142	·901943	·888996	·876297	863838	3
4	·960980	·942184	·923845	·914843	·905951	·897166	·888487	·871442	·854804	·838561	·822702	4
5	·951466	·928260	·905731	·894712	·883854	·873154	·862609	·841973	·821927	·802451	·783526	5
6	·942045	·914542	·887971	·875024	·862297	·849785	·837484	·813501	·790315	·767896	·746215	6
7	·932718	·901027	·870560	·855769	·841265	·827041	·813092	·785991	·759918	·734828	·710681	7
8	·923483	·887711	·853490	·836938	·820747	·804906	·789409	·759412	·730690	·703185	·676839	8
9	·914340	·874592	·836755	·818522	·800728	·783364	·766417	·733731	·702587	·672904	·644609	9
10	·905287	·861667	·820348	·800510	·781198	·762398	·744094	·708919	·675564	·643928	·613913	10
11	·896324	·848933	·804263	·782895	·762145	·741993	·722421	·684946	·649581	·616199	·584679	11
12	·887449	·836387	·788493	·765667	·743556	·722134	·701380	·661783	·624597	·589664	·556837	12

13	·878663	·824027	·773083	·748819	·725420	·702807	·680951	·639404	·600574	·564272	·530321	13
14	·869963	·811849	·757875	·732341	·707727	·683997	·661118	·617782	·577475	·539973	·505068	14
15	·861349	·799852	·743015	·716226	·690466	·665691	·641862	·596891	·555264	·516720	·481017	15
16	·852821	·788081	·728446	·700466	·673625	·647874	·623167	·576706	·533908	·494469	·458112	16
17	·844377	·776385	·714163	·685052	·657195	·630535	·605016	·557204	·513373	·473176	·436297	17
18	·836017	·764912	·700159	·669978	·641166	·613659	·587395	·538361	·493628	·452800	·415521	18
19	·827740	·753607	·686431	·655235	·625528	·597235	·570286	·520156	·474642	·433302	·395784	19
20	·819544	·742470	·672971	·640816	·610271	·581251	·553676	·502566	·456387	·414643	·376889	20
21	·811430	·731498	·659776	·626715	·595386	·565694	·537549	·485571	·438834	·396787	·358942	21
22	·803396	·720688	·646839	·612925	·580865	·550554	·521893	·469151	·421955	·379701	·341850	22
23	·795442	·710037	·634156	·599437	·566697	·535819	·506692	·453286	·405726	·363350	·325571	23
24	·787566	·699544	·621721	·586247	·552875	·521478	·491934	·437957	·390121	·347708	·310068	24
25	·779768	·689206	·609531	·573346	·539391	·507521	·477606	·423147	·375117	·332731	·295303	25
26	·772048	·679021	·597579	·560730	·526235	·493936	·463695	·408838	·360689	·318402	·281241	26
27	·764404	·668986	·585862	·548391	·513400	·480718	·450189	·395012	·346816	·304691	·267848	27
28	·756836	·659099	·574375	·536324	·500878	·467852	·437077	·381654	·333477	·291571	·255094	28
29	·749342	·649359	·563112	·524522	·488661	·455331	·424346	·368748	·320651	·279015	·242946	29
30	·741923	·639762	·552071	·512980	·476743	·443141	·411987	·356278	·308319	·267000	·231377	30

TABLE XV

Amount of an annuity of 1 at the end of n periods : $s_{\overline{n}|}$

n	1%	1½%	2%	2¼%	2½%	2¾%	3%	3½%	4%	5%	n
1	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000	1
2	2·01000	2·01500	2·02000	2·02250	2·02500	2·02750	2·03000	2·03500	2·04000	2·05000	2
3	3·03010	3·04523	3·06040	3·06801	3·07562	3·08326	3·09090	3·10623	3·12160	3·15250	3
4	4·06040	4·09090	4·12161	4·13704	4·15252	4·16805	4·18363	4·21494	4·24646	4·31013	4
5	5·10101	5·15227	5·20404	5·23012	5·25633	5·28267	5·30914	5·36247	5·41632	5·52563	5
6	6·15202	6·22955	6·30812	6·34780	6·38774	6·42794	6·46841	6·55015	6·63298	6·80191	6
7	7·21354	7·32299	7·43428	7·49062	7·54743	7·60471	7·66246	7·77941	7·89829	8·14201	7
8	8·28567	8·43284	8·58297	8·65916	8·73612	8·81384	8·89234	9·05169	9·21423	9·54911	8
9	9·36853	9·55933	9·75463	9·85399	9·95452	10·05622	10·15911	10·36850	10·58280	11·02656	9
10	10·46221	10·70272	10·94972	11·07571	11·20338	11·33276	11·46388	11·73139	12·00611	12·57789	10
11	11·56683	11·86326	12·16872	12·32491	12·48347	12·64442	12·80780	13·14199	13·48635	14·20679	11
12	12·68250	13·04121	13·41209	13·60222	13·79555	13·99214	14·19203	14·60196	15·02581	15·91713	12

13	13·80933	14·23683	14·68033	14·90827	15·14044	15·37692	15·61779	16·11303	16·62684	17·71298	13
14	14·94742	15·45038	15·97394	16·24371	16·51895	16·79979	17·08632	17·67699	18·29191	19·59863	14
15	16·09690	16·68214	17·29342	17·60919	17·93193	18·26178	18·59891	19·29568	20·02359	21·57856	15
16	17·25786	17·93237	18·63928	19·00540	19·38022	19·76398	20·15688	20·97103	21·82453	23·65749	16
17	18·43044	19·20136	20·01207	20·43302	20·86473	21·30749	21·76159	22·70501	23·69751	25·84037	17
18	19·61475	20·48938	21·41231	21·89276	22·38635	22·89344	23·41444	24·49969	25·64541	28·13238	18
19	20·81090	21·79672	22·84056	23·38535	23·94601	24·52301	25·11687	26·35718	27·67123	30·53900	19
20	22·01900	23·12367	24·29737	24·91152	25·54466	26·19740	26·87037	28·27968	29·77808	33·06595	20
21	23·23919	24·47052	25·78332	26·47203	27·18327	27·91783	28·67649	30·26947	31·96920	35·71925	21
22	24·47159	25·83758	27·29898	28·06765	28·86286	29·68557	30·53678	32·32890	34·24797	38·50521	22
23	25·71630	27·22515	28·84496	29·69917	30·58443	31·50192	32·45288	34·46041	36·61789	41·43048	23
24	26·97346	28·63352	30·42186	31·36740	32·34904	33·36822	34·42647	36·66653	39·08260	44·50200	24
25	28·24320	30·06302	32·03030	33·07317	34·15776	35·28585	36·45926	38·94986	41·64591	47·72710	25
26	29·52563	31·51397	33·67090	34·81732	36·01171	37·25621	38·55304	41·31310	44·31174	51·11345	26
27	30·82089	32·98668	35·34432	36·60071	37·91200	39·28075	40·70963	43·75906	47·08421	54·66913	27
28	32·12910	34·48148	37·05121	38·42422	39·85980	41·36098	42·93092	46·29063	49·96758	58·40258	28
29	33·45039	35·99870	38·79223	40·28877	41·85630	43·49840	45·21885	48·91080	52·96629	62·32271	29
30	34·78489	37·53868	40·56808	42·19526	43·90270	45·69461	47·57542	51·62267	56·08494	66·43885	30

TABLE XVI

Present Value of an Immediate Annuity of 1 for n periods : $a_{\overline{n}|}$

n	1%	1½%	2%	2¼%	2½%	2¾%	3%	3½%	4%	5%	n
1	0·990099	0·985222	0·980392	0·977995	0·975610	0·973236	0·970874	0·966184	0·961539	0·952381	1
2	1·970395	1·955883	1·941561	1·934470	1·927424	1·920424	1·913470	1·899694	1·886095	1·859410	2
3	2·940985	2·912200	2·883883	2·869897	2·856024	2·842262	2·828611	2·801637	2·775091	2·723248	3
4	3·901966	3·854385	3·807729	3·784740	3·761974	3·739428	3·717098	3·673079	3·629895	3·545951	4
5	4·853431	4·782645	4·713460	4·679453	4·645829	4·612582	4·579707	4·515052	4·451822	4·329477	5
6	5·795477	5·697187	5·601431	5·554477	5·508125	5·462367	5·417191	5·328553	5·242137	5·075692	6
7	6·728195	6·598214	6·471961	6·410246	6·349391	6·289408	6·230283	6·114544	6·002055	5·786373	7
8	7·651678	7.485925	7·325481	7·247185	7·170137	7·094314	7·019692	6·873956	6·732745	6·463213	8
9	8·566018	8·360517	8·162237	8·065706	7·970866	7·877678	7·786109	7·607687	7·435332	7·107822	9
10	9·471305	9·222185	8·982585	8·866216	8·752064	8·640076	8·530203	8·316605	8·110896	7·721735	10
11	10·367625	10·071118	9·786848	9·649111	9·514209	9·382069	9·252624	9·001551	8·760477	8·306414	11
12	11·255078	10·907505	10·575341	10·414779	10·257765	10·104204	9·954004	9·663334	9·385074	8·863252	12
13	12·133740	11·731532	11·348374	11·163598	10·983185	10·807011	10·634955	10·302739	9·985648	9·393573	13

14	13·003703	12·543382	12·106249	11·895939	11·690012	11·491008	11·296073	10·920520	10·563123	9·898641	14
15	13·865053	13·343233	12·849264	12·612166	12·381378	12·156699	11·937935	11·517411	11·118387	10·379658	15
16	14·717874	14·131264	13·577709	13·312631	13·055003	12·804573	12·561102	12·094117	11·652296	10·837770	16
17	15·562251	14·907649	14·291872	13·997683	13·712198	13·435108	13·166119	12·651321	12·165669	11·274066	17
18	16·398269	15·672561	14·992031	14·667661	14·353364	14·048767	13·753513	13·189682	12·659297	11·689587	18
19	17·226009	16·426168	15·678462	15·322896	14·978891	14·646002	14·323799	13·709837	13·133939	12·085321	19
20	18·045553	17·168639	16·351433	15·963712	15·589162	15·227252	14·877475	14·212403	13·590326	12·462210	20
21	18·856983	17·900137	17·011209	16·590428	16·184549	15·792946	15·415024	14·697974	14·029160	12·821153	21
22	19·660379	18·620525	17·658048	17·203352	16·765413	16·343500	15·936917	15·167125	14·451115	13·163003	22
23	20·455821	19·330362	18·292204	17·802790	17·332111	16·879319	16·443608	15·620411	14·856842	13·488574	23
24	21·243387	20·030405	18·913926	18·389036	17·884986	17·400797	16·935542	16·058368	15·246963	13·798642	24
25	22·023156	20·719611	19·523457	18·96233	18·424376	17·908318	17·413148	16·481515	15·622080	14·093945	25
26	22·795204	21·398632	20·121036	19·523113	18·950611	18·402256	17·876842	16·890352	15·982769	14·375185	26
27	23·559608	22·067618	20·706898	20·071504	19·464011	18·882974	18·327032	17·285365	16·329586	14·643034	27
28	24·316443	22·726717	21·281272	20·607828	19964889	19·350826	18·764108	17·667019	16·663063	14·898127	28
29	25·065785	23·376076	21·844385	21·132350	20·453550	19·806157	19·188455	18·035767	16·983715	15·141074	29
30	25·807708	24·015838	22·396456	21·645330	20·930293	20·249301	19.600441	18·392045	17·292033	15·372451	30

TABLE XVII

Table of Yield on Investments

Price %	Dividends per centum per annum					
Stock	2¾%	3%	3½%	4%	4½%	5%	5½%

Stock	2¾%	3%	3½%	4%	4½%	5%	5½%
80	3·43	3·75	4·37	5·00	5·62	6·25	6·87
80½	3·41	3·73	4·34	4·97	5·59	6·21	6·83
81	3·39	3·70	4·32	4·94	5·55	6·17	6·79
81½	3·37	3·68	4·29	4·91	5·52	6·13	6·75
82	3·35	3·66	4·27	4·88	5·49	6·10	6·70
82½	3·33	3·63	4·24	4·85	5·45	6·06	6·67
83	3·31	3·61	4·21	4·82	5·42	6·02	6·63
83½	3·29	3·59	4·19	4·79	5·39	5·99	6·58
84	3·27	3·57	4·17	4·76	5·36	5·95	6·55
84½	3·25	3·55	4·14	4·73	5·32	5·91	6·51
85	3·24	3·53	4·12	4·71	5·29	5·88	6·47
85½	3·21	3·50	4·10	4·68	5·27	5·85	6·43
86	3·20	3·49	4·07	4·65	5·23	5·81	6·40
86½	3·18	3·47	4·05	4·62	5·20	5·78	6·36
87	3·16	3·45	4·02	4·59	5·18	5·74	6·33
87½	3·14	3·42	4·00	4·57	5·14	5·71	6·29
88	3·12	3·41	3·98	4·55	5·11	5·68	6·25
88½	3·10	3·39	3·95	4·52	5·09	5·65	6·21
89	3·09	3·37	3·93	4·49	5·05	5·61	6·18
89½	3·07	3·35	3·91	4·47	5·03	5·58	6·14
90	3·06	3·33	3·88	4·44	5·00	5·56	6·11
90½	3·04	3·31	3·87	4·42	4·97	5·52	6·08
91	3·02	3·30	3·85	4·40	4·94	5·49	6·04
91½	3·00	3·28	3·83	4·37	4·92	5·46	6·01
92	2·99	3·26	3·80	4·35	4·89	5·43	5·98
92½	2·97	3·24	3·78	4·32	4·86	5·41	5·95

93	2·96	3·23	3·76	4·30	4·84	5·38	5·91
93½	2·94	3·21	3·74	4·28	4·81	5·35	5·88
94	2·93	3·19	3·73	4·26	4·79	5·32	5·85
94½	2·91	3·17	3·70	4·23	4·76	5·29	5·82
95	2·89	3·16	3·68	4·21	4·73	5·26	5·79
95½	2·88	3·14	3·66	4·19	4·71	5·24	5·76
96	2·86	3·13	3·65	4·16	4·68	5·20	5·72
96½	2·85	3·11	3·63	4·15	4·66	5·18	5·70
97	2·83	3·09	3·61	4·12	4·64	5·15	5·67
97½	2·82	3·08	3·59	4·10	4·61	5·13	5·64
98	2·81	3·06	3·57	4·08	4·59	5·10	5·61
98½	2·79	3·05	3·55	4·06	4·57	5·08	5·58
99	2·78	3·03	3·53	4·04	4·55	5·05	5·56
99½	2·76	3·02	3·51	4·02	4·52	5·03	5·53
100	2·75	3·00	3·50	4·00	4·50	5·00	5·50
100½	2·73	2·98	3·48	3·98	4·48	4·98	5·47
101	2·72	2·97	3·46	3·96	4·46	4·95	5·44
101½	2·71	2·96	3·45	3·94	4·43	4·93	5·41
102	2·70	2·94	3·43	3·92	4·41	4·90	5·39
102½	2·69	2·92	3·41	3·90	4·39	4·88	5·36
103	2·67	2·91	3·40	3·88	4·37	4·85	5·34
103½	2·66	2·90	3·38	3·86	4·35	4·83	5·31
104	2·64	2·88	3·36	3·85	4·33	4·81	5·28
104½	2·63	2·87	3·35	3·82	4·31	4·78	5·26
105	2·62	2·86	3·33	3·81	4·29	4·76	5·23
105½	2·61	2·84	3·32	3·79	4·27	4·74	5·21
106	2·59	2·83	3·30	3·77	4·25	4·72	5·18
106½	2·58	2·82	3·28	3·76	4·22	4·69	5·16
107	2·57	2·80	3·27	3·74	4·20	4·67	5·14
107½	2·56	2·79	3·25	3·72	4·18	4·65	5·11
108	2·55	2·78	3·24	3·70	4·16	4·63	5·09
108½	2·54	2·76	3·22	3·69	4·14	4·60	5·06
109	2·52	2·75	3·21	3·68	4·12	4·58	5·04
109½	2·51	2·74	3·19	3·65	4·11	4·56	5·02
110	2·50	2·72	3·18	3·64	4·09	4·54	5·00

TABLE XVIII

LOGARITHMS OF NUMBERS

											Mean Differences								
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	00000	00432	00860	01284	01703						42	85	127	170	212	254	297	339	381
						02119	02531	02938	03342	03743	40	81	121	162	202	242	283	323	364
11	04139	04532	04922	05308	05690						37	77	116	154	193	232	270	309	348
						06070	06446	06819	07188	07555	37	74	111	148	185	222	259	296	333
12	07918	08279	08636	08991	09342						36	71	106	142	177	213	248	284	319
						09691	10037	10380	10721	11059	34	68	102	136	170	204	238	272	307
13	11394	11727	12057	12385	12710						33	66	98	131	164	197	229	262	295
						13033	13354	13672	13988	14301	32	63	95	126	158	190	221	253	284
14	14613	14922	15229	15534	15836						30	61	91	122	152	183	213	244	274
						16137	16435	16732	17026	17319	29	59	88	118	147	177	206	236	265
15	17609	17898	18184	18469	18752						28	57	85	114	142	171	199	228	256
						19033	19312	19590	19866	20140	28	56	83	110	138	165	193	221	248
16	20412	20683	20952	21219	21484						27	53	80	107	134	160	187	214	240
						21748	22011	22272	22531	22789	26	52	78	104	130	156	182	208	233
17	23045	23300	23553	23805	24055						26	50	76	101	126	151	176	201	227
						24304	24551	24797	25042	25285	25	49	73	98	122	147	171	196	220
18	25527	25768	26007	26245	26482						24	48	71	95	119	143	167	190	214
						26717	26951	27184	27416	27646	23	46	69	93	116	139	162	185	208
19	27875	28103	28330	28556	28780						23	45	68	90	113	135	158	180	203
						29003	29226	29447	29667	29885	22	44	66	88	110	132	154	176	198

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
20	30103	30320	30535	30750	30963	31175	31387	31597	31806	32015	21	43	64	85	106	127	148	170	190
21	32222	32428	32634	32838	33041	33244	33445	33646	33846	34044	20	40	60	80	100	120	140	160	180
22	34242	34439	34635	34830	35025	35218	35411	35603	35793	35984	20	39	58	77	97	116	135	154	174
23	36173	36361	36549	36736	36922	37107	37291	37475	37658	37840	19	37	56	74	93	111	130	148	167
24	38021	38202	38382	38561	38739	38917	39094	39270	39445	39620	18	35	53	71	89	106	124	142	159
25	39794	39967	40140	40312	40483	40654	40824	40993	41162	41330	17	34	51	68	85	102	119	136	153
26	41497	41664	41830	41996	42160	42325	42488	42651	42813	42975	16	33	49	66	82	98	115	131	148
27	43136	43297	43457	43616	43775	43933	44091	44248	44404	44560	16	32	47	63	79	95	111	126	142
28	44716	44871	45025	45179	45332	45484	45637	45788	45939	46090	15	30	46	61	76	91	107	122	137
29	46240	46389	46538	46687	46835	46982	47129	47276	47422	47567	15	29	44	59	74	88	103	118	132
30	47712	47857	48001	48144	48287	48430	48572	48714	48855	48996	14	29	43	57	72	86	100	114	129
31	49136	49276	49415	49554	49693	49831	49969	50106	50243	50379	14	28	41	55	69	83	97	110	124
32	50515	50650	50786	50920	51054	51188	51322	51455	51587	51720	13	27	40	54	67	80	94	107	121
33	51851	51983	52114	52244	52375	52504	52634	52763	52892	53020	13	26	39	52	65	78	91	104	117
34	53148	53275	53403	53529	53656	53782	53908	54033	54158	54283	13	25	38	50	63	76	88	101	113
35	54407	54531	54654	54777	54900	55023	55145	55267	55388	55509	12	24	37	49	61	73	85	98	110
36	55630	55751	55871	55991	56110	56229	56348	56467	56585	56703	12	24	36	48	60	71	83	95	107
37	56820	56937	57054	57171	57287	57403	57519	57634	57749	57864	12	23	35	46	58	70	81	93	104
38	57978	58092	58206	58320	58433	58546	58659	58771	58883	58995	11	23	34	45	57	68	79	90	102
39	59106	59218	59329	59439	59550	59660	59770	59879	59988	60097	11	22	33	44	55	66	77	88	99
40	60206	60314	60423	60531	60638	60746	60853	60959	61066	61172	11	22	33	44	55	66	77	88	99
41	61278	61384	61490	61595	61700	61805	61909	62014	62118	62221	10	21	31	42	53	63	74	84	95
42	62325	62428	62531	62634	62737	62839	62941	63043	63144	63246	10	20	31	41	51	61	71	82	92
43	63347	63448	63548	63649	63749	63849	63949	64048	64147	64246	10	20	30	40	50	60	70	80	90
44	64345	64444	64542	64640	64738	64836	64933	65031	65128	65225	10	20	29	39	49	59	68	78	88
45	65321	65418	65514	65610	65706	65801	65896	65992	66087	66181	10	19	29	38	48	57	67	76	86
46	66276	66370	66464	66558	66652	66745	66839	66932	67025	67117	9	19	28	37	47	56	65	74	84
47	67210	67302	67394	67486	67578	67669	67761	67852	67943	68034	9	18	27	36	46	55	64	73	82
48	68124	68215	68305	68395	68485	68574	68664	68753	68842	68931	9	18	27	36	45	53	63	72	81
49	69020	69108	69197	69285	69373	69461	69548	69636	69723	69810	9	18	26	35	44	53	62	70	79

LOGARITHMS OF NUMBERS

	0	1	2	3	4	5	6	7	8	9	Mean Differences								
											1	2	3	4	5	6	7	8	9
50	69897	69984	70070	70157	70243	70329	70415	70501	70586	70672	9	17	26	34	43	52	60	69	77
51	70757	70842	70927	71012	71096	71181	71265	71349	71433	71517	8	17	25	34	42	50	59	67	76
52	71600	71684	71767	71850	71933	72016	72099	72181	72263	72346	8	17	25	33	42	50	58	66	75
53	72428	72509	72591	72673	72754	72835	72916	72997	73078	73159	8	16	24	32	41	49	57	65	73
54	73239	73320	73400	73480	73560	73640	73719	73799	73878	73957	8	16	24	32	40	48	56	64	72
55	74036	74115	74194	74273	74351	74429	74507	74586	74663	74741	8	16	23	31	39	47	55	63	70
56	74819	74896	74974	75051	75128	75205	75282	75358	75435	75511	8	15	23	31	39	46	54	62	69
57	75587	75664	75740	75815	75891	75967	76042	76118	76193	76268	8	15	23	30	38	45	53	60	68
58	76343	76418	76492	76567	76641	76716	76790	76864	76938	77012	7	15	22	30	37	44	52	59	67
59	77085	77159	77232	77305	77379	77452	77525	77597	77670	77743	7	15	22	29	37	44	51	58	66
60	77815	77887	77960	78032	78104	78176	78247	78319	78390	78462	7	14	22	29	36	43	50	58	65
61	78533	78604	78675	78746	78817	78888	78958	79029	79099	79169	7	14	21	28	36	43	50	57	64
62	79239	79309	79379	79449	79518	79588	79657	79727	79796	79865	7	14	21	28	35	41	48	55	62
63	79934	80003	80072	80140	80209	80277	80346	80414	80482	80550	7	14	20	27	34	41	48	54	61
64	80618	80686	80754	80821	80889	80956	81023	81090	81158	81224	7	13	20	27	34	40	47	54	60
65	81291	81358	81425	81491	81558	81624	81690	81757	81823	81889	7	13	20	26	33	40	46	53	59
66	81954	82020	82086	82151	82217	82282	82347	82413	82478	82543	7	13	20	26	33	39	46	52	59
67	82607	82672	82737	82802	82866	82930	82995	83059	83123	83187	6	13	19	26	32	38	45	51	58
68	83251	83315	83378	83442	83506	83569	83632	83696	83759	83822	6	13	19	25	32	38	44	50	57
69	83885	83948	84011	84073	84136	84198	84261	84323	84386	84448	6	12	19	25	31	37	43	50	56
70	84510	84572	84634	84696	84757	84819	84880	84942	85003	85065	6	12	19	25	31	37	43	50	56
71	85126	85187	85248	85309	85370	85431	85491	85552	85612	85673	6	12	18	24	31	37	43	49	55
72	85733	85794	85854	85914	85974	86034	86094	86153	86213	86273	6	12	18	24	30	36	42	48	54
73	86332	86392	86451	86510	86570	86629	86688	86747	86806	86864	6	12	18	24	30	35	41	47	53
74	86923	86982	87040	87099	87157	87216	87274	87332	87390	87448	6	12	17	23	29	35	41	46	52

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
75	87506	87564	87622	87679	87737	87795	87852	87910	87967	88024	6	12	17	23	29	35	41	46	52
76	88081	88138	88195	88252	88309	88366	88423	88480	88536	88593	6	11	17	23	29	34	40	46	51
77	88649	88705	88762	88818	88874	88930	88986	89042	89098	89154	6	11	17	22	28	34	39	45	50
78	89209	89265	89321	89376	89432	89487	89542	89597	89653	89708	6	11	17	22	28	33	39	44	50
79	89763	89818	89873	89927	89982	90037	90091	90146	90200	90255	6	11	17	22	28	33	39	44	50
80	90309	90363	90417	90472	90526	90580	90634	90687	90741	90795	5	11	16	22	27	32	38	43	49
81	90848	90902	90956	91009	91062	91116	91169	91222	91275	91328	5	11	16	21	27	32	37	42	48
82	91381	91434	91487	91540	91593	91645	91698	91751	91803	91855	5	11	16	21	27	32	37	42	48
83	91908	91960	92012	92064	92117	92169	92221	92273	92324	92376	5	10	16	21	26	31	36	42	47
84	92428	92480	92531	92583	92634	92686	92737	92788	92840	92891	5	10	15	20	26	31	36	41	46
85	92942	92993	93044	93095	93146	93197	93247	93298	93349	93399	5	10	15	20	26	31	36	41	46
86	93450	93500	93551	93601	93651	93702	93752	93802	93852	93902	5	10	15	20	25	30	35	40	45
87	93952	94002	94052	94101	94151	94201	94250	94300	94349	94399	5	10	15	20	25	30	35	40	45
88	94448	94498	94547	94596	94645	94694	94743	94792	94841	94890	5	10	15	20	25	29	34	39	44
89	94939	94988	95036	95085	95134	95182	95231	95279	95328	95376	5	10	15	19	24	29	34	39	44
90	95424	95472	95521	95569	95617	95665	95713	95761	95809	95856	5	10	14	19	24	29	34	38	43
91	95904	95952	95999	96047	96095	96142	96190	96237	96284	96332	5	9	14	19	24	28	33	38	42
92	96379	96426	96473	96520	96567	96614	96661	96708	96755	96802	5	9	14	19	24	28	33	38	42
93	96848	96895	96942	96988	97035	97081	97128	97174	97220	97267	5	9	14	18	23	28	32	38	42
94	97313	97359	97405	97451	97497	97543	97589	97635	97681	97727	5	9	14	18	23	28	32	37	42
95	97772	97818	97864	97909	97955	98000	98046	98091	98137	98182	5	9	14	18	23	27	32	36	41
96	98227	98272	98318	98363	98408	98453	98498	98543	98588	98632	5	9	14	18	23	27	32	36	41
97	98677	98722	98767	98811	98856	98900	98945	98989	99034	99078	4	9	13	18	22	27	31	36	40
98	99123	99167	99211	99255	99300	99344	99388	99432	99476	99520	4	9	13	18	22	26	31	35	40
99	99564	99607	99651	99695	99739	99782	99826	99870	99913	99957	4	9	13	17	22	26	31	35	39
100	00000	00043	00087	00130	00173	00217	00260	00303	00346	00389	4	9	13	17	22	26	30	34	39

TABLE XIX
ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean differences								
											1	2	3	4	5	6	7	8	9
·00	10000	10023	10046	10069	10093	10116	10139	10162	10186	10209	2	5	7	9	12	14	16	19	21
·01	10233	10257	10280	10304	10328	10351	10375	10399	10423	10447	2	5	7	10	12	14	17	19	21
·02	10471	10495	10520	10544	10568	10593	10617	10641	10666	10691	2	5	7	10	12	15	17	20	22
·03	10715	10740	10765	10789	10814	10839	10864	10889	10914	10940	3	5	8	10	13	15	18	20	23
·04	10965	10990	11015	11041	11066	11092	11117	11143	11169	11194	3	5	8	10	13	15	18	20	23
·05	11220	11246	11272	11298	11324	11350	11376	11402	11429	11455	3	5	8	11	13	16	18	21	24
·06	11482	11508	11535	11561	11588	11614	11641	11668	11695	11722	3	5	8	11	13	16	19	21	24
·07	11749	11776	11803	11830	11858	11885	11912	11940	11967	11995	3	5	8	11	14	16	19	22	25
·08	12023	12050	12078	12106	12134	12162	12190	12218	12246	12274	3	6	8	11	14	17	20	22	25
·09	12303	12331	12359	12388	12417	12445	12474	12503	12531	12560	3	6	9	11	14	17	20	23	26
·10	12589	12618	12647	12677	12706	12735	12764	12794	12823	12853	3	6	9	12	15	18	21	24	26
·11	12882	12912	12942	12972	13002	13032	13062	13092	13122	13152	3	6	9	12	15	18	21	24	27
·12	13183	13213	13243	13274	13305	13335	13366	13397	13428	13459	3	6	9	12	15	18	21	25	28
·13	13490	13521	13552	13583	13614	13646	13677	13709	13740	13772	3	6	9	13	16	19	22	25	28
·14	13804	13836	13868	13900	13932	13964	13996	14028	14060	14093	3	6	10	13	16	19	22	26	29
·15	14125	14158	14191	14223	14256	14289	14322	14355	14388	14421	3	7	10	13	16	20	23	26	30
·16	14454	14488	14521	14555	14588	14622	14655	14689	14723	14757	3	7	10	13	17	20	24	27	30
·17	14791	14825	14859	14894	14928	14962	14997	15031	15066	15101	3	7	10	14	17	21	24	28	31
·18	15136	15171	15205	15241	15276	15311	15346	15382	15417	15453	4	7	11	14	18	21	25	28	32
·19	15488	15524	15560	15596	15631	15668	15704	15740	15776	15812	4	7	11	14	18	22	25	29	32
·20	15849	15885	15922	15959	15996	16032	16069	16106	16144	16181	4	7	11	15	18	22	26	30	33
·21	16218	16255	16293	16331	16368	16406	16444	16482	16520	16558	4	8	11	15	19	23	26	30	34
·22	16596	16634	16672	16711	16749	16788	16827	16866	16904	16943	4	8	12	[illegible]	19	23	27	31	35
·23	16982	17022	17061	17100	17140	17179	17219	17258	17298	17338	4	8	12	16	20	24	28	32	36
·24	17378	17418	17458	17498	17539	17579	17620	17660	17701	17742	4	8	12	16	20	24	28	32	36

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·25	17783	17824	17865	17906	17947	17989	18030	18072	18113	18155	4	8	12	17	21	25	29	33	37
·26	18197	18239	18281	18323	18365	18408	18450	18493	18535	18578	4	8	13	17	21	25	30	34	38
·27	18621	18664	18707	18750	18793	18836	18880	18923	18967	19011	4	9	13	17	22	26	30	35	39
·28	19055	19099	19143	19187	19231	19275	19320	19364	19409	19454	4	9	13	18	22	26	31	35	40
·29	19498	19543	19588	19634	19679	19724	19770	19815	19861	19907	5	9	14	18	23	27	32	36	41
·30	19953	19999	20045	20091	20137	20184	20230	20277	20324	20370	5	9	14	19	23	28	33	37	42
·31	20417	20464	20512	20559	20606	20654	20701	20749	20797	20845	5	10	14	19	24	29	33	38	43
·32	20893	20941	20989	21038	21086	21135	21184	21232	21281	21330	5	10	15	19	24	29	34	39	44
·33	21380	21429	21478	21528	21577	21627	21677	21727	21777	21827	5	10	15	20	25	30	35	40	45
·34	21878	21928	21979	22029	22080	22131	22182	22233	22284	22336	5	10	15	20	25	31	36	41	46
·35	22387	22439	22491	22542	22594	22646	22699	22751	22803	22856	5	10	16	21	26	31	37	42	47
·36	22909	22961	23014	23067	23121	23174	23227	23281	23336	23388	5	11	16	21	27	32	37	43	48
·37	23442	23496	23550	23605	23659	23714	23768	23823	23878	23933	5	11	16	22	27	33	38	44	49
·38	23988	24044	24099	24155	24210	24266	24322	24378	24434	24491	6	11	17	22	28	34	39	45	50
·39	24547	24604	24660	24717	24774	24831	24889	24946	25003	25061	6	11	17	23	29	34	40	46	51
·40	25119	25177	25236	25293	25351	25410	25468	25527	25586	25645	6	12	18	23	29	35	41	47	53
·41	25704	25763	25823	25882	25942	26002	26062	26122	26182	26242	6	12	18	24	30	36	42	48	54
·42	26303	26363	26424	26485	26546	26607	26669	26730	26792	26853	6	12	18	24	31	37	43	49	55
·43	26915	26977	27040	27102	27164	27227	27290	27353	27416	27479	6	13	19	25	31	38	44	50	56
·44	27542	27606	27669	27733	27797	27861	27925	27990	28054	28119	6	13	19	26	32	39	45	51	58
·45	28184	28249	28314	28379	28445	28510	28576	28642	28708	28774	7	13	20	26	33	39	46	52	59
·46	28840	28907	28973	29040	29107	29174	29242	29309	29376	29444	7	13	20	27	34	40	47	54	60
·47	29512	29580	29648	29717	29785	29854	29923	29992	30061	30130	7	14	21	28	34	41	48	55	62
·48	30200	30269	30339	30409	30479	30549	30620	30690	30761	30832	7	14	21	28	35	42	49	56	63
·49	30903	30974	31046	31117	31189	31261	31333	31405	31477	31550	7	14	22	29	36	43	50	58	65

ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean differences. 1	2	3	4	5	6	7	8	9
·50	31623	31696	31769	31842	31916	31989	32063	32137	32211	32285	7	15	22	29	37	44	52	59	66
·51	32359	32434	32509*	32584	32659	32735	32809	32885	32961	33037	8	15	23	30	38	45	53	60	68
·52	33113	33189	33266	33343	33420	33497	33574	33651	33729	33806	8	15	23	31	39	46	54	62	69
·53	33884	33963	34041	34119	34198	34277	34356	34435	34514	34594	8	16	24	32	40	47	55	63	71
·54	34674	34754	34834	34914	34995	35075	35156	35237	35318	35400	8	16	24	32	40	48	56	65	73
·55	35481	35563	35645	35727	35810	35892	35975	36058	36141	36224	8	16	25	33	41	50	58	66	74
·56	36308	36392	36475	36559	36644	36728	36813	36898	36983	37068	8	17	25	34	42	51	59	68	76
·57	37154	37239	37325	37411	37497	37584	37670	37757	37844	37931	9	17	26	35	43	52	61	69	78
·58	38019	38107	38194	38282	38371	38459	38548	38637	38726	38815	9	18	27	35	44	53	62	71	80
·59	38905	38994	39084	39174	39264	39355	39446	39537	39628	39719	9	18	27	36	45	54	63	72	82
·60	39811	39902	39994	40087	40179	40272	40365	40458	40551	40644	9	19	28	37	46	56	65	74	83
·61	40738	40832	40926	41020	41115	41210	41305	41400	41495	41591	9	19	28	38	47	57	66	76	85
·62	41687	41783	41879	41976	42073	42170	42267	42364	42462	42560	10	19	29	39	49	58	68	78	87
·63	42658	42756	42855	42954	43053	43152	43251	43351	43451	43551	10	20	30	40	50	60	70	80	89
·64	43652	43752	43853	43954	44055	44157	44259	44361	44463	44566	10	20	30	41	51	61	71	81	91
·65	44668	44771	44875	44978	45082	45186	45290	45394	45499	45604	10	21	31	42	52	62	73	83	94
·66	45709	45814	45920	46026	46132	46238	46345	46452	46559	46666	11	21	32	43	53	64	75	85	96
·67	46774	46881	46989	47098	47206	47315	47424	47534	47643	47753	11	22	33	44	54	65	76	87	98
·68	47863	47973	48084	48195	48306	48417	48529	48641	48753	48865	11	22	33	45	56	67	78	89	100
·69	48978	49091	49204	49317	49431	49545	49659	49774	49888	50003	11	23	34	46	57	68	80	91	103
·70	50119	50234	50350	50466	50582	50699	50816	50933	51050	51168	12	23	35	47	58	70	82	93	105
·71	51286	51404	51523	51642	51761	51880	52000	52119	52240	52360	12	24	36	48	60	72	84	96	108
·72	52481	52602	52723	52845	52966	53088	53211	53333	53456	53580	12	24	37	49	61	73	85	98	110
·73	53703	53827	53951	54075	54200	54325	54450	54576	54702	54828	13	25	38	50	63	75	88	100	113
·74	54954	55081	55208	55336	55463	55590	55719	55847	55976	56105	13	26	38	51	64	77	90	102	115

·75	56234	56364	56494	56624	56754	56885	57016	57148	57280	57412	13	26	39	52	66	79	92	105	118
·76	57544	57677	57810	57943	58076	58210	58345	58479	58614	58749	13	27	40	54	67	80	94	107	121
·77	58884	59020	59156	59293	59429	59566	59704	59841	59979	60117	14	27	41	55	69	82	96	110	123
·78	60256	60395	60534	60674	60814	60954	61094	61235	61376	61518	14	28	42	56	70	84	98	112	126
·79	61659	61802	61944	62087	62230	62373	62517	62661	62806	62951	14	29	43	58	72	86	101	115	130
·80	63096	63241	63387	63533	63680	63826	63973	64121	64269	64417	15	29	44	59	74	88	103	118	132
·81	64565	64714	64863	65013	65163	65313	65464	65615	65766	65917	15	30	45	60	75	90	105	120	135
·82	66069	66222	66374	66527	66681	66834	66988	67143	67298	67453	15	31	46	62	77	92	108	123	139
·83	67608	67764	67920	68077	68234	68391	68549	68707	68865	69024	16	32	47	63	79	95	110	126	142
·84	69183	69343	69503	69663	69823	69984	70146	70307	70469	70632	16	32	48	64	81	97	113	129	145
·85	70795	70958	71121	71285	71450	71614	71779	71945	72111	72277	17	33	50	66	83	99	116	132	149
·86	72444	72611	72778	72946	73114	73282	73451	73621	73790	73961	17	34	51	68	85	101	118	135	152
·87	74131	74302	74473	74645	74817	74989	75162	75336	75509	75683	17	35	52	69	87	104	121	138	156
·88	75858	76033	76208	76384	76560	76736	76913	77090	77268	77446	18	35	53	71	89	107	125	142	159
·89	77625	77804	77983	78163	78343	78524	78705	78886	79068	79250	18	36	54	72	91	109	127	145	163
·90	79433	79616	79799	79983	80168	80353	80538	80724	80910	81096	19	37	56	74	93	111	130	148	167
·91	81283	81470	81658	81846	82035	82224	82414	82604	82794	82985	19	38	57	76	95	113	132	151	170
·92	83176	83368	83560	83753	83946	84140	84333	84528	84723	84918	19	39	58	78	97	116	136	155	175
·93	85114	85310	85507	85704	85901	86099	86298	86497	86696	86896	20	40	60	79	99	119	139	158	178
·94	87096	87297	87498	87700	87902	88105	88308	88512	88716	88920	20	41	61	81	102	122	142	162	183
·95	89125	89331	89536	89743	89950	90157	90365	90573	90782	90991	21	42	62	83	104	125	146	166	187
·96	91201	91411	91622	91833	92045	92257	92470	92683	92897	93111	21	42	64	85	106	127	149	170	191
·97	93325	93541	93756	93972	94189	94406	94624	94842	95060	95280	22	43	65	87	109	130	152	174	195
·98	95499	95719	95940	96161	96383	96605	96828	97051	97275	97499	22	44	67	89	111	133	155	178	200
·99	97724	97949	98175	98401	98628	98855	99083	99312	99541	99770	23	46	68	91	114	137	160	182	205
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

TABLE XX

RECIPROCALS OF NUMBERS. From 1 to 10.

Numbers in difference columns to be subtracted, not added.

	0	1	2	3	4	5	6	7	8	9	Mean Differences.								
											1	2	3	4	5	6	7	8	9
1·0	1·00000	99010	98039	97087	96154	95238	94340	93458	92593	91743									
1·1	·90909	90090	89286	88496	87719	86957	86207	85470	84746	84034									
1·2	·83333	82645	81967	81301	80645	80000	79365	78740	78125	77519									
1·3	·76923	76336	75758	75188	74627	74074	73529	72993	72464	71942									
1·4	·71429	70922	70423	69930	69444	68966	68493	68027	67568	67114									
1·5	·66667	66225	65789	65359	64935	64516	64103	63694	63291	62893									
1·6	·62500	62112	61728	61350	60976	60606	60241	59880	59524	59172	37	74	110	147	184	221	258	294	331
1·7	·58824	58480	58140	57803	57471	57143	56818	56497	56180	55866	33	65	98	131	164	196	229	262	294
1·8	·55556	55249	54945	54645	54348	54054	53763	53476	53191	52910	29	58	88	117	146	175	204	234	263
1·9	·52632	52356	52083	51813	51546	51282	51020	50761	50505	50251	26	53	79	105	132	158	184	210	237
2·0	·50000	49751	49505	49261	49020	48780	48544	48309	48077	47847	24	48	71	95	119	143	167	190	214
2·1	·47619	47393	47170	46948	46729	46512	46296	46083	45872	45662	22	43	65	86	108	130	151	173	194
2·2	·45455	45249	45045	44843	44643	44444	44248	44053	43860	43668	20	40	59	79	99	119	139	158	178
2·3	·43478	43290	43103	42918	42735	42553	42373	42194	42017	41841	18	36	54	72	91	109	127	145	163
2·4	·41667	41494	41322	41152	40984	40816	40650	40486	40323	40161	17	33	50	67	84	100	117	134	150
2·5	·40000	39841	39683	39526	39370	39216	39063	38911	38760	38610	15	31	46	62	77	92	108	123	139
2·6	·38462	38314	38168	38023	37879	37736	37594	37453	37313	37175	14	29	43	57	72	86	100	114	129
2·7	·37037	36900	36765	36630	36496	36364	36232	36101	35971	35842	13	26	40	53	66	79	92	106	119
2·8	·35714	35587	35461	35336	35211	35088	34965	34843	34722	34602	12	25	37	49	62	74	86	98	111
2·9	·34483	34364	34247	34130	34014	33898	33784	33670	33557	33445	12	23	35	46	58	69	81	92	104

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
3·0	·33333	33223	33113	33003	32895	32787	32680	32573	32468	32362	11	22	33	43	54	65	76	86	97
3·1	·32258	32154	32051	31949	31847	31746	31646	31546	31447	31348	10	20	30	40	51	61	71	81	91
3·2	·31250	31153	31056	30960	30864	30769	30675	30581	30488	30365	10	19	29	38	48	57	67	76	86
3·3	·30303	30211	30120	30030	29940	29851	29762	29674	29586	29499	9	18	27	36	45	53	62	71	80
3·4	·29412	29326	29240	29155	29070	28986	28902	28818	28736	28653	8	17	25	34	42	50	59	67	76
3·5	·28571	28490	28409	28329	28249	28169	28090	28011	27933	27855	8	16	24	32	40	47	55	63	71
3·6	·27778	27701	27624	27548	27473	27397	27322	27248	27174	27100	8	15	23	30	38	45	53	60	68
3·7	·27027	26954	26882	26810	26738	26667	26596	26525	26455	26385	7	14	21	28	36	43	50	57	64
3·8	·26316	26247	26178	26110	26042	25974	25907	25840	25773	25707	7	14	20	27	34	41	48	54	61
3·9	·25641	25575	25510	25445	25381	25316	25253	25189	25126	25063	6	13	19	26	32	38	45	51	58
4·0	·25000	24938	24876	24814	24752	24691	24631	24570	24510	24450	6	12	18	24	31	37	43	49	55
4·1	·24390	24331	24272	24213	24155	24096	24038	23981	23923	23866	6	12	17	23	29	35	41	46	52
4·2	·23810	23753	23697	23641	23585	23529	23474	23419	23364	23310	6	11	17	22	28	33	39	44	50
4·3	·23256	23202	23148	23095	23041	22989	22936	22883	22831	22779	5	11	16	21	27	32	37	42	48
4·4	·22727	22676	22624	22573	22523	22472	22422	22371	22321	22272	5	10	15	20	26	31	36	41	46
4·5	·22222	22173	22124	22075	22026	21978	21930	21882	21834	21786	5	10	14	19	24	29	34	38	43
4·6	·21739	21692	21645	21598	21552	21505	21459	21413	21368	21322	5	9	14	18	23	28	32	37	41
4·7	·21277	21231	21186	21142	21097	21053	21008	20964	20921	20877	4	9	13	18	22	26	31	35	40
4·8	·20833	20790	20747	20704	20661	20619	20576	20534	20492	20450	4	9	13	17	22	26	30	34	39
4·9	·20408	20367	20325	20284	20243	20202	20161	20121	20080	20040	4	8	12	16	20	25	29	33	37
5·0	·20000	19960	19920	19881	19841	19802	19763	19724	19685	19646	4	8	12	16	20	24	27	31	35
5·1	·19608	19569	19531	19493	19455	19417	19380	19342	19305	19268	4	8	11	15	19	23	26	30	34
5·2	·19231	19194	19157	19120	19084	19048	19011	18975	18939	18904	4	7	11	15	18	22	25	29	33
5·3	·18868	18832	18797	18762	18727	18691	18657	18622	18587	18553	3	7	10	14	18	21	24	28	31
5·4	·18519	18484	18450	18416	18382	18349	18315	18282	18248	18215	3	7	10	14	17	20	24	27	30

RECIPROCALS OF NUMBERS. From 1 to 10.

Numbers in difference columns to be subtracted, not added

	0	1	2	3	4	5	6	7	8	9	Mean Differences. 1	2	3	4	5	6	7	8	9
5·5	·18182	18149	18116	18083	18051	18018	17986	17953	17921	17889	3	7	10	13	16	20	23	26	29
5·6	·17857	17825	17794	17762	17731	17699	17668	17637	17606	17575	3	6	9	13	16	19	22	25	28
5·7	·17544	17513	17483	17452	17422	17391	17361	17331	17301	17271	3	6	9	12	15	18	21	24	27
5·8	·17241	17212	17182	17153	17123	17094	17065	17036	17007	16978	3	6	9	12	15	18	20	23	26
5·9	·16949	16920	16892	16863	16835	16807	16779	16750	16722	16694	3	6	9	11	14	17	20	23	25
6·0	·16667	16639	16611	16584	16556	16529	16502	16474	16447	16420	3	6	8	11	14	16	19	22	25
6·1	·16393	16367	16340	16313	16287	16260	16234	16207	16181	16155	3	5	8	11	13	16	19	21	24
6·2	·16129	16103	16077	16051	16026	16000	15974	15949	15924	15898	3	5	8	10	13	15	18	21	23
6·3	·15873	15848	15823	15798	15773	15748	15723	15699	15674	15649	2	5	7	10	12	15	17	20	22
6·4	·15625	15601	15576	15552	15528	15504	15480	15456	15432	15408	2	5	7	10	12	14	17	19	22
6·5	·15385	15361	15337	15314	15291	15267	15244	15221	15198	15175	2	5	7	9	12	14	16	19	21
6·6	·15152	15129	15106	15083	15060	15038	15015	14993	14970	14948	2	5	7	9	11	14	16	18	20
6·7	·14925	14903	14881	14859	14837	14815	14793	14771	14749	14728	2	4	7	9	11	13	15	18	20
6·8	·14706	14684	14663	14641	14620	14599	14577	14556	14535	14514	2	4	6	9	11	13	15	17	19
6·9	·14493	14472	14451	14430	14409	14388	14368	14347	14327	14306	2	4	6	8	10	12	14	17	[illegible]
7·0	·14286	14265	14245	14225	14205	14184	14164	14144	14124	14104	2	4	6	8	10	12	14	16	[illegible]
7·1	·14085	14065	14045	14025	14006	13986	13966	13947	13928	13908	2	4	6	8	10	12	14	16	[illegible]
7·2	·13889	13870	13850	13831	13812	13793	13774	13755	13736	13717	2	4	6	8	10	11	13	15	[illegible]
7·3	·13699	13680	13661	13643	13624	13605	13587	13569	13550	13532	2	4	6	7	9	11	13	15	17
7·4	·13514	13495	13477	13459	13441	13423	13405	13387	13369	13351	2	4	5	7	9	11	13	15	16

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
7·5	·13333	13316	13298	13280	13263	13245	13228	13210	13193	13175	2	4	5	7	9	11	12	14	16
7·6	·13158	13141	13123	13106	13089	13072	13055	13038	13021	13004	2	3	5	7	9	10	12	14	15
7·7	·12987	12970	12953	12937	12920	12903	12887	12870	12853	12837	2	3	5	7	8	10	12	13	15
7·8	·12821	12804	12788	12771	12755	12739	12723	12706	12690	12674	2	3	5	7	8	10	11	13	[illegible]
7·9	·12658	12642	12626	12610	12594	12579	12563	12547	12531	12515	2	3	5	6	8	9	11	13	[illegible]
8·0	·12500	12484	12469	12453	12438	12422	12407	12392	12376	12361	2	3	5	6	8	9	11	12	[illegible]
8·1	·12346	12330	12315	12300	12285	12270	12255	12240	12225	12210	2	3	5	6	8	9	11	12	14
8·2	·12195	12180	12165	12151	12136	12121	12107	12092	12077	12063	1	3	4	6	7	9	10	12	13
8·3	·12048	12034	12019	12005	11990	11976	11962	11947	11933	11919	1	3	4	6	7	9	10	11	13
8·4	·11905	11891	11876	11862	11848	11834	11820	11806	11792	11779	1	3	4	6	7	8	10	11	13
8·5	·11765	11751	11737	11723	11710	11696	11682	11669	11655	11641	1	3	4	5	7	8	10	11	12
8·6	·11628	11614	11601	11587	11574	11561	11547	11534	11521	11507	1	3	4	5	7	8	9	11	12
8·7	·11494	11481	11468	11455	11442	11429	11416	11403	11390	11377	1	3	4	5	7	8	9	10	12
8·8	·11364	11351	11338	11325	11312	11299	11287	11274	11261	11249	1	3	4	5	6	8	9	10	12
8·9	·11236	11223	11211	11198	11186	11173	11161	11148	11136	11123	1	3	4	5	6	8	9	10	11
9·0	·11111	11099	11086	11074	11062	11050	11038	11025	11013	11001	1	2	4	5	6	7	9	10	11
9·1	·10989	10977	10965	10953	10941	10929	10917	10905	10893	10881	1	2	4	5	6	7	8	10	11
9·2	·10870	10858	10846	10834	10823	10811	10799	10787	10776	10764	1	2	4	5	6	7	8	9	11
9·3	·10753	10741	10730	10718	10707	10695	10684	10672	10661	10650	1	2	4	5	6	7	8	9	10
9·4	·10638	10627	10616	10604	10593	10582	10571	10560	10549	10537	1	2	3	4	6	7	8	9	10
9·5	·10526	10515	10504	10493	10482	10471	10460	10449	10438	10428	1	2	3	4	5	7	8	9	10
9·6	·10417	10406	10395	10384	10373	10363	10352	10341	10331	10320	1	2	3	4	5	6	8	9	10
9·7	·10309	10299	10288	10277	10267	10256	10246	10235	10225	10215	1	2	3	4	5	6	7	8	10
9·8	·10204	10194	10183	10173	10163	10152	10142	10132	10121	10111	1	2	3	4	5	6	7	8	9
9·9	·10101	10091	10081	10070	10060	10050	10040	10030	10020	10010	1	2	3	4	5	6	7	8	9

TABLE XXI

Calcutta Stock Exchange Association.

Schedule of Brokerage.

Rate	Description of Stock or Share	Remarks
1 anna % 2 annas %	3% and 3½% Government Paper Other Government Stock	Minimum Charge Rupee 1
¼%	Municipal and Port Trust Debentures	
½%	Joint Stock Debentures	
2 annas per share	Shares worth up to Rs 5 each	Nominal value of Rs 25 each or less
4 annas per share	Shares worth up to Rs 50 each	Ditto
8 annas per share	Shares worth up to Rs 100 each	Ditto
8 annas per every Rs 100 consideration money or part thereof	Shares worth over Rs 100 each	*Ditto
Rupee 1 per share	Shares worth up to Rs 200 each	Nominal value of Rs 75 and above
8 annas per every Rs 100 consideration money or part thereof	Shares worth above Rs 200 each	Ditto

N. B.* Rates proportionate to the last two are charged in respect of Shares of nominal value between Rs 25 and Rs 75 each.